Iterative waterfilling algorithm with sub-channel selection for the coexistence of multiple cognitive tactical radio networks

Vincent Le Nir, Bart Scheers

***Abstract*—In 2002, the work of Yu et al. has shown that the distributed power control problem in a frequency selective interference channel can be modeled as a non-cooperative game and can be solved efficiently by the iterative waterfilling algorithm (IWFA). The convergence of the algorithm has been initially established for two users and later extended to any number of users in a digital subscriber line (DSL) scenario. In a wireless scenario, multiple Nash equilibrium solutions exist and no theoretical proof of convergence can be obtained. In this paper, we study the convergence behavior of the IWFA in parallel Gaussian quasi-static Rayleigh interference channels for the coexistence of multiple cognitive tactical radio networks. We investigate the addition of expert rules to the networks, more specifically the opportunity to select a subset of contiguous sub-channels. In this case, the networks can allocate power only over a subset of the available sub-channels, thereby limiting the maximum number of sub-channels needed for transmission. Moreover, the networks can only choose a group of contiguous sub-channels. A first advantage is to lower the complexity of the IWFA by allocating power only over a subset of the available sub-channels. A second advantage is to lower the complexity of the physical layer in the case of a multi-carrier waveform with non-overlapping sub-channels. A third advantage is to give the networks more facility to avoid each other for high target rates and to improve the convergence of the IWFA in wireless channels. Without loss of generality, the sub-channel selection can also be applied to various variants of the IWFA.**

***Index Terms*—Iterative waterfilling algorithm, interference channels, sub-channel selection**

I. INTRODUCTION

The distributed power control problem in a frequency selective interference channel has been introduced by Yu et al. [1]. This problem can be modeled as a non-cooperative game and can be solved efficiently by the iterative waterfilling algorithm (IWFA). The existence and uniqueness of the Nash equilibrium has been established for two users in a digital subscriber lines (DSL) scenario which exhibits diagonal dominant channel conditions. In [2], the distributed power control problem has been reformulated into an equivalent linear complementary problem (LCP), proving the linear convergence of the IWFA in a DSL scenario for arbitrary symmetric interference environment as well as for diagonally dominant asymmetric channel conditions with any number of users. However, in a wireless scenario in which the channel gains of the interferers could be as high as the channel gains of the direct link, multiple Nash equilibrium solutions exist and no theoretical proof of convergence can be obtained. Moreover, the initial assumption of quasi-static fading channels might be no longer valid in a wireless scenario, therefore robust versions of the IWFA have been introduced in [3], [4] by considering imperfect channel and noise variances information. Simulation results were made under diagonal dominant channel conditions to guarantee the existence and uniqueness of the Nash equilibrium.

In this paper, we study the convergence behavior of the IWFA in parallel Gaussian quasi-static Rayleigh interference channels for the coexistence of multiple cognitive tactical radio networks. For instance, parallel channels represent multiple orthogonal sub-carriers as used in orthogonal frequency division multiplexing (OFDM), or multiple non-overlapping sub-channels. We assume that the links between the transmitters and the receivers exhibit quasi-static fading, i.e. in which the coherence times of the fading channels are larger than the time necessary to compute the algorithm. Such an assumption is motivated by the fact that tactical radio networks using VHF and low UHF bands exhibit long coherence times for low mobility patterns. It is observed that the IWFA shows a good convergence behavior for low target rates but sometimes fails to converge for high target rates. This difficulty is inherent to IWFA because at each iteration some power is poured in the best sub-channels regardless of the interference caused to the other networks, while they have a better benefit avoiding each other by taking different sub-channels. However, an optimal multiple access scheme would require some level of coordination in a centralized approach. This motivates the addition of expert rules to the networks, more specifically the opportunity to select a subset of contiguous sub-channels. In this case, the networks can allocate power only over a subset of the available sub-channels, thereby limiting the maximum number of sub-channels needed for transmission. Moreover, the networks can only choose a group of contiguous sub-channels. A first advantage is to lower the complexity of the IWFA by allocating power only over a subset of the available sub-channels. A second advantage is to lower the complexity of the physical layer in the case of a multi-carrier waveform with non-overlapping sub-channels. A third advantage is to give the networks more facility to avoid each other for high target rates and to improve the convergence of the IWFA in wireless channels. Without loss of generality, the sub-channel selection can also be applied to the robust versions of the IWFA for parallel Gaussian interference channels [3], [4] and to the IWFA for parallel Gaussian broadcast channels with only common information [5], [6], [7].

This paper is organized as follows. First, the system model is presented in Section II. The IWFA is presented along with the ability to select a subset of contiguous sub-channels at each iteration of the inner loop. Extensive simulation results are provided in Section III for the classic IWFA and the IWFA with sub-channel selection. Finally, Section IV concludes the paper.

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II. SYSTEM MODEL

The coexistence of multiple cognitive tactical radio networks is shown on Figure 1. Each network is composed of one transmitter and one receiver. The transmission range

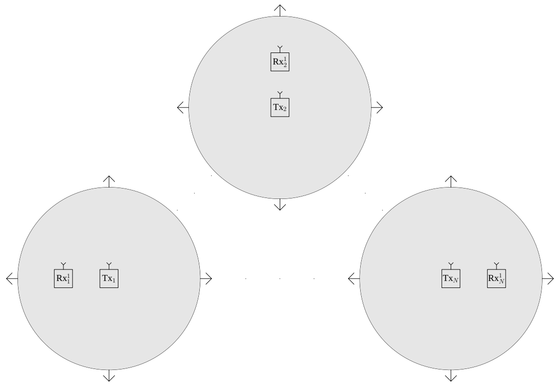

Fig. 1. Scenario considered for the coexistence between tactical radio networks

is represented by the gray area around the transmitter. The different networks can interfere with each other, causing transmission losses if dynamic spectrum management techniques are not implemented. Our goal is to alleviate this problem by equipping each terminal with an algorithm which gives the possibility to optimize its transmission power for each sub-channel. We assume that the links between the transmitters and the receivers exhibit quasi-static fading, i.e. in which the coherence times of the fading channels are larger than the time necessary to compute the algorithm. Such an assumption is motivated by the fact that tactical radio networks using VHF and low UHF bands exhibit long coherence times for low mobility patterns. The received signals $y_{j,i}$ can be modeled as

$$y_{j,i} = h_{i,jj}x_{ij} + \sum_{k \neq j}^{N} h_{i,jk}x_{ik} + n_{ij} \quad \begin{array}{l} i = 1 \ldots N_c, \\ j = 1 \ldots N, \end{array} \tag{1}$$

where N_c is the number of sub-channels, N the number of networks, n_{ij} the complex noise with variance σ_{ij}^2 for network j on sub-channel i, x_{ij} the transmitted signal for network j on sub-channel i, and $h_{i,jk}$ the complex channel coefficient from network k to j on sub-channel i.

A. Classical IWFA

We consider the maximization of the sum rate subject to a total power constraint per network

$$\begin{array}{l} \max\limits_{\underline{\underline{\phi}}} \sum\limits_{j=1}^{N} R_j(\underline{\phi}_j) \\ \text{subject to} \sum\limits_{i=1}^{N_c} \phi_{ij} = P_j^{tot} \ \forall j \end{array} \tag{2}$$

with

$$R_j(\underline{\phi}_j) = \Delta f \sum_{i=1}^{N_c} \log_2(1 + \frac{|h_{i,jj}|^2 \phi_{ij}}{\Gamma(\sigma_{ij}^2 + \sum\limits_{k \neq j} |h_{i,jk}|^2 \phi_{ik})}) \tag{3}$$

and $\underline{\underline{\phi}}$ the power allocation among all sub-channels and networks, $\underline{\phi}_j$ the power allocation among all sub-channels for network j , $\phi_{ij} = E[|x_i|^2]$ the variance of the input signal on sub-channel i for network j, P_j^{tot} the total power constraint for network j, Δf the sub-channel bandwidth, and Γ the SNR gap which measures the loss with respect to theoretically optimum performance [8]. Maximization of the sum rate subject to a total power constraint per network in a centralized algorithm is an extensive task, since it requires the knowledge of the sub-channel gains from any transmitter to any receiver $|h_{i,jk}|^2 \ \forall i, j, k$. Although sub-optimal, a distributed algorithm only requires the knowledge of the sub-channel gains from a transmitter to its own receiver ($|h_{i,jj}|^2$, $\forall i, j$), as well as noise variances of its receiver estimated by spectrum sensing ($\tilde{\sigma}_{ij}^2 = \sigma_{ij}^2 + \sum\limits_{k \neq j} |h_{i,jk}|^2 \phi_{ik}$). The distributed algorithm called IWFA iteratively updates the power allocation of each network while considering all other network's crosstalk as noise [1]. This process is updated regularly between all the different networks until they reach a Nash equilibrium. Finally, an outer loop minimizes the power while maintaining a target rate for all networks. Note that some more robust IWFA can also be applied in case of imperfect channel and noise variance information [3], [4]. In this case the SNR gap is increased to assure reliable communication under operating conditions all the time. Considering the classical IWFA, the Lagrangian function can be written as

$$\begin{aligned} L(\underline{\lambda}, \underline{\underline{\phi}}) = \sum_{i=1}^{N_c} \left(\Delta f \sum_{j=1}^{N} \log_2(1 + \frac{|h_{i,jj}|^2 \phi_{ij}}{\Gamma \tilde{\sigma}_{ij}^2}) - \sum_{j=1}^{N} \lambda_j \phi_{ij} \right) \\ + \sum_{j=1}^{N} \lambda_j P_j^{tot} \end{aligned} \tag{4}$$

in which $\underline{\lambda}$ are the Lagrange multipliers for all networks. According to [9], the Karush-Kuhn-Tucker (KKT) conditions of the optimization problem can be solved by taking the derivative of the Lagrangian function with respect to ϕ_{ij}

$$\frac{\partial L(\underline{\lambda}, \underline{\underline{\phi}})}{\partial \phi_{ij}} = \frac{\Delta f}{\ln 2} \; \frac{1}{\frac{\Gamma \tilde{\sigma}_{ij}^2}{|h_{i,jj}|^2} + \phi_{ij}} - \lambda_j \ . \tag{5}$$

Nulling the derivative gives

$$\frac{\partial L(\lambda, \underline{\underline{\phi}})}{\partial \phi_{ij}} = 0 \Rightarrow \frac{1}{\frac{\Gamma \tilde{\sigma}_{ij}^2}{|h_{i,jj}|^2} + \phi_{ij}} = \underbrace{\lambda_j \frac{\ln 2}{\Delta f}}_{\tilde{\lambda}_j} \ . \tag{6}$$

The optimal power allocation corresponds to Gallager's water-filling strategy for parallel Gaussian channels [10]:

$$\phi_{ij}^{opt} = \left[\frac{1}{\tilde{\lambda}_j} - \frac{\Gamma \tilde{\sigma}_{ij}^2}{|h_{i,jj}|^2} \right]^+ \tag{7}$$

B. Classical IWFA with distributed power control

The classical IWFA maximizes the sum rate subject to a total power constraint per network. In practice we want to minimize the power subject to a target rate per network. This can be achieved by distributed power control using the same power allocation as the classical IWFA [1]. Figure 2 shows the distributed power control for multiple networks. An inner loop determines iteratively for each network the power allocation maximizing the rate and satisfying its total power constraint. Then, an outer loop minimizes the total powers of the different networks individually such that a target rate R^{target} is achieved. Algorithm 1 provides the power allocation for power minimization subject to a target rate constraint. The inner loop and the outer loop correspond to lines 2-10 and 1-16 respectively.

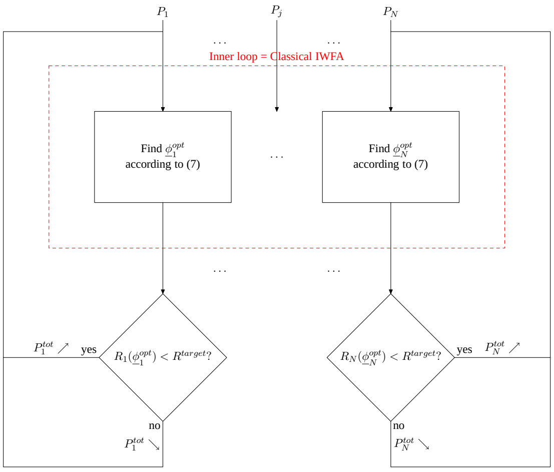

Fig. 2. Distributed power control with IWFA

Algorithm 1 Classical IWFA with distributed power control

1 repeat
2   repeat
3   repeat
4   for $j = 1$ to N
5   Calculate ϕ_{ij}^{opt} $\forall i$ according to (7)
6   if $\sum_{i=1}^{N_c} \phi_{ij}^{opt} < P_j^{tot}$ decrease $\tilde{\lambda}_j$
7   if $\sum_{i=1}^{N_c} \phi_{ij}^{opt} > P_j^{tot}$ increase $\tilde{\lambda}_j$
8   end for
9     $\times$ times
10   until the desired accuracy is reached
11   for $j = 1$ to N
12   Calculate $R_j(\underline{\phi}_j^{opt}) = \Delta f \sum_{i=1}^{N_c} \log_2(1 + \frac{|h_{i,jj}|^2 \phi_{ij}^{opt}}{\Gamma \tilde{\sigma}_{ij}^2})$
13   if $R_j(\underline{\phi}_j^{opt}) < R^{target}$ increase P_j^{tot}
14   if $R_j(\underline{\phi}_j^{opt}) > R^{target}$ decrease P_j^{tot}
15   end for
16 until the desired accuracy is reached

C. IWFA with sub-channel selection

In this Section, the addition of expert rules to the networks is investigated, more specifically the opportunity to select a subset of contiguous sub-channels. In this case, the networks can allocate power only over a subset of the available sub-channels, thereby limiting the maximum number of sub-channels needed for transmission. Moreover, the networks can only choose a group of contiguous sub-channels. A first advantage is to lower the complexity of the IWFA by allocating power only over a subset of the available sub-channels. A second advantage is to lower the complexity of the physical layer in the case of a multi-carrier waveform with non-overlapping sub-channels. A third advantage is to give the networks more facility to avoid each other for high target rates and to improve the convergence of the IWFA in wireless channels.

The sub-channel selection is described as follows. At each iteration of the inner loop in the IWFA, a network can only use L contiguous sub-channels, with $L \in \{1, N_c\}$. In fact, the network j chooses the subset of contiguous sub-channels exhibiting the maximum rate

$$l_j^{opt} = \underset{l_j}{max} \ \Delta f \sum_{i=l_j}^{l_j+L-1} \log_2(1 + \frac{|h_{i,jj}|^2 P_j^{tot}}{L\Gamma \tilde{\sigma}_{ij}^2}) \tag{8}$$

The optimal subset of sub-channels to be used for network j is therefore determined by

$$\mathcal{A}_j = \{l_j^{opt}, l_j^{opt} + L - 1\} \tag{9}$$

Figure 3 shows the distributed power control for multiple networks. An inner loop determines iteratively for each network the best subset of contiguous sub-channels and the power allocation maximizing the rate and satisfying its total power constraint. Then, an outer loop minimizes the total powers of the different networks individually such that a target rate R^{target} is achieved. Algorithm 2 provides the proposed power allocation for power minimization subject to a target rate constraint with sub-channel selection. The inner loop and the outer loop correspond to lines 3-12 and 2-18 respectively. Note that if $L = N_c$, Algorithm 2 reduces to Algorithm 1 and IWFA with sub-channel selection becomes the classical IWFA with distributed power control.

The IWFA with/without sub-channel selection are non-optimal solutions of the centralized problem. Similarly to the convergence of the IWFA in non-diagonally dominant channel conditions, the convergence of the IWFA with sub-channel selection cannot be proven theoretically. Therefore, the convergence of the IWFA in wireless channels with/without sub-channel selection will be studied through simulations using Monte Carlo trials of multiple channel realizations and locations of the radio nodes and the networks, as well as an implementation in OMNeT++/MiXiM.

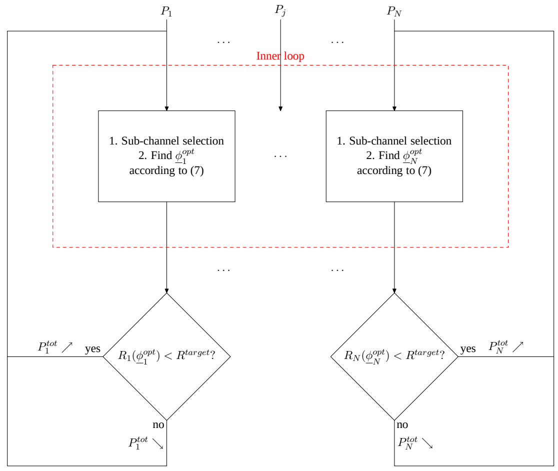

Fig. 3. Distributed power control with IWFA and sub-channel selection

Algorithm 2 IWFA with sub-channel selection

initialize $L \in \{1, N_c\}$
repeat
  repeat
    repeat
      for $j = 1$ to N
        $l_j^{opt} = \underset{l_j}{max} \;\; \Delta f \sum_{i=l_j}^{l_j+L-1} \log_2(1 + \frac{|h_{i,jj}|^2 P_j^{tot}}{L\Gamma\tilde{\sigma}_{ij}^2})$
        Calculate ϕ_{ij}^{opt}, $i \in \mathcal{A}_j$ according to (7)
        if $\sum_{i=1}^{N_c} \phi_{ij}^{opt} < P_j^{tot}$ decrease $\tilde{\lambda}_j$
        if $\sum_{i=1}^{N_c} \phi_{ij}^{opt} > P_j^{tot}$ increase $\tilde{\lambda}_j$
      end for
    $\times$ times
  until the desired accuracy is reached
  for $j = 1$ to N
    Calculate $R_j(\underline{\phi}_j^{opt}) = \Delta f \sum_{i=l_j^{opt}}^{l_j^{opt}+L-1} \log_2(1 + \frac{|h_{i,jj}|^2 \phi_{ij}^{opt}}{\Gamma\tilde{\sigma}_{ij}^2})$
    if $R_j(\underline{\phi}_j^{opt}) < R^{target}$ increase P_j^{tot}
    if $R_j(\underline{\phi}_j^{opt}) > R^{target}$ decrease P_j^{tot}
  end for
until the desired accuracy is reached

III. Simulation Results

For the simulations, the log-distance path loss model is used to measure the path loss between the transmitter and the receivers [11]:

$$PL(dB) = PL(d_0) + 10n\log_{10}(\frac{d}{d_0}) \tag{10}$$

with n the path loss exponent, d is the distance between the transmitter and the receiver, and d_0 the close-in reference distance in kilometers. The reference path loss is calculated using the free space path loss formula

$$PL(d_0) = -32.44 - 20\log_{10}(f_c) - 20\log_{10}(d_0) \tag{11}$$

where f_c is the carrier frequency in MHz. The transmitter and the receivers are placed randomly in a circle area of 1 km^2. The carrier frequency is chosen to be in the very high frequency (VHF) band ($f_c = 80$ MHz). The SNR gap for an uncoded quadrature amplitude modulation (QAM) to operate at a symbol error rate 10^{-7} is $\Gamma = 9.8$ dB. The sub-channel bandwidth is $\Delta f = 25$ kHz, the path loss exponent is $n = 4$, reference distance $d_0 = 0.02$ kilometers and thermal noise with the following expression

$$\sigma_{ij}^2 = -204\text{dBW/Hz} + 10\log_{10}(\Delta f) \qquad \forall i, j. \tag{12}$$

In the first set of simulations, we compare Algorithm 1 with Algorithm 2 for the minimization of the power subject to a target rate constraint, and with $N = 2$ tactical radio networks and $N_c = 2$ sub-channels. Sub-channel attenuations are added to the log-distance path loss model (10) to take into account the multipath propagation. The complex channel coefficients of the sub-channels follow a quasi-static Rayleigh fading model calculated from random complex numbers whose real and imaginary components are independently and identically distributed (i.i.d.) Gaussian. The accuracy for the target rate is defined as

$$\frac{|R_j(\underline{\phi}_j^{opt}) - R^{target}|}{\Delta f} < 10^{-10} \qquad \forall j. \tag{13}$$

Whenever a realization doesn't achieve the desired accuracy, the result of the realization is deleted and considered as an error (no convergence achieved).

Figure 4 shows the results of the power minimization subject to a target rate constraint ranging from $R^{target} = 2$ kbps to $R^{target} = 256$ kbps over 10^4 Monte Carlo trials (left part of the figure) and the corresponding errors in percentage (right part of the figure). The IWFA with sub-channel selection of a single sub-channel (WF1) leads to about 8.9 dB improvement in average compared to the classical IWFA (WF2) at $R^{target} = 64$ kbps. This difficulty is inherent to IWFA because at each iteration some power is poured in the best channels regardless of the interference caused to the other networks, while they have a better benefit avoiding each other by taking different channels. The classical IWFA shows a good convergence behavior for low target rates but sometimes fails to converge for high target rates (3.12% errors at $R^{target} = 256$ kbps). On the contrary, the IWFA with sub-channel selection of a single sub-channel shows a good convergence behavior for high target rates but sometimes fails to converge for low target rates (6.41% errors at $R^{target} = 8$ kbps). This difficulty is inherent to the sub-channel selection because at each iteration a new sub-channel can be chosen regardless of the choices made by the other networks, while they have a better benefit keeping the same sub-channel. This effect can be reduced by freezing the sub-channel selection before reducing the power in the outer loop.

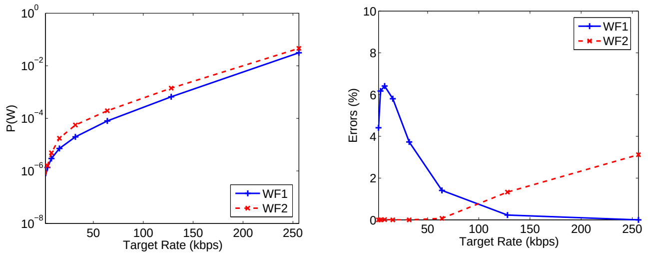

Fig. 4. Results with $N = 2$ networks and $N_c = 2$ sub-channels

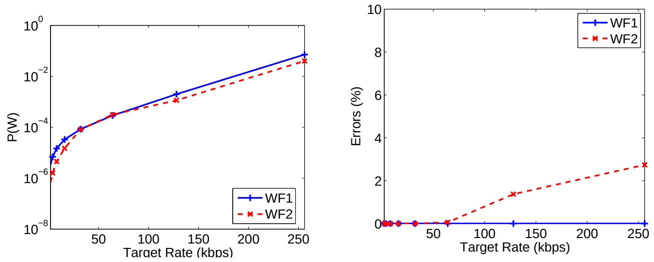

Fig. 5. Results with $N = 2$ networks and $N_c = 2$ sub-channels

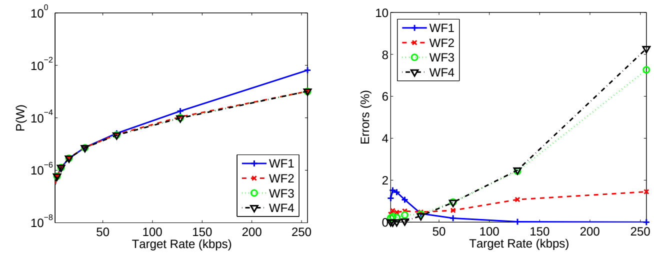

Fig. 6. Results with $N = 2$ networks and $N_c = 4$ sub-channels

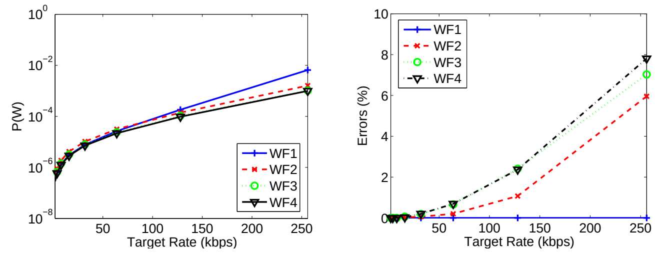

Fig. 7. Results with $N = 2$ networks and $N_c = 4$ sub-channels

Figure 5 shows the results of the same setup ($N = 2$ and $N_c = 2$) with some additional expert rules to reduce the previous observed effects. For the classical IWFA, the sub-channels which are active but not participating significantly in the data rate are switched off. These sub-channels rather cause interference to the other networks, therefore the sub-channel i of network j is canceled whenever the condition

$$0 < \log_2(1 + \frac{|h_{i,jj}|^2 \phi_{ij}^{opt}}{\Gamma \tilde{\sigma}_{ij}^2}) < \frac{R^{target}}{\Delta f} \times 10^{-6} \qquad (14)$$

is met. For the IWFA with sub-channel selection, the choice of the sub-channel satisfying the target rate is kept during the simulation before reducing the power in the outer loop. As seen on the figure, there are no more errors for the IWFA with sub-channel selection at a price of an increased power, and the classical IWFA is able to reduce slightly the number of errors with similar power results as Figure 4.

In the second set of simulations, we compare Algorithm 1 with Algorithm 2 with $N = 2$ tactical radio networks and $N_c = 4$ sub-channels. Figure 6 shows the results of the power minimization subject to a target rate constraint ranging from $R^{target} = 2$ kbps to $R^{target} = 256$ kbps over 10^4 Monte Carlo trials (left part of the figure) and the corresponding errors in percentage (right part of the figure). The classical IWFA (WF4) and the IWFA with sub-channel selection of two (WF2) and three (WF3) sub-channels show similar performance. The IWFA with sub-channel selection of a single sub-channel (WF1) has worse performance than the other IWFA for high target rates but has similar performance as the other IWFA for low target rates. The IWFA with sub-channel selection of a single sub-channel sometimes fails to converge for low target rates (1.54% at $R^{target} = 4$ kbps) and shows a good convergence behavior for high target rates. The classical IWFA shows a good convergence behavior for low target rates but sometimes fails to converge for high target rates (8.57% at $R^{target} = 256$ kbps). The IWFA with sub-channel selection of two and three sub-channels sometimes fails to converge for low target rates (0.38% and 0.22% respectively at $R^{target} = 2$ kbps) and for high target rates (1.45% and 7.26% respectively at $R^{target} = 256$ kbps).

Figure 7 shows the results of the same setup ($N = 2$ and $N_c = 4$) with the same additional expert rules presented for Figure 5. We can see that freezing the sub-channel selection reduces the number of errors to zero when one sub-channel has to be selected. The IWFA with sub-channel selection of two or three sub-channels take the advantages and disadvantages from the two expert rules introduced. Therefore, simulation results show that sub-channel selection does not affect drastically the performance of IWFA and in some cases can lead to better performance and a better convergence behavior in wireless channels. This is especially true for IWFA with sub-channel selection of a single sub-channel showing no errors of convergence, which could be seen as an enhanced version of a simple "detect and avoid" strategy.

In the third set of simulations, we compare Algorithm 1 with Algorithm 2 with $N = 2$ tactical radio networks $N_c = 2$ sub-channels by an implementation in the event-driven simulator OMNeT++/MiXiM. OMNeT++ is an extensible, modular, component-based C++ simulation library and framework, primarily for building network simulators [12]. MiXiM is an OMNeT++ modeling framework created for mobile and fixed wireless networks (wireless sensor networks, body area networks, ad-hoc networks, vehicular networks, etc.) [13]. In this simulation, we have extended the OMNeT++/MiXiM implementation of the classical IWFA [14] to the IWFA with sub-channel selection.

Figure 8 shows the scenario used for the simulation. The first network is mobile and follows a pre-defined trajectory with a constant velocity v (about 90 km/h). In this network, node 1 broadcasts a common information to node 0 at 64 kbps. The second network remains at the same location during the simulation. In this network, node 3 broadcasts a common information to node 2 at 64 kbps. The most critical configuration is obviously reached when the two networks are close to each other, and the interference is maximal. The time interval between two inner loops is set to 0.1s in each network, while the time interval between two outer loops is set to 0.5s with power updates of $10\log_{10}(0.9) \sim 0.46$ dB. The complex channel coefficients of the sub-channels follow a quasi-static Rayleigh fading model calculated from random complex numbers whose real and imaginary components are independently and identically distributed (i.i.d.) Gaussian. We assume that the complex channel coefficients of the direct channels (node 1 $\rightarrow$ node 0 and node 3 $\rightarrow$ node 2) do not change during the simulation as the relative doppler is zero (coherence time $t_c = \infty$). However, the complex channel coefficients of the interference channels (node 1 $\rightarrow$ node 2 and node 3 $\rightarrow$ node 2) have a relative doppler shift $f_d = vf_c/c = 6.67$ Hz, c being the speed of light. The coherence times of the complex channel coefficients of the interference channels are given by $t_c = k/f_d$, k begin a constant value between 0.25 and 0.5. Assuming a quasi-static Rayleigh fading model for the the complex channel coefficients of the interference channelsTherefore, they are updated according to their coherence time every $t_c = 0.05$s.

Figure 9 shows the evolution of the data rate, the total power and the occupation of the sub-channels versus time of both networks for the classical IWFA. At the beginning of the simulation, the total power of both networks is maximal, i.e. 10 W. In this case, the difference between the sub-channel gains is negligeable compared to the power being waterfilled, leading to 50% occupation between the sub-channels. As the power is decreasing and as the networks are getting close to each others, the first network tends to take the first sub-channel and the second network tends to take the second sub-channel.

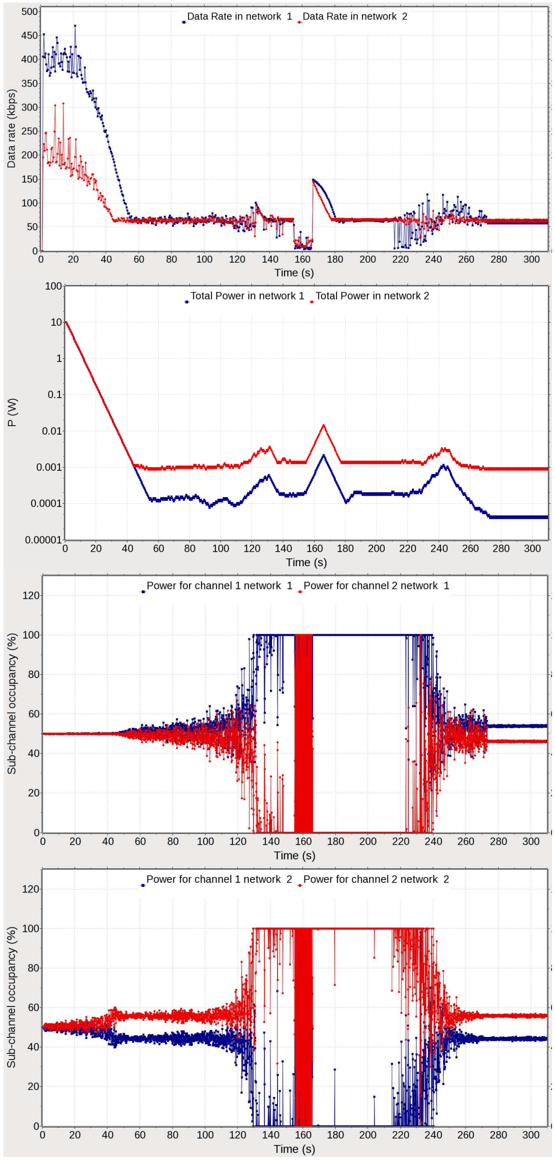

Fig. 9. Data rate, total power and sub-channel occupation for the classical IWFA with $N = 2$ radio networks, $N_c = 2$ sub-channels

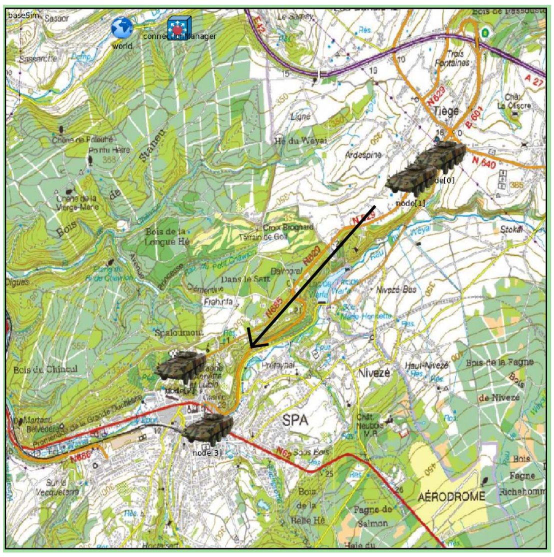

Fig. 8. Scenario used for the simulation

However, this transition is a rather slow process and can cause an increase of the total power and a difficulty to stabilize the data rate for both networks. As the interference is strong enough to make both networks choose different sub-channels, the total power and the data rate converge towards stable values althoug the sub-channel occupation shows some convergence issues. These convergence issues are due to the doppler effect on the sub-channel gains of the interference channels and the existence of multiple Nash equilibria. Indeed, these channels are not quasi-static fading channels since their coherence times are lower than the time interval between two inner loops. When the first network moves away from the second, we also see an increase of the total power and a difficulty to stabilize the data rate for both networks.

Figure 10 shows the evolution of the data rate, the total power and the occupation of the sub-channels versus time of both networks for the IWFA with sub-channel selection of

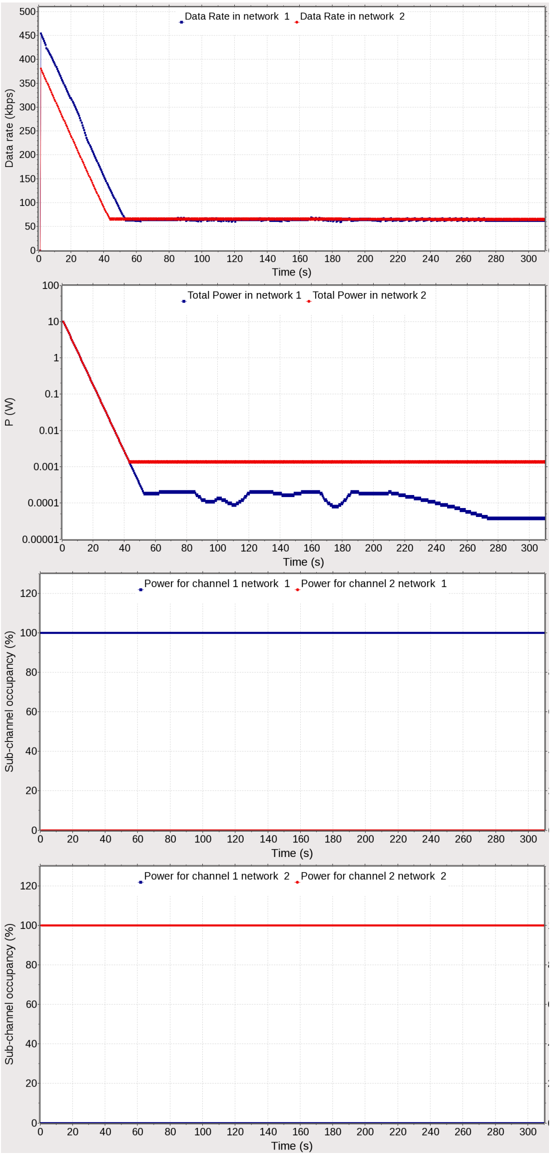

Fig. 10. Data rate, total power and sub-channel occupation for the IWFA with sub-channel selection of one sub-channel with $N = 2$ radio networks and $N_c = 2$ sub-channels

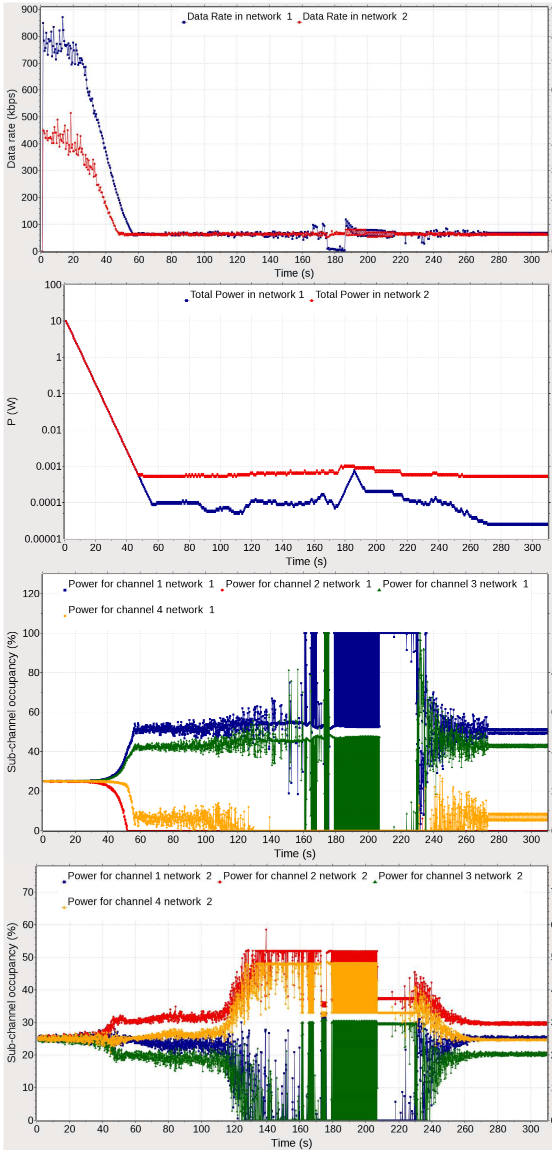

Fig. 11. Data rate, total power and sub-channel occupation for the classical IWFA with $N = 2$ tactical radio networks and $N_c = 4$ sub-channels

a single sub-channel. It is observed that the data rate and the total power are very stable for both networks. The first network takes the first sub-channel while the second network takes the second sub-channel. In this case, the system is converging rapidly towards one Nash equilibrium and there is no more convergence issues due to the doppler effect on the sub-channel gains of the interference channels.

In the fourth set of simulations, we compare Algorithm 1 with Algorithm 2 with $N = 2$ tactical radio networks and $N_c = 4$ sub-channels using the same scenario. Figure 11 shows the evolution of the data rate, the total power and the occupation of the sub-channels versus time of both networks for the classical IWFA. The data rate and the total power shows better convergence with $N_c = 4$ sub-channels compared to $N_c = 2$ sub-channels on Figure 9. Moreover, the averaged power necessary to achieve the required data rate is lower with $N_c = 4$ than $N_c = 2$ sub-channels owing to the degrees of freedom introduced by the sub-channels. However, the management of the sub-channel powers is more complex as seen on the figures showing the sub-channel occupation versus time. Moreover, some convergence issues appear due to the doppler effect on the sub-channel gains of the interference channels and the existence of multiple Nash equilibria.

The convergence issues can be reduced using a more robust IWFA such as [3], [4]. However, these algorithms either trade performance with robustness or assume a specific distribution of the error process. In [15], [16], the authors proposed to heuristically address the impact of such time-varying uncertainty by introducing a memory parameter at each iteration $\alpha \in (0, 1]$ in the calculation of the transmission power levels $\underline{\phi}_j^{opt}(t+1) = (1-\alpha)\underline{\phi}_j^{opt}(t) + \alpha\underline{\phi}_j^{opt}\ \forall j$. However, the choice of the memory parameter α is crucial for the convergence and there is no method to find the optimal value. Recently,

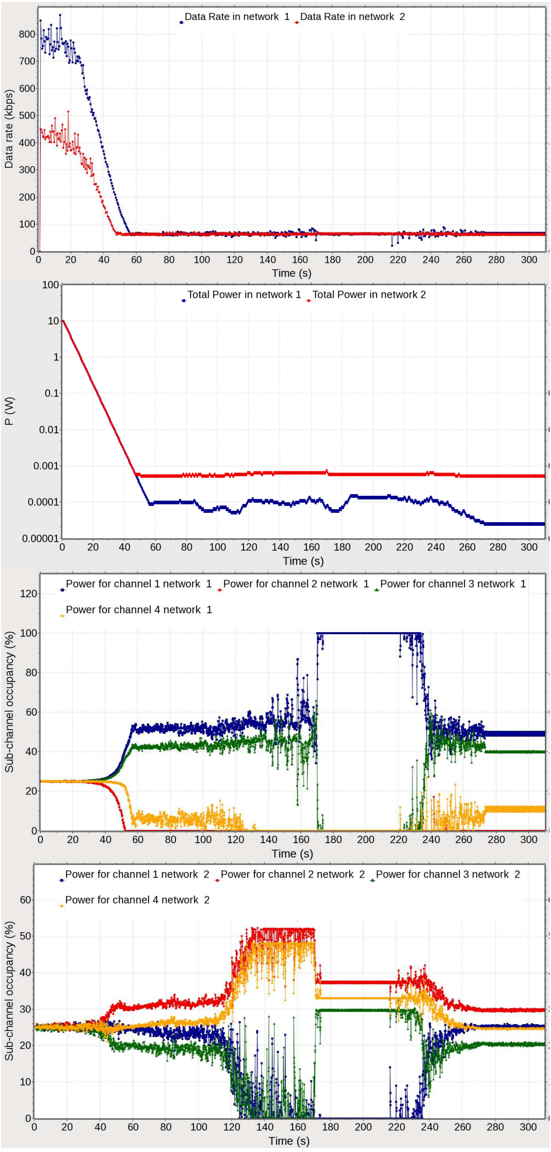

Fig. 12. Data rate, total power and sub-channel occupation for the averaged IWFA with $N = 2$ tactical radio networks and $N_c = 4$ sub-channels

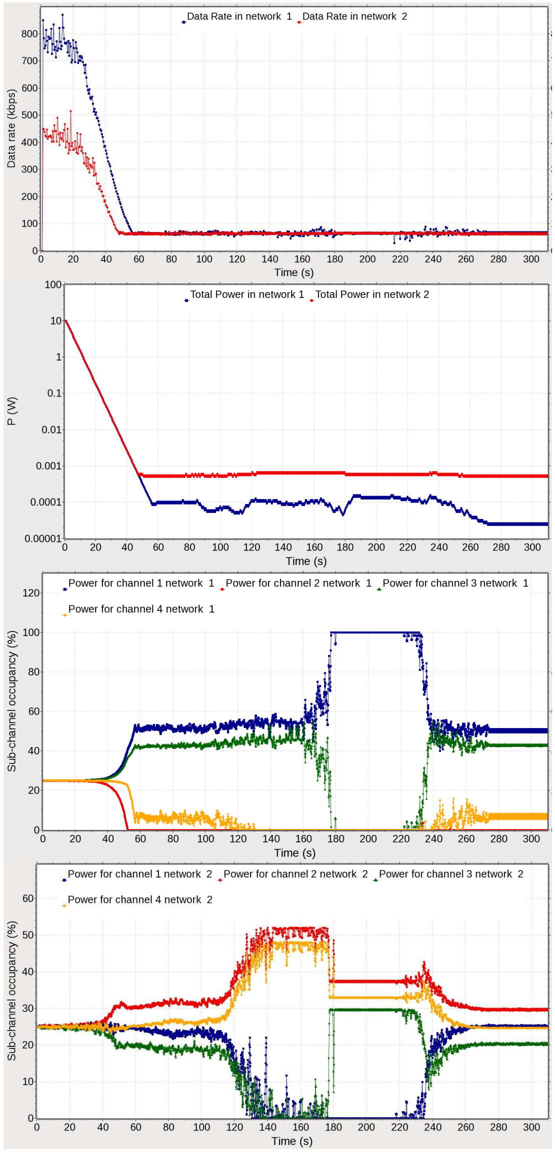

Fig. 13. Data rate, total power and sub-channel occupation for the circular averaged IWFA with $N = 2$ tactical radio networks and $N_c = 4$ sub-channels

[17] proposed the averaged IWFA for improved robustness and convergence, showing that if the memory parameter is chosen as a time sequence $\alpha_t = \dfrac{1}{1+t}$, the transmission power levels are averaged $\underline{\phi}_j^{opt}(T+1) = \frac{1}{T+1}\sum_{t=0}^{T}\underline{\phi}_j^{opt}(t+1)$ $\forall j$. As shown on Figure 12, the averaged IWFA has better convergence properties than the classical IWFA. As the outer loop is executed every 0.5s and the inner loop ever 0.1s, we choose $T = 4$ to avoid averaging across multiple total power constraints. Indeed, the averaged IWFA is better suited for maximizing the data rates subject to a total power constraint (inner loop of the IWFA) than minimizing the powers subject to data rate constraints (outer loop of the IWFA). Therefore, each memory parameter sequence should be restarted at $t = 0$ whenever the total power constraint is modified. To average across multiple total power constraints, we propose to feed a circular buffer of length $T+1$ with the transmission power levels $\underline{\phi}_j^{opt}(T+i+1) = \frac{1}{T+1}\sum_{t=i}^{T+i}\underline{\phi}_j^{opt}(t+1)$ $\forall i, j$. We call this algorithm the circular averaged IWFA. As shown on Figure 13, the circular averaged IWFA with $T = 4$ has also better convergence properties than the classical IWFA and similar convergence properties as the averaged IWFA.

Figure 14 shows the evolution of the data rate, the total power and the occupation of the sub-channels versus time of both networks for the IWFA with sub-channel selection of a single sub-channel. The first network takes the first sub-channel while the second network takes the second sub-channel. The system is also converging rapidly towards one Nash equilibrium and there is no more convergence issues due to the doppler effect on the sub-channel gains of the interference channels and the existence of multiple Nash equilibria. One can observe an increased power for both networks of

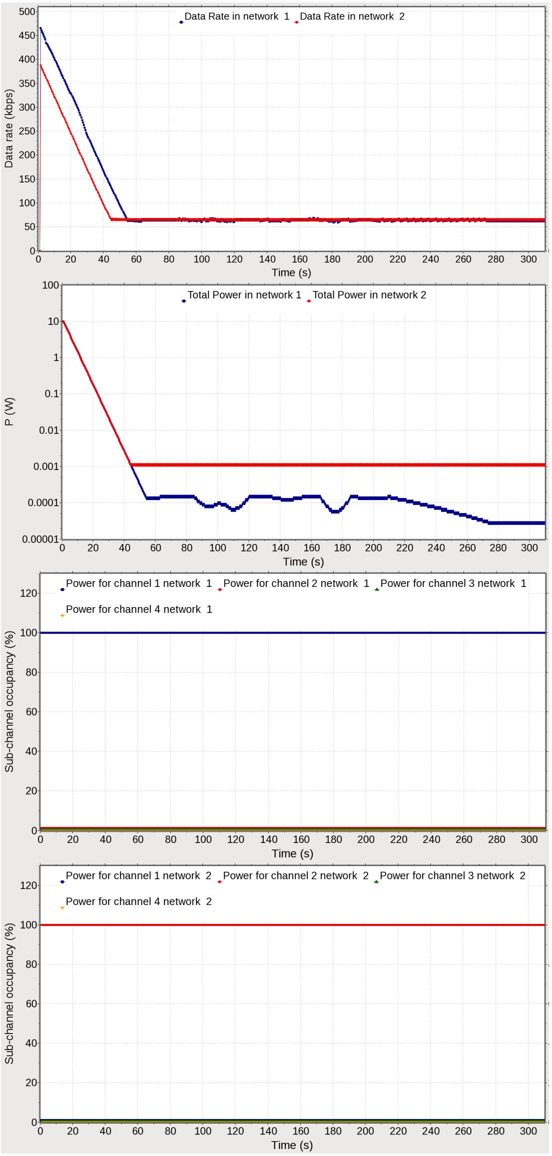

Fig. 14. Data rate, total power and sub-channel occupation for the IWFA with sub-channel selection of one sub-channel with $N = 2$ tactical radio networks and $N_c = 4$ sub-channels

about 1 dB in average compared to Figure 12 and 13 because of the exploitation of a single sub-channel instead of four sub-channels. Therefore, in order to reduce the power when a higher number of sub-channels N_c is available, one might consider to exploit several sub-channels for the IWFA with sub-channel selection.

IV. Conclusion

In this paper, we have studied the convergence behavior of the IWFA in parallel Gaussian quasi-static Rayleigh interference channels for the coexistence of multiple cognitive tactical radio networks. We have investigated the addition of expert rules to the networks, more specifically the opportunity to select a subset of contiguous sub-channels during the IWFA. A first advantage is to lower the complexity of the IWFA by allocating power only over a subset of the available sub-channels. A second advantage is to lower the complexity of the physical layer in the case of a multi-carrier waveform with non-overlapping sub-channels. A third advantage is to give the networks more facility to avoid each other for high target rates and to improve the convergence of the IWFA in wireless channels. In a wireless scenario, multiple Nash equilibrium solutions of the IWFA exist and no theoretical proof of convergence can be obtained. Therefore, the convergence of the IWFA with/without expert rules have been studied through extensive simulation results using Monte Carlo trials of multiple channel realizations and locations of the radio nodes and the networks, as well as an implementation in OMNeT++/MiXiM. Simulation results show that sub-channel selection does not affect drastically the performance of IWFA and in some cases can lead to better performance and a better convergence behavior in wireless channels. This is especially true for IWFA with sub-channel selection of a single sub-channel showing no errors of convergence, which could be seen as an enhanced version of a simple “detect and avoid” strategy.

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