STRUCTURED COVARIANCE MATRIX ESTIMATION FOR THE RANGE-DEPENDENT PROBLEM IN STAP

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ABSTRACT

We propose a method to compute an estimate of the clutter-plus-noise covariance matrix in bistatic radar configurations. The estimation is based on the computation of the clutter scattering coefficients based on a single data snapshot at each range using a model of the received signal. The covariance matrix of the data is modeled as a structured covariance matrix with the scattering coefficients as unknown parameters. The method is based on the computation of the maximum likelihood. We use the Expectation-Maximization method as estimation benchmark. Since the problem is ill-posed, regularization is mandatory. This regularization is performed by spatial smoothing. The method we propose, unlike the Expectation Maximization, is not iterative and is thus less computationally demanding.

The obtained covariance matrix estimate is used to compute the matched filter in order to perform target detection. The performance of the proposed estimation method is evaluated in terms of signal to interference-plus-noise ratio (SINR) losses and is found to be almost indistinguishable from the performance of the clairvoyant case.

KEY WORDS

Bistatic radar, STAP, maximum likelihood, structured covariance matrix estimation, range dependence, Expectation-Maximization

1 Introduction

Slow-moving target echoes received by airborne ground moving target indicator (GMTI) radars typically compete with very strong clutter return. Space-Time Adaptive Processing (STAP) can be used to discriminate between target and clutter signals [1, 2]. The clutter-plus-noise covariance matrix (CM) at the range of interest is required in order to compute the optimum STAP filter. An estimate of this CM is typically obtained from secondary data, i.e., data at neighboring ranges around the range of interest. To provide an accurate estimate, the secondary data needs to be independent and needs to have the same distribution as the clutter-plus-noise signal at the range of interest.

The two main factors that affect the statistical distribution of the clutter signal are ground cover and radar geometry (position and velocity vector of the transmitter and the receiver, antenna array geometry and element radiation pattern, ground relief). Assuming a uniform ground cover and a flat terrain, clutter echoes at different ranges are typically independent and identically distributed (IID) for monostatic sidelooking configurations with a uniform linear array (ULA) and a few other particular bistatic configurations [3]. This is however not the case for most bistatic configurations [3, 4] and most multistatic configurations [5].

If the clutter echoes at different ranges are IID and Gaussian and if no particular structure for the CM other than hermitian semi-positive definite is assumed, the maximum likelihood (ML) estimate of the clutter CM can be obtained by computing the average of the sample CM at each range. If the clutter echoes at different ranges are not IID, other methods have to be considered to compensate for the range-dependence of the clutter statistics prior to averaging (see for instance [6, 7, 8]). These methods can be interpreted as performing an averaging in the spectral domain. However, they do not exploit the fact that there is a direct relation between the spectral domain and the physical domain and thus have difficulties to cope with spatially inhomogeneous clutter such as in coastal areas or in areas of changing ground cover.

To be able to cope with inhomogeneous clutter and be able to introduce spatial regularity as a constraint on the solution, we want to estimate the parameter of the physical process at the origin of the scattering, i.e., the scattering coefficients of the clutter patches on the ground. We show that the problem can be cast as the estimation of a structured CM which is classically solved by maximizing the likelihood and requires the resolution of a non-linear equation. One of the possible method to maximize the likelihood is the Expectation-Maximization algorithm [9]. However this method is very computationally intensive, we thus propose an approximation of this method and show that the corresponding result are almost as good. Moreover, given the low-rank nature of the problem, the inverse problem is ill-posed and needs to be regularized.

The paper is organized as follows. In Section 2, we introduce the signal model and the resulting CM model. Based on this model, in Section 3, we review the likelihood and derive the expression to be maximized. In Section 4, we briefly describe the Expectation-Maximization algorithm applied to our problem and show that it can be interpreted as a hill-climbing method. The regularization methods are discussed in Section 5. In Section 6, we describe the approximation of the ML solution and how the scattering coefficients can be estimated over the whole operating range of the radar. In this section we also compare the proposed method to other methods found in the literature. Section 7 presents the end-to-end performance of the method in terms of SINR losses. Finally, Section 8 concludes the paper.

2 Signal model

We consider a pulse-Doppler radar with arbitrary transmitter and receiver locations. An arbitrary antenna array is used on the receiver. At each coherent processing interval (CPI), the echo signal from the M pulses received on the N antenna array elements is sampled at the range of interest. The resulting lexically-ordered samples form a data snapshot vector of size NM denoted by $\mathbf{y}$. The signal due to the clutter patches can be modeled as a sum of the contribution over a finite number L of clutter patches [1, 10]

$$\mathbf{y} = \sum_{i=1}^{L} a_i \mathbf{v}_{c_i} + \mathbf{n}, \tag{1}$$

where a_i denotes the (complex) reflectivity of clutter patch i and is assumed zero mean circular complex Gaussian with unknown variance r_{a_i}. $\mathbf{v}_{c_i}$ is the steering vector corresponding to the clutter patch i, and is usually expressed as $\mathbf{v}_{c_i} = \mathbf{c}_i \circ \mathbf{v}_i$ where $\mathbf{c}_i$ is a factor that groups the geometric factors of the radar equation (range attenuation, element radiation pattern, ...), $\mathbf{v}_i$ is the usual normalized steering vector corresponding to clutter patch i, and $\circ$ denotes the Hadamar product (element-wise multiplication). $\mathbf{n}$ denotes the thermal noise, assumed Gaussian with CM $R_n = \sigma_n^2 I$. We assume σ_n known, e.g., based on direct measurements [11].

Equation (1) can be rewritten in matrix form as

$$\mathbf{y} = V\mathbf{a} + \mathbf{n}, \tag{2}$$

where $\mathbf{a} = \{a_1, \ldots, a_L\}^T$ and $V = \{\mathbf{v}_{c_1}, \ldots, \mathbf{v}_{c_L}\}$.

Using this model, the clutter-plus-noise CM $R_y = E\{\mathbf{y}\mathbf{y}^\dagger\}$ is

$$R_y = V R_a V^\dagger + R_n, \tag{3}$$

where $R_a = \mathrm{diag}\{\mathbf{r}_a\}$ with $\mathbf{r}_a = \{r_{a_1}, \ldots, r_{a_L}\}^T$ and $r_{a_i} = E\{a_i a_i^*\}$.

3 The likelihood

We want to find the estimate of R_a which is the most compatible with the measurements $\mathbf{y}$. Hence, we want to maximize the probability $p(R_a|\mathbf{y})$ where $p(\alpha|\beta)$ denotes the conditional probability of α knowing β. Using the Bayes identity we can write

$$p(R_a|\mathbf{y}) = \frac{p(\mathbf{y}|R_a)p(R_a)}{p(\mathbf{y})}, \tag{4}$$

where $p(R_a)$ is some a priori probability for R_a, and $p(\mathbf{y})$ is the probability of the measurement $\mathbf{y}$ and is independent of R_a. Using a flat a priori probability for R_a and noting that $p(\mathbf{y})$ is independent of R_a, the optimum estimate of R_a can be found by maximizing $p(\mathbf{y}|R_a)$ which is classically called the likelihood of R_a. Since $\mathbf{y}$ is assumed Gaussian with CM R_y given by (3), one has

$$p(\mathbf{y}|R_a) \propto \frac{1}{|R_y|} e^{-\mathbf{y}^\dagger R_y^{-1} \mathbf{y}}, \tag{5}$$

where $|R_y|$ denotes the determinant of the matrix R_y. Since $\mathbf{y}^\dagger R_y^{-1}\mathbf{y} = \mathrm{tr}(R_y^{-1}\mathbf{y}\mathbf{y}^\dagger)$, the logarithm of the likelihood $\Lambda(R_a|\mathbf{y})$ is given by

$$\Lambda(R_a|\mathbf{y}) = \ln p(\mathbf{y}|R_a) = -\ln|R_y| - \mathrm{tr}(R_y^{-1}\mathbf{y}\mathbf{y}^\dagger) + C. \tag{6}$$

where C is a known constant, independent of R_a.

A necessary condition at the maximum of $\Lambda(R_a|\mathbf{y})$ is

$$\frac{d\Lambda(R_a|\mathbf{y})}{dr_{a_i}} = 0 \tag{7}$$

for all i. Following [12, 13], we have $\partial \ln|B|/\partial b = \mathrm{tr}(B^{-1}\partial B/\partial b)$ and $\partial B^{-1}/\partial b = -B^{-1}(\partial B/\partial b)B^{-1}$ and hence, with $\{B\}_d$ denoting the column vector consisting of the diagonal elements of B, we have

$$\frac{\partial \ln|R_y|}{\partial \mathbf{r}_a} = \{V^\dagger R_y^{-1} V\}_d \tag{8}$$

and

$$\frac{\partial\, \mathrm{tr}(R_y^{-1}\mathbf{y}\mathbf{y}^\dagger)}{\partial \mathbf{r}_a} = -\{V^\dagger R_y^{-1}\mathbf{y}\mathbf{y}^\dagger R_y^{-1} V\}_d, \tag{9}$$

and the derivative of (6) finally becomes

$$\frac{\partial\Lambda(R_a|\mathbf{y})}{\partial \mathbf{r}_a} = \{V^\dagger R_y^{-1}(\mathbf{y}\mathbf{y}^\dagger - R_y)R_y^{-1}V\}_d. \tag{10}$$

Equation (7) can thus be rewritten for all i in vector form as

$$\{V^\dagger R_y^{-1}(\mathbf{y}\mathbf{y}^\dagger - R_y)R_y^{-1}V\}_d = 0, \tag{11}$$

which is equivalent to the trace equation [14] when all admissible variations of $\mathbf{r}_a$ are considered. This equation must be solved for R_a. It is non linear in R_a and no closed-form solution is known.

Noting, as in [13],

$$F = R_a V^\dagger R_y^{-1}, \tag{12}$$

and decomposing R_y according to (3), (10) can be rewritten as

$$\begin{aligned} \mathbf{r}_a \circ \tfrac{\partial\Lambda(R_a|\mathbf{y})}{\partial \mathbf{r}_a} \circ \mathbf{r}_a &= \{F\mathbf{y}\mathbf{y}^\dagger F^\dagger\}_d \\ &-\{FVR_aV^\dagger F^\dagger\}_d - \{FR_nF^\dagger\}_d. \end{aligned} \tag{13}$$

Similarly as in [13], defining

$$T = \mathrm{diag}\{\{FVR_aV^\dagger F^\dagger\}_d\}R_a^{-1}, \tag{14}$$

from (13), the maximum satisfies

$$\mathbf{r}_a = T^{-1}(\{F\mathbf{y}\mathbf{y}^\dagger F^\dagger\}_d - \{FR_nF^\dagger\}_d). \tag{15}$$

Consequently, the following iterative solution is proposed [13]

$$\begin{aligned} \mathbf{r}_a^{(k+1)} &= \mathbf{r}_a^{(k)} \\ &+ \tau\left(T^{-1}(\{F\mathbf{y}\mathbf{y}^\dagger F^\dagger\}_d - \{FR_nF^\dagger\}_d) - \mathbf{r}_a\right)\Big|_{\mathbf{r}_a^{(k)}}. \end{aligned} \tag{16}$$

Taking (13) into account, this last equation can be rewritten as

$$\mathbf{r}_a^{(k+1)} = \mathbf{r}_a^{(k)} + \tau \left(T^{-1}\mathbf{r}_a \circ \frac{\partial \Lambda(R_a|\mathbf{y})}{\partial \mathbf{r}_a} \circ \mathbf{r}_a \right)\Bigg|_{\mathbf{r}_a^{(k)}} . \tag{17}$$

This shows that the iterative scheme proposed in [13] is equivalent to a modified gradient descent where the gradient is multiplied by a matrix. Moreover, taking $\tau = 1$ in (17) results in

$$\mathbf{r}_a^{(k+1)} = \left(T^{-1}(\{F\mathbf{y}\mathbf{y}^\dagger F^\dagger\}_d - \{FR_nF^\dagger\}_d)\right)|_{\mathbf{r}_a^{(k)}}. \tag{18}$$

The fixed points of this equation satisfy (15) and this might be interpreted as an iterative solution to (15). Notice that we do not claim convergence of (18).

4 Expectation-Maximization

The Expectation-Maximization (EM) algorithm can be used to find the value of R_a that maximizes (6) [9]. In the EM algorithm, the coefficients $\mathbf{a}$ are called the complete data while $\mathbf{y}$ are the incomplete data and (2) provides a mapping between them.

The EM algorithm is an iterative algorithm that consists in a succession of an expectation (E) step and a maximization (M) step. In the E-step, the expectation of the log-likelihood of the complete data $\Lambda_{cd}(R_a|\mathbf{a})$ conditioned on an estimate of R_a and on the incomplete data $\mathbf{y}$ is computed,

$$Q(R_a|R_a^{(k)}) = E\{\Lambda_{cd}(R_a|\mathbf{a})|R_a^{(k)}, \mathbf{y}\}. \tag{19}$$

In the M-step, the value of R_a that maximizes $Q(R_a|R_a^{(k)})$ is computed

$$R_a^{(k+1)} = \underset{R_a}{\operatorname{argmax}}\, Q(R_a|R_a^{(k)}). \tag{20}$$

These two steps are repeated until convergence.

Since $\mathbf{a}$ is complex Gaussian distributed with CM R_a, we have

$$\Lambda_{cd}(R_a|\mathbf{a}) = -\ln|R_a| - \operatorname{tr}(R_a^{-1}\mathbf{a}\mathbf{a}^\dagger) + C \tag{21}$$

where C is a known constant independent of R_a and (19) can be evaluated as

$$Q(R_a|R_a^{(k)}) = -\ln|R_a| - \operatorname{tr}(R_a^{-1}E\{\mathbf{a}\mathbf{a}^\dagger|R_a^{(k)}, \mathbf{y}\}) + C. \tag{22}$$

Since the term $E\{\mathbf{a}\mathbf{a}^\dagger|R_a^{(k)}, \mathbf{y}\}$ does not depend on R_a, taking the derivative of $Q(R_a|R_a^{(k)})$ with respect to $\mathbf{r}_a$ yields

$$\frac{\partial Q(R_a|R_a^{(k)})}{\partial \mathbf{r}_a} = -\{R_a^{-1}\}_d + \{R_a^{-1}R_a^{-1}E\{\mathbf{a}\mathbf{a}^\dagger|R_a^{(k)}, \mathbf{y}\}\}_d \tag{23}$$

and setting this derivative equal to zero to find the maximum of $Q(R_a|R_a^{(k)})$ yields

$$\mathbf{r}_a^{(k+1)} = \{E\{\mathbf{a}\mathbf{a}^\dagger|R_a^{(k)}, \mathbf{y}\}\}_d, \tag{24}$$

which is a set of L decoupled equations. Equation i of this set is

$$r_{a_i}^{(k+1)} = E\{|a_i|^2|R_a^{(k)}, \mathbf{y}\}. \tag{25}$$

The term $E\{\mathbf{a}\mathbf{a}^\dagger|R_a^{(k)}, \mathbf{y}\}$ appearing in (24) can be expressed as a function of the conditional mean $\bar{\mathbf{a}} = E\{\mathbf{a}|R_a^{(k)}, \mathbf{y}\}$ and the conditional covariance of $\mathbf{a}$

$$E\{\mathbf{a}\mathbf{a}^\dagger|R_a^{(k)}, \mathbf{y}\} = \bar{\mathbf{a}}\bar{\mathbf{a}}^\dagger + \operatorname{cov}\{\mathbf{a}|R_a^{(k)}, \mathbf{y}\} \tag{26}$$

where the conditional mean and the conditional covariance are well-known results from estimation theory [15, 16] and, with $R_y = R_y(R_a)|_{R_a^{(k)}}$, are respectively given by

$$\bar{\mathbf{a}} = E\{\mathbf{a}|R_a^{(k)}, \mathbf{y}\} = R_a^{(k)}V^\dagger R_y^{-1}\mathbf{y} \tag{27}$$

and by

$$\operatorname{cov}\{\mathbf{a}|R_a^{(k)}, \mathbf{y}\} = R_a^{(k)} - R_a^{(k)}V^\dagger R_y^{-1}VR_a^{(k)}. \tag{28}$$

Hence, (24) can be rewritten as

$$\begin{aligned} \mathbf{r}_a^{(k+1)} = \;& \{R_a^{(k)}V^\dagger R_y^{-1}\mathbf{y}\mathbf{y}^\dagger R_y^{-1}VR_a^{(k)}\}_d \\ & + \{R_a^{(k)}\}_d - \{R_a^{(k)}V^\dagger R_y^{-1}VR_a^{(k)}\}_d \end{aligned} \tag{29}$$

or, using the expression for $\frac{\partial \Lambda(R_a|\mathbf{y})}{\partial \mathbf{r}_a}$ obtained in the previous section, we finally can rewrite one iteration of the EM algorithm as

$$\mathbf{r}_a^{(k+1)} = \mathbf{r}_a^{(k)} + \mathbf{r}_a^{(k)} \circ \left.\frac{\partial \Lambda(R_a|\mathbf{y})}{\partial \mathbf{r}_a}\right|_{R_a^{(k)}} \circ \mathbf{r}_a^{(k)}. \tag{30}$$

This shows that — at least in the context in which we use it — the EM algorithm can be interpreted as a modified gradient descent method to find the value of R_a that maximizes the log-likelihood $\Lambda(R_a|\mathbf{y})$.

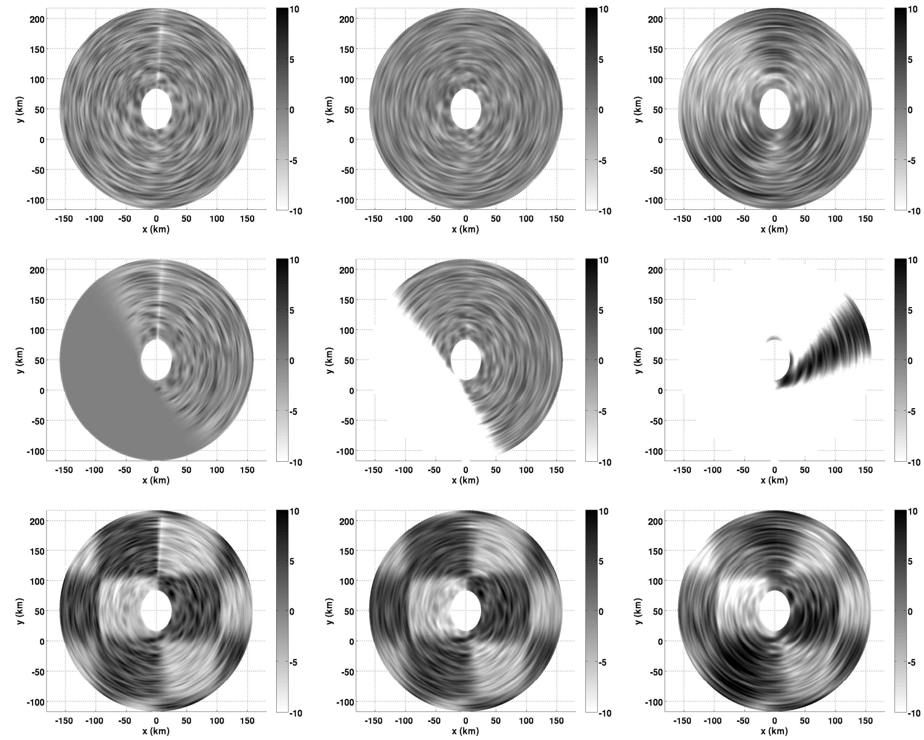
5 Regularization

In the intended application, $VR_aV^\dagger$ is not full rank, which implies that there is no unique solution to the inverse problem. This kind of problem is called "ill conditioned" and regularization is necessary to obtain a solution [17]. Regularization is obtained by introducing a priori knowledge about the solution R_a. This can be done, e.g., by directly imposing an a priori probability density of R_a in the likelihood to be minimized [13, 18] or by restricting the space in which the solution is looked for, e.g., by decomposing the CM R_a in a family of CM [12, 19]. A review of various regularization methods applied to the EM algorithm can be found in [20].

A particular regularization is one that consists in adding a smoothing step after computation of the M-step of the EM algorithm. In [13, 20], it is shown that, in some particular cases, there is an equivalence between the imposition of some prior probability density $p(R_a)$ and the addition of a spatial smoothing step. The Tikhonov regularization introduced in [13] results in the modification of the matrix T in (17), which introduces a spatial smoothing. This motivates the introduction of a spatial smoothing step in (18) as

$$\mathbf{r}_a^{(k+1)} = W\left(T^{-1}(\{F\mathbf{y}\mathbf{y}^\dagger F^\dagger\}_d - \{FR_nF^\dagger\}_d)\right)|_{R_a^{(k)}} \tag{31}$$

and in (30) as



Expectation-Maximization Single iteration Crude matched filter

Figure 1. Comparison of the scattering coefficient maps obtained using different estimation methods (each column correspond to one method) for different scenarios (each line corresponds to a different scenario). The graphs share the same color scale.

$$\mathbf{r}_a^{(k+1)} = W \left(\mathbf{r}_a^{(k)} + \mathbf{r}_a^{(k)} \circ \left. \frac{\partial \Lambda(R_a|\mathbf{y})}{\partial \mathbf{r}_a} \right|_{R_a^{(k)}} \circ \mathbf{r}_a^{(k)} \right), \tag{32}$$

where the square matrix W is a circulant smoothing matrix. The matrix W is circulant because the scattering coefficients are located along a closed isorange on the ground. The effect of W is to perform a weighted averaging of r_{a_i} with the neighboring scattering coefficients.

6 Application to scattering coefficient maps estimation

Equations (2) and (3) can be written down for each range-gate. Notice that for bistatic scenarios, V typically depends on the range. If the measurements $\mathbf{y}$ at different ranges are uncorrelated, i.e., if the range sidelobes are negligible, the estimation of the unregularized R_a is decoupled in range, what simplifies the computations.

However, the physics clearly imposes a spatial constraint on the scattering coefficients r_{a_i} in the sense that a smooth evolution of the scattering coefficients should be favored, both in azimuth (cross-range) and in range. Hence the smoothing step discussed in the previous section will be extended to smooth the estimate of $r_{a_i}^{(k)}$ also across different ranges.

Equations (31) and (32), while able to provide a solution to the problem, result in an extremely computationally intensive algorithm. We thus propose to consider a single iteration of (31) as estimator. The motivation is that, if the initial estimate $R_a^{(0)}$ is close enough to the true value, a single iteration might be sufficient. Moreover, if the clutter-to-noise ratio is high enough, the contribution of the second term in (31) may be neglected. At one range, an estimate of R_a is thus obtained by computing

$$\bar{\mathbf{r}}_a = W \left. \left(T^{-1}\{\bar{\mathbf{a}}\bar{\mathbf{a}}^\dagger\}_d \right) \right|_{R_a^{(0)}}, \tag{33}$$

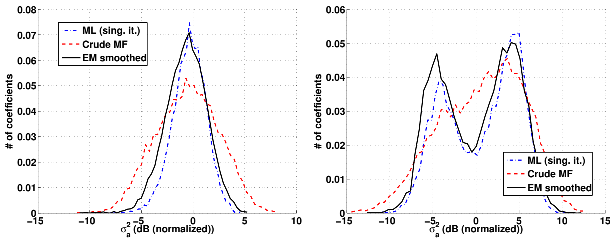
where $\bar{\mathbf{a}}$ is the ML estimate of $\mathbf{a}$ conditioned on $R_a^{(0)}$ and $\mathbf{y}$ is given by (27).

The algorithm we propose thus consists in two steps. The first step estimates $\bar{\mathbf{a}}$ at each range and computing $\bar{\mathbf{r}}_a = T^{-1}\{\bar{\mathbf{a}}\bar{\mathbf{a}}^\dagger\}_d$ at each range independently. The second step performs a local spatial averaging of the estimated scattering coefficients.

As a comparison, we consider the estimator used in [8], which roughly results from taking $F = V^\dagger$. In order to avoid the introduction of a trivial bias on that crude estimator, a normalization factor [13] is used to correct the mean value. We will denote this estimator as the crude matched filter (MF).

We compared the performances of the different estimation methods for different scenarios. In all cases, we considered a bistatic setup, where the transmitter is located at the origin and the receiver is located at $(0, 100)$. The transmitter platform is flying east while the receiver is flying north. Unless otherwise specified, the clutter-to-noise ratio is 20dB. The initial value of the covariance matrix

$R_a^{(0)}$ is taken equal to rI where r is +30dB offset from the true average value. The resulting scattering coefficient maps are illustrated in Fig. 1. Each column corresponds to a different method and each row to a different scenario. The first column is the EM algorithm (32) where the smoothing step is extended in range as discussed above. The middle column is the single-iteration algorithm described above and the third column is the crude MF.



(a) (b)

Figure 2. Scattering coefficient histogram for (a) uniform scattering coefficients and (b) the checkerboard patterned scattering coefficients.

The first scenario (first row) corresponds to uniform scattering coefficients. The fact that the estimate obtained using the crude MF exhibits spatial artifacts indicates that a constant correction factor does not completely compensate the bias. It should also be noted that the EM-based estimator and the single-iteration estimator provide a very similar scattering coefficients map. This is also visible in Fig. 2(a) showing the histogram of the scattering coefficients obtained using the different methods.

In the scenario of the second row, a sinc-shaped transmit antenna without backlobe pattern was considered. The scattering coefficients in the backlobe of the antenna have no influence on the measured data and, thus, cannot be estimated. The values for those coefficients thus results from the regularization. Again, clearly, the crude MF fails to provide a useful estimate. The crude MF is directly affected by the antenna diagram through c_i. Following a similar reasoning as in the previous sections, a similar expression to the crude MF has been used [8] to obtain an estimate of $E\{|a_i c_i|^2\}$, but in that case, the prior knowledge is difficult to express due to the mixture of the geometric effects c_i with knowledge about a_i what makes regularization difficult whenever c_i is not constant.

Finally, in the last scenario, non-uniform scattering coefficients were considered. A difference of 10dB was considered between the high and low scattering coefficients. This is the typical difference that is observed between (monostatic) ground backscattering and monostatic backscattering by the sea-surface in C-band [21], a demanding scenario in STAP. As can be seen, despite the non-uniformity, the pattern is still easily recognizable. Moreover, the histogram of Fig. 2(b) is clearly bimodal with peaks around -5dB and +5dB, i.e. the exact values. While the pattern is still present in the crude MF estimate, the corresponding histogram does not exhibit the bimodality, thus denoting a large estimation error.

7 End-to-end results

Figure 3 presents a comparison of the performance of various CM estimation methods in terms of SINR loss.

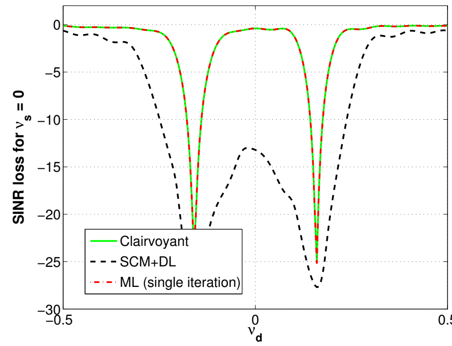


Figure 3. Comparison of the SINR loss for different covariance matrix estimation methods.

The scenario considered is the same as the first scenario of Fig. 1. In all cases, the filter considered has the same expression,

$$\mathbf{w} = R_y^{-1}\mathbf{v} \tag{34}$$

where $\mathbf{v}$ is the target steering vector and the CM R_y is replaced with an estimate or the clairvoyant CM. The classical sample CM [22], complemented with diagonal loading to cope with the limited number of training samples, is denoted by SCM+DL [10]. The estimate obtained using the proposed method, where the clutter reflectivity is estimated using a single iteration of the ML iterative solution, is denoted by “ML (single iteration)”.

As can be seen, the SCM+DL method fails due to the range-dependence of the clutter distribution and due to the very limited number of available training snapshots. The proposed method results in a SINR loss that is almost identical to that of the clairvoyant case.

8 Summary and conclusion

We propose a new method to estimate the clutter covariance matrix (CM) in the presence of range-dependence effects (due both the ground cover variation and to geometric effects). The method relies on the knowledge of the structure of the CM to obtain an approximation of the ML estimate of the CM of the data. An estimate of the true ML solution is be obtained as the results of an iterative hill climbing method, using, e.g., the Expectation-Maximization algorithm. We approach this using a single iteration. Due to the ill-posed nature of the problem, the estimation needs to be regularized. We achieve this by performing a spatial averaging. The estimated clutter scattering coefficients are very close to the estimated ML solution. Moreover, a significant reduction of the computational burden is achieved by using a single iteration.

The method relies on a model and will of course fail if the model is inaccurate. This is the case in the presence of target and the corresponding ranges should be removed before proceeding to the spatial averaging step.

Finally, the proposed method is shown to provide a CM estimate that performs nearly as well as the clairvoyant CM in terms of SINR loss, which indicates that the resulting filter will have nearly-optimal detection performance.

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