Directionally adaptive image restoration

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Abstract— **A directional selective restoration method is described. The adaptivity is based on the estimation of the local SNR of the ideal image. In order to achieve directional adaptivity, a directional selective transform, based on a discrete short time Fourier transform is presented. The results obtained are compared with a non-directional selective restoration method.**

I. Introduction

IMAGE restoration aims at removing blur in presence of observation noise. The inversion problem being ill-posed, regularisation procedures are commonly used to make the deblurring well-behaved.

Many restoration methods have been proposed [1], [2] but most of these consider space-invariant restoration.

Adaptive restoration methods, that consist in computing an inversion filter depending on the local properties of the image [3], [4] have only recently been proposed. These methods often involve the use of local image descriptions using a development in basis functions with a local support like wavelets, windowed polynomials or Gabor functions. In [3], adaptivity is attained by estimating the local SNR of the original image and by selecting the restoration filter based on its value.

Hence, this adaptive scheme always considers the image a isotropic, what is obviously not the case in the presence of an edge for instance. This results in a well deblurred edge transition, but much noise is introduced along the edge. This could be solved by introducing directional adaptivity, i.e. estimating the SNR locally in various directions, allowing to catch the anisotropies in the image.

In order to carry out this directional selectivity, a directional selective transform has to be devised. A modified version of the STFT was selected and is detailed in section 1. The core of the restoration method, i.e. the computation of the restoration filters is explained in section 2 while the next section shows how these filters can be used to perform an adaptive restoration. Finally section 4 shows some results.

II. Directional selective transform

Since directional selectivity is of great importance, separable filters, that confuse patterns in direction α and $-\alpha$, can not be used.

Good candidates transforms are the STFT or the Gabor transform. These two transforms are widely used in pattern recognition, where they are used to detect and classify patterns according to their response in the $(\overline{x}, \overline{\omega})$ plane.

In contrast with pattern recognition applications, we need to reconstruct the original signal from the coefficients of the transform. Unfortunately, the Gabor transform suffers from reconstruction instabilities when critically sampled. Therefore, overlapping window functions will be used.

Considering discrete signals, let $V(i,j)$ be the localisation window, the signal $L(i,j)$ localised at location $(k\Delta_i, l\Delta_j)$ equals

$$L(i,j)V(i-k\Delta_i, j-l\Delta_j). \quad (1)$$

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Defining a weighting function $W(i,j) = \sum_{k,l} V(i-k\Delta_i, j-l\Delta_j)$, and provided it is non zero $\forall i, j$, the signal can be reconstructed from the localised versions using the following formula

$$L(i,j) = \frac{\sum_{k,l} L(i,j)V(i-k\Delta_i, j-l\Delta_j)}{W(i,j)}, \quad (2)$$

hence avoiding stability problems when reconstructing the original signal.

The Gabor transform can be viewed as the Fourier transform of a localised signal, where the localisation function is a Gaussian. This approach provides an easy mean to compute the inverse transform: it consists in computing the inverse Fourier transform of the localised signal and reconstruct the whole signal as described here above.

If we consider a window function of finite dimension (N,M), a binomial function for instance

$$V^2(i,j) = C_{N-1}^{i+N/2} C_{M-1}^{j+M/2}, \quad (3)$$

the windowed signals are also of finite length, and can be decomposed using the DFT.

The discrete Fourier transform consists in fact in decomposing the original signal in a basis of plane waves $e^{-2j\pi(kr/N+ls/M)}$, where k and l determine the frequency and the orientation of the waves. But since the original signal is real, some simplifications may be conducted, leading to a transform that closely resemble to a Fourier-series decomposition. The basis function of this decomposition are

$$\cos 2\pi\left(\frac{kr}{N} + \frac{ls}{M}\right) \quad \text{and} \quad \sin 2\pi\left(\frac{kr}{N} + \frac{ls}{M}\right). \quad (4)$$

Due to the parity-properties of the sin and cos functions, the functions corresponding the (k,l) and to $(-k,-l)$ are not independent. Moreover, the sin function corresponding to $(k,l) = (0,0)$ is also useless.

For the convenience of the notation, these basis functions will be denoted by $C_{n,m}(r,s)$, where the odd n will corresponds to functions with $k > 0$, while even n will corresponds to function with $k < 0$ and $n = 0$ will correspond to $k = 0$. Something similar holds for m, where m odd corresponds to cos basis functions, and m even corresponds to sin basis functions. Hence, up to a constant multiplicative factor, the functions $C_{n,m}$ define an orthonormal basis on $\mathcal{R}^{N\times M}$.

Defining

$$D_{n,m}(i,j) = V(-i,-j)C_{n,m}(-i,-j) \quad (5)$$

the decomposition of the localised original signal in the basis functions $C_{n,m}$ can be carried out by sampling the original signal filtered using filters $D_{n,m}$

$$L_{n,m}(k\Delta_i, l\Delta_j) = [L(i,j) * D_{n,m}(i,j)]_{(k\Delta_i, l\Delta_j)}. \quad (6)$$

Similarly, defining

$$P_{n,m}(i,j) = \frac{C_{n,m}(i,j)V(i,j)}{W(i,j)} \quad (7)$$

the reconstruction of the original signal can be carried out by interpolating the coefficients $L_{n,m}(k\Delta_i, l\Delta_j)$ using the pattern functions $P_{n,m}$

$$L(i,j) = \sum_{k,l}\sum_{n,m} P_{n,m}(i-k\Delta_i, j-l\Delta_j)L_{n,m}(k\Delta_i, l\Delta_j) \quad (8)$$

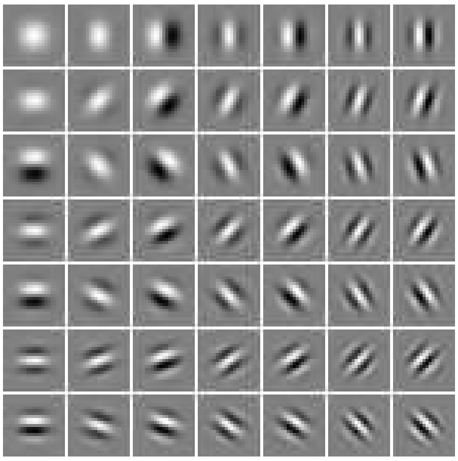

Fig. 1. $D_{n,m}$ filter functions for $n, m = 0, \ldots, 6$

Fig. 2. $|\hat{D}_{n,m}|$ filter functions for $n, m = 0, \ldots, 6$

The analysis functions $D_{n,m}$ are depicted in figure 1 together with their transmittance in fig 2. The directional selectivity of these filter is clearly illustrated in this last figure. It should be noticed that, while showing some similarities with the DCT, the described decomposition exhibits a true directional selectivity, which is not the case of the DCT.

Moreover, due to the effect of the window function $V(i,j)$, the DC component of the even filters (i.e. cos) is not zero.

III. Computation of the restoration filters

The degradation model considered is a linear blur filter with additive white noise. Hence, if L_b denotes the blurred signal and L the ideal signal, we may write

$$L_b(i,j) = L(i,j) * B(i,j) + Q(i,j) \tag{9}$$

where $B(i,j)$ is the blurring filter and $Q(i,j)$ represents the noise.

Assuming that the ideal signal can be described locally with the coefficients of the transform described in section 2, image restoration will be obtained by computing an estimate of the coefficients of the transform of the ideal image, starting from the degraded image. We assume that these estimates can be obtained from the blurred image using filters $H_{n,m}$. Hence, an estimate of the coefficient $L_{n,m}(k\Delta_i, l\Delta_j)$ will be obtained by computing

$$\hat{L}_{n,m}(k\Delta_i, l\Delta_j) \quad = [L_b(i,j) * H_{n,m}(i,j)]_{(k\Delta_i, l\Delta_j)} \tag{10}$$

Those estimates are then used to reconstruct an estimation of the ideal signal using (8).

The coefficients of the filters $H_{n,m}$ are obtained by minimising the mean squared error (MSE) between the unknown coefficients and their estimate

$$\epsilon^2_{n,m} = E\left((\hat{L}_{n,m} - L_{n,m})^2\right). \tag{11}$$

If the noise is assumed to be zero-mean white and uncorrelated with the ideal signal, the only noise characteristic of interest is its variance

$$E(Q(i,j)^2) = s_Q^2. \tag{12}$$

Moreover, the autocorrelation function of the ideal signal in the neighbourhood of $(k\Delta_i, l\Delta_j)$ is given by

$$R_{k,l}(p,q;r,s) = E(L(k\Delta_i, l\Delta_j)L(k\Delta_i, l\Delta_j)) \tag{13}$$

and if the signal is assumed to be stationary in the same neighbourhood, i.e. if the statistical characteristics of the signal are shift-invariant in this neighbourhood, the autocorrelation becomes

$$R_{k,l}(p,q;r,s) = R_{k,l}(p-r, q-s) = R_{k,l}(r-p, s-q) \tag{14}$$

and the MSE can be rewritten as

$$\begin{aligned} \varepsilon^2_{n,m} = \; & s_Q^2 \sum_{i,j} H^2_{n,m}(i,j) \\ & + [D_{n,m}(-p,-q) * D_{n,m}(p,q) * R_{k,l}(p,q)]_{(0,0)} \\ & -2 \sum_{i,j} H_{n,m}(i,j) \\ & \quad [D_{n,m}(p,q) * B(-p,-q) * R_{k,l}(p,q)]_{(i,j)} \\ & + \sum_{i,j} \sum_{i',j'} H_{n,m}(i,j) H_{n,m}(i',j') \\ & \quad [B(-p,-q) * B(p,q) * R_{k,l}(p,q)]_{(i-i',j-j')} \end{aligned} \tag{15}$$

At the minimum of this error, the partial derivative of $\epsilon^2_{n,m}$ with respect to $H_{n,m}(i,j)$ equals zero, which yields

$$\begin{aligned} & \sum_{i',j'} H_{n,m}(i',j') \left(s_Q^2 \delta_{ii'} \delta_{jj'} + \right. \\ & \qquad \left. [B(-p,-q) * B(p,q) * R_{k,l}(p,q)]_{(i-i',j-j')} \right) \\ & \qquad = [D_{n,m}(p,q) * B(-p,-q) * R_{k,l}(p,q)]_{(i,j)} \end{aligned} \tag{16}$$

Since the latter equality must be satisfied for all values of (i,j), a system of linear equations is obtained whose solution gives the coefficients of the desired filters $H_{n,m}$. It should be noticed that the matrix in the left-hand term of eq. (16) is independent of the order (n,m) of the filter, hence only one matrix inversion will be required to compute all filters $H_{n,m}$.

In order to simplify these equations, the signal autocorrelation is assumed to be much smaller than the blurring kernel $B(i,j)$, i.e.

$$B(i,j) * R_{k,l}(p,q) = s^2_{k,l} B(p,q) + m^2_{k,l} \sum_{r,s} B(r,s) \tag{17}$$

where $s^2_{k,l}$ is the local signal variance and $m_{k,l}$ its local mean. Even though this hypothesis is not realistic for the whole image, it remains a good approximation where the deblurring will have the largest effect, i.e. where there are large intensity changes and hence a small correlation length.

If a normalisation condition is enforced

$$\sum_{i',j'} H_{n,m}(i',j') = \frac{\sum_{p,q} D_{n,m}(p,q)}{\sum_{p,q} B(p,q)} \quad (18)$$

equation (16) finally becomes

$$\sum_{i',j'} H_{n,m}(i',j') \left(\frac{s_Q^2}{s_{k,l}^2} \delta_{ii'}\delta_{jj'} + [B(p,q) * B(-p,-q)]_{(i-i',j-j')} \right) = [D_{n,m}(p,q) * B(-p,-q)]_{(i',j')} \quad (19)$$

The size of the system to be solved depends on the support size of the filter kernel $H_{n,m}(i,j)$ which in turn depends on the width of the localisation window and on the support size of the blurring kernel.

Due to the presence of noise, it will not be possible to estimate accurately the coefficients $\hat{L}_{n,m}$ of high order.

IV. Adaptive restoration

The filters $H_{n,m}$ described in previous section depend on the local signal variance $s_{k,l}^2$ of the image. Non adaptive restoration consisted in taking a constant value for the variance used when computing the filters $H_{n,m}$. This constant value resulted from a compromise between good restoration where possible and little noise amplification in constant areas of the image.

By locally adapting the restoration filter $H_{n,m}$ to the local signal variance, optimal restoration can be achieved, taking into account the difference in information content at different places in the image.

In order to evaluate what can be gained from an adaptive restoration, let us first compute the estimation error on the coefficients $\hat{L}_{n,m}$ in function of the local SNR, for a given restoration filter $H_{n,m}$. From (15), taking into account the small correlation length of the signal with respect to the width of the impuls response of B and $D_{n,m}$, introducing the relations (19) together with the normalisation condition (18), we obtain

$$\begin{aligned} \varepsilon_{n,m}^2 = \; & s_Q^2 \sum_{i,j} H_{n,m}^2(i,j) \\ & + s_{k,l}^2 \Bigg([D_{n,m}(-p,-q) * D_{n,m}(p,q)]_{(0,0)} \\ & - \sum_{i,j} H_{n,m}(i,j)\,[D_{n,m}(p,q) * B(-p,-q)]_{(i,j)} \\ & - \frac{s_Q^2}{s^2} \sum_{i,j} H_{n,m}(i,j) \Bigg) \end{aligned} \quad (20)$$

where s^2 is the value of the signal variance that was used when computing the filters $H_{n,m}$. Defining the following amplification factors

$$\begin{aligned} A_Q(n,m) = \; & \sum_{i,j} H_{n,m}^2(i,j) \\ A_F(n,m) = \; & \sum_{i,j} H_{n,m}(i,j)\,[D_{n,m}(p,q) * B(-p,-q)]_{(i,j)} \\ A_S(n,m) = \; & [D_{n,m}(-p,-q) * D_{n,m}(p,q)]_{(0,0)} \\ & - A_F(n,m) - \frac{s_Q^2}{s^2} A_Q(n,m) \end{aligned} \quad (21)$$

eq. (20) finally becomes

$$\frac{\varepsilon_{n,m}^2}{s_Q^2} = A_Q(n,m) + \frac{s_{k,l}^2}{s_Q^2} A_S(n,m). \quad (22)$$

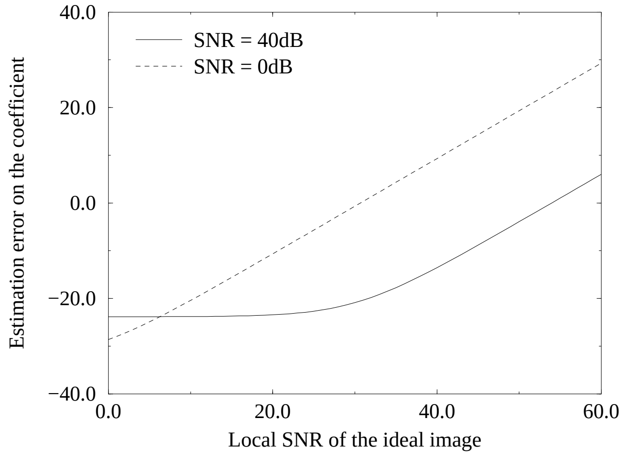

Fig. 3. Coefficients'MSE for $(n,m) = (0,0)$

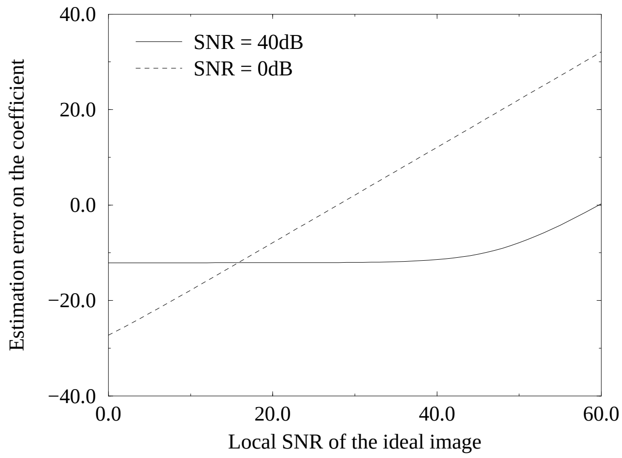

Fig. 4. Coefficients'MSE for $(n,m) = (4,0)$

This equation gives the reduced estimation error on the coefficient $\hat{L}_{n,m}(k\Delta_i, l\Delta_j)$ in function of the local SNR, when using restoration filters computed using a given signal variance s^2.

The evolution of the coefficient estimation error is depicted in figures 3 for the coefficient $\hat{L}_{0,0}$ computed using filters optimised for respectively a low SNR ($\frac{s^2}{s_Q^2} = 0dB$) and a high SNR ($\frac{s^2}{s_Q^2} = 30dB$). The same data are plotted in figure 4 for the coefficient $\hat{L}_{4,0}$. On these figures, we can see that the filter optimised for a high SNR will only give an improvement with respect to the filter optimised for a low SNR if the local SNR ($\frac{s_{k,l}^2}{s_Q^2}$) exceeds a certain threshold. This threshold increases for higher order. Moreover, for coefficients of low order, this threshold will be very small, hence low order coefficients will always be computed using filters optimised for high SNR. Conversely, for high-order coefficients, the filters optimised for low SNR will always be used. For intermediate orders, we will switch between the two filters, based on the local SNR.

An estimate of the local SNR of the ideal signal can be obtained from the estimates of the coefficients of the development.

Indeed, from (10) we may write

$$E\left(\hat{L}_{n,m}^2\right) = s_Q^2 \sum_{i,j} H_{n,m}^2(i,j) + \sum_{i,j}\sum_{i',j'} H_{n,m}(i,j)H_{n,m}(i',j') [B(p,q) * B(-p,-q) * R_{k,l}(p,q)]_{(i-i',j-j')} \quad (23)$$

or, taking the hypothesis of the small autocorrelation length (17) together with the normalisation condition (18) into account and introducing the amplification factors defined in (21), we finally obtain

$$E\left(\hat{L}_{n,m}^2\right) = s_Q^2 A_Q(n,m) + s_{k,l}^2 \left(A_F(n,m) - \frac{s_Q^2}{s^2} A_Q(n,m)\right) + m_{k,l}^2 \left(\sum_{p,q} D_{n,m}(p,q)\right)^2, \quad (24)$$

giving a mean to compute an estimate of the local signal variance of the ideal signal using an estimation of the coefficients of the development of the ideal image. The term in $m_{k,l}^2$ in eq. (24) vanishes if the DC component of the corresponding filter $D_{n,m}$ is zero. For the simplicity, we will assume that the above equation is only used when this is verified, i.e. for the sin based filters. Summing eq. (24) for (n,m) corresponding to filters $D_{n,m}$ with similar directional selectivity α, we obtain an estimate of the signal variance in the corresponding direction

$$s_{k,l}^2 = \frac{\sum_{(n,m)\in\alpha} E\left(\hat{L}_{n,m}^2\right) - s_Q^2 \sum_{(n,m)\in\alpha} A_Q(n,m)}{\sum_{(n,m)\in\alpha} \left(A_F(n,m) - \frac{s_Q^2}{s^2} A_Q(n,m)\right)}. \quad (25)$$

This estimation of the local signal variance could be used to compute the corresponding optimal restoration filters $H_{n,m}$, but this has one drawback without providing a great performance enhancement. The drawback is obviously the computation load, since for each filter-family, the inversion of a (large) matrix is required. The influence of the local SNR in the computation of the restoration filters $H_{n,m}$ lies in the extend to which the high-frequencies of the ideal signal will be recovered. For low SNR, the high-frequencies of the blurred signal will be considered as being covered by noise, while for high SNR, these high frequencies will be recovered by the restoration filters. In other words, filters optimised for a low SNR will always return zero HF coefficients (high order coefficients), while giving the same coefficients as the filters optimised for high SNR for low order coefficients.

Hence the adaptive restoration method is as follows: compute a family of restoration filters optimised for a high SNR (30dB for instance) and compute an estimate of the low-order coefficients in all directions and for all window positions. These estimates are then used to compute an estimate of the local signal variance, and at the window positions where that local signal variance in certain directions exceeds a threshold, compute higher order coefficients corresponding to this direction/location. All estimated coefficients are then used to reconstruct an estimate of the ideal image before degradation.

V. Results

The blurred image (fig. 5) used as test image was taken with an out of focus CCD camera. It should be noticed that doing the blur is quite severe and that the CCD camera used was very noisy. The blurring filter that must be known in order to apply this algorithm was determined starting with a parametrised model of an out of focus camera. The parameters of the model were determined by trial and error.

Fig. 5. Degraded image

The figure 6 presents the same picture restored using the restoration scheme described in [3] where adaptivity with respect to the local image SNR, but that

Fig. 6. Simple adaptive method

doesn't consider directional adaptivity. Although the noise

Fig. 7. Directional adaptive method

in the constant areas of the image is smoothed out, artifacts appears near large intensity transitions.

Figure 7 presents a restoration conducted using the directional adaptive method described in this paper.

VI. Conclusions

A directional adaptive restoration method is presented. This method is based on the estimation of the local SNR of the ideal image. To achieve directional selectivity, a directional selective decomposition scheme based on a variant of the discrete short-time Fourier transform is developed. The major improvement of this method with respect to non directional adaptive restoration methods is particularly well visible along edges, where the noise in the direction of the edge is greatly reduced.

VII. Acknowledgement

This work was supported by an IBM grant from the NFWO (Belgian National Fund for Scientific Research).

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