Université Libre de Bruxelles
OPERA-Wireless Communication Group
B - 1050 Brussels, Belgium

Royal Military Academy
Communication Information Systems and
Sensors Department
B - 1000 Brussels, Belgium

Cooperative Spectrum Sensing Algorithms For Cognitive Radio Networks

Djamel TEGUIG

Thesis presented in partial fulfillment of the requirements
for the PhD degree in Engineering Sciences and Technology

PhD Committee:

Prof. Bart SCHEERS, *Supervisor* (Royal Military Academy, Belgium)
Prof. François HORLIN, *Supervisor* (Université Libre de Bruxelles, Belgium)
Prof. Jean-Michel DRICOT, (Université Libre de Bruxelles, Belgium)
Prof. Philippe DE DONCKER, *President* (Université Libre de Bruxelles, Belgium)
Prof. Sofie POLLIN, (KU Leuven, Belgium)
Prof. Vincent LE NIR, (Royal Military Academy, Belgium)

October 2015

Acknowledgments

This doctoral thesis was realized under the joint supervision program between the Royal Military Academy (Belgium) and the Université Libre de Bruxelles (Belgium) under the support of the Ecole Militaire Polytechnique (Algeria). Although my name appears as the sole author of the thesis, in reality a number of people are responsible for the work. First and foremost, I would like to thank my thesis advisors, Bart SCHEERS, and François HORLIN for the guidance, understanding, motivation, creative ideas and excellent support during the time of this thesis. Thanks for the confidence you have placed in me.

I would also like to express my gratitude to my committee members: Professor Jean-Michel DRICOT, Professor Philippe DE DONCKER, Professor Sofie POLLIN and Professor Vincent LE NIR, for accepting to assess this modest work. There are too many people who have helped me during my journey that I need to thank. However, there are some people that definitely deserve a place in my thesis, because without them, everything would be more difficult, namely Vincent LE NIR and Muhammad Hafeez Chaudhary. I must thank Vincent LE NIR for the discussions on my work and his efforts in making my papers and my thesis readable.

I cannot forget to thank all researchers of CISS department of RMA for their great hospitality and help that make my life in Brussels very easy.

Finally, and most importantly, I would like to express gratitude to my family. Even thousands of kilometers apart, they have been present through every step of my life, providing support in difficult times. They have been a constant source of inspiration, and this thesis is dedicated to them, especially my parents, my wife, my brothers and my sisters.

Djamel TEGUIG, Brussels, October, 2015

Contents

Notations and Acronyms **xiv**

General Introduction **1**

0.1 Motivation 2

0.2 Spectrum Sensing Challenges 3

0.3 Objectives and contributions 4

0.3.1 Objectives 4

0.3.2 Key Contributions 5

0.4 Thesis Outline 5

1 Introduction to Cognitive Radio **9**

1.1 Wireless Communications 9

1.2 Cognitive Radio 11

1.2.1 Software Defined Radio to Cognitive Radio 11

1.2.2 Definitions of a cognitive radio 12

1.2.3 Functions and components of Cognitive Radio 13

1.2.4 Cognition Cycle 14

1.2.5 Cognitive Radio Networks Architecture 15
1.3 Dynamic Spectrum Access and Management 16
1.3.1 Dynamic Exclusive Use Model 17
1.3.2 Open Sharing Model 18
1.3.3 Hierarchical Access Model 18
1.4 Cognitive Radio Standardization 19
1.5 Cognitive Radio Applications 22
1.6 Conclusion 23

2 Spectrum Sensing for Cognitive Radio **25**
2.1 Introduction 25
2.2 Overview of Spectrum Sensing Algorithms 26
2.3 Statistical Detection Techniques 28
2.3.1 Maximum A Posteriori Detection (MAP) 29
2.3.2 Maximum Likelihood Detection (ML) 29
2.3.3 The Neyman-Pearson Detection 29
2.4 Detection performance 30
2.5 Energy Detection Based Spectrum Sensing 31
2.5.1 Energy Detector 32
2.5.2 Noise Power Uncertainty in Energy Detection 37
2.6 Matched Filter Based Spectrum Sensing 38
2.7 Cyclostationary Based Spectrum Sensing 40
2.7.1 Cyclostationary Analysis 40

2.7.2 Cyclostationary Feature Detection for CR 42

2.7.3 Cyclostationary based spectrum sensing limitations . . . 43

2.8 Eigenvalue based Spectrum Sensing 44

2.8.1 Computation of the sample covariance matrix 45

2.8.2 Implementation of Maximum-Minimum Eigenvalues ratio detector (MME) 45

2.9 Spectrum Sensing Methods between strength and weakness . . 47

2.10 Other Spectrum Sensing Methods 47

2.10.1 Covariance Based Spectrum Sensing 48

2.10.2 Wavelet Based Spectrum Sensing 49

2.10.3 Filter Bank Based Spectrum Sensing 50

2.10.4 Multitaper Method Based Spectrum Sensing (MTM) . . . 51

2.10.5 High-order Statistics Based Spectrum Sensing 51

2.11 Cooperative Spectrum Sensing 52

2.12 Conclusion 54

3 Optimization of Centralized Cooperative Spectrum Sensing for Cognitive Radio Networks 55

3.1 Introduction 55

3.2 Related Works 58

3.3 Issues in Cooperative Spectrum Sensing 59

3.4 System Model 60

3.5 Fusion Rules 62

3.5.1 Hard fusion rules 62

3.5.2 Soft data fusion 64

3.5.3 Quantized data fusion 67

3.6 Cognitive Radio Transmission Scenarios 70

3.6.1 Combining Rules for CSS under CR Transmission Scenarios 71

3.6.2 Performances detection of CSS under CPUP and CSUSU Transmission mode 76

3.7 Throughput Optimization for Cooperative Spectrum Sensing in CRN 79

3.7.1 Throughput Optimization under CR Transmission Scenarios 79

3.7.2 Capacity Optimization detection for CSS under CPUP and CSUSU Transmission mode 80

3.8 Conclusion 83

4 Blind Spectrum Sensing Based on Statistic test (GoF test) 85

4.1 Introduction 85

4.2 Goodness of Fit Tests 86

4.3 Spectrum Sensing method based on GoF test using chi-square distribution 88

4.3.1 Performance comparison of existing GoF sensing methods 90

4.4 Adaptation of existing GoF tests for spectrum sensing 92

4.4.1 Modified AD GoF sensing 92

4.4.2 Chi-square GoF test for spectrum sensing 94

4.4.3 Order Statistic (OS) GoF sensing method 96

4.5 Spectrum Sensing Based on The Likelihood Ratio Goodness of Fit test . . . 98

4.5.1 Likelihood based Goodness of fit test . . . 99

4.5.2 The proposed spectrum sensing (LLR-GoF sensing) . . . 100

4.6 GoF Sensing Under Non Gaussian Noise and Noise Uncertainty 102

4.6.1 Non Gaussian noise (GM Model) . . . 102

4.6.2 Noise uncertainty . . . 106

4.7 New proposed GoF sensing method . . . 110

4.7.1 AD sensing method based on sub-blocks . . . 110

4.7.2 Spectrum Sensing Method Based on The new GoF statistic test . . . 112

4.8 Wide-band Spectrum Sensing based on GoF testing . . . 119

4.8.1 Result on Synthetic Data . . . 120

4.9 Conclusion . . . 123

5 Distributed Consensus Spectrum Sensing For CRN 125

5.1 Introduction . . . 125

5.2 Related Works . . . 126

5.3 Network Model for Distributed Spectrum sensing . . . 128

5.4 Spectrum sensing Model . . . 128

5.5 The Consensus Algorithms for Distributed Spectrum Sensing . 131

5.6 Weighted Average Consensus for Distributed Spectrum Sensing 133

5.7 Test the optimality of the proposed weighted consensus DSS scheme . . . 137

5.7.1 Exhaustive Search (ES) based algorithm 138

5.7.2 GA based GoF cooperative spectrum sensing 139

5.7.3 Simulation results and comparison 140

5.8 Conclusion 143

6 Conclusions and Future Work 145

6.1 Conclusions 145

6.2 Future Work 147

List of Publications 149

Bibliography 151

List of Figures

1.1 Spectrum utilization [1]. 10

1.2 The main functionalities of cognitive radios 13

1.3 Cognitive cycle as introduced by Joseph Mitola [2]. 15

1.4 Cognitive Radio Networks Architecture [1]. 16

1.5 A taxonomy of dynamic spectrum access [3]. 17

1.6 Summary of international standardization on CRN [4] 21

2.1 Classification of spectrum sensing techniques. 28

2.2 Threshold in ED: trade off between missed detection and false alarm. 31

2.3 Energy detector: (a) time domain (b) frequency domain 32

2.4 Complementary ROC curves for the energy detection under AWGN and Rayleigh fading channels 35

2.5 ROC curves for the energy detection under AWGN and Rayleigh fading channels 36

2.6 ROC curves for the energy detection with Gaussian approximation 37

2.7 Spectral correlation density for BPSK with a signal to noise ratio of $2dB$ estimated over 50 BPSK symbols 42

2.8 ROC curves for MME method under different SNR for 10000 simulation Monte Carlo 46

2.9 The PSD structure of a wideband signal with N bands [5] 50

2.10 Schematic illustration of centralized cooperative spectrum sensing scheme 53

2.11 Schematic illustration of distributed cooperative spectrum sensing scheme 54

3.1 Sensing problems (receiver uncertainty, multipath and shadowing). 56

3.2 Elements of cooperative spectrum sensing [6]. 58

3.3 ROC for the hard fusion rules under AWGN channel, $SNR = -2dB$, $K = 3$ users, and energy detection over 1000 samples. . . 64

3.4 ROC for soft fusion rules under AWGN channel with K=3 users, and energy detection with m=5. 67

3.5 Principle of three-bit hard combination scheme. 68

3.6 ROC curves for quantized data fusion under AWGN channel with $SNR = -2dB$, $K = 3$ CR users and $N = 1000$ samples. . . 69

3.7 ROC for combining fusion rules under AWGN channel with $K = 3$ users, $SNR = -2dB$ using energy detection with $N = 1000$ samples. 70

3.8 The 4 energies regions for the two-bit combination scheme. . . . 74

3.9 Probability of false alarm versus sensing time under CPUP scenario using different combining rules (K=10, $\bar{Q}_d = 0.95$) 77

3.10 Probability of detection versus sensing time under CSUSU scenario using different combining rules (K=10, $\bar{Q}_f = 0.05$) 78

3.11 Normalized capacity versus sensing time under CPUP scenario using different combining rules (K=10, $\bar{Q}_d = 0.95$) 82

3.12 Normalized capacity versus sensing time under CSUSU scenario using different combining rules (K=10, $\bar{Q}_f = 0.05$) 83

4.1 Detection probability versus false alarm probability of various GOF test based sensing at $SNR = -6dB$ and $n = 80$ samples . . 91

4.2 Detection probability versus SNR for different GOF tests based sensing with $Pfa = 0.05$ and $n = 80$ samples 92

4.3 Detection probability versus SNR for modified AD GoF sensing with $Pfa = 0.05$ and $n = 80$ samples 94

4.4 Detection probability versus SNR for chi-square GoF sensing over AWGN channels with $Pfa = 0.05$ and n=80 samples 96

4.5 Detection probability versus SNR for OS sensing with $Pfa = 0.05$ and n=80 samples 98

4.6 Detection probability versus false alarm probability over AWGN channels with $SNR = -6$ dB and $n = 80$ samples 101

4.7 Detection probability versus SNR over AWGN channels with $Pfa = 0.05$ and n=80 samples 102

4.8 probability distribution function (pdf) of GM noise $\alpha = 0.9$, $\beta = 5$ and $\sigma = 1$ 103

4.9 Detection probability versus SNR under Gaussian and non Gaussian noise for AD-GoF, with $Pfa = 0.05$ and $n = 80$ samples 104

4.10 Detection probability versus SNR under Gaussian and non Gaussian noise for LLR-GoF, with $Pfa = 0.05$ and $n = 80$ samples 105

4.11 Detection probability versus SNR under Gaussian and non Gaussian noise for ED, with $Pfa = 0.05$ and $n = 80$ samples . . 106

4.12 Impact of noise uncertainty on ED with $Pfa = 0.05$ and $n = 80$ samples 108

4.13 Impact of noise uncertainty on GoF test based sensing with $Pfa = 0.05$ and $n = 50$ samples 109

4.14 A new AD sensing method block diagram 110

4.15 Detection probability versus SNR over AWGN channels with $Pfa = 0.01$ for the AD GoF sensing based on sub-blocks 112

4.16 Noise power area 113

4.17 Detection probability versus SNR for the proposed GoF sensing under different weights, with $Pfa = 0.05$ and n=80 samples . . 118

4.18 Wideband sensing method block diagram [7]. 120

4.19 Empirical CDF for every frequency bin: in blue the CDFs in the H_0 hypothesis, in red the CDFs in the H_1 hypothesis. The CDF F_0 is represented in green [7]. 122

4.20 Wide-band sensing result on the 2 low SNR signals: $N = 1024$, $K = 40$, $\lambda = 3.89$ [7]. 123

5.1 Centralized Cooperative Spectrum Sensing (left) and Distributed Cooperative Spectrum Sensing (right) 126

5.2 The network with 50 CR users and fixed graph 135

5.3 Convergence of the network for conventional consensus based GoF test 136

5.4 Detection probability versus false alarm probability for proposed weighted consensus based DSS using GoF for local sensing 137

5.5 Detection probability versus false alarm probability for some optimal schemes using GoF for local sensing 141

5.6 Detection probability versus false alarm probability for proposed weighted consensus based DSS using GoF for local sensing 142

List of Tables

1.1 Components of the IEEE 1900 standards 19

2.1 Numerical table for the Tracy-Wisdom distribution of order 1 [8] 46

4.1 Threshold values for some given Pfa with $n = 80$ samples (OS Sensing) 98

4.2 Threshold values for some given Pfa and n=80 samples 118

5.1 Threshold values for some given Pfa 134

Notations and Acronyms

Notations

$\approx$	is approximately equal to.
j	imaginary unit, $j = \sqrt{-1}$; also used as an index variable.
x	a scalar.
$x^\dagger$	the complex conjugate of x.
$\lvert x \rvert$	the absolute value of x.
$\mathbf{x}$	a column vector, $\mathbf{x} \triangleq [x_1, \ldots, x_N]^T$.
$\mathbf{x}^T$	the transpose of vector $\mathbf{x}$.
$\mathbf{X}$	a matrix.
$\mathbf{X}_{[m,n]}$	the mth row and nth column entry of matrix $\mathbf{X}$.
$\mathbf{X}_j$	A matrix with unity at the (j, j)th place, whereas all other elements being zero.
$\mathbf{X}^T$	the transpose of matrix $\mathbf{X}$.
$\mathbf{X}^{-1}$	the inverse of matrix $\mathbf{X}$.
$\mathrm{tr}\,\{\mathbf{X}\}$	the trace of matrix $\mathbf{X}$.
$\mathrm{diag}\,(\mathbf{x})$	the diagonal matrix whose entries are the elements of vector $\mathbf{x}$.
$\mathbf{I}$	the identity matrix—a subscript may be used to indicate the matrix dimension.
$\mathbb{E}[x]$	the expected value (or mathematical expectation) of x.
$\exp(x)$	the exponential function: $\exp(x) = e^x$.
$\log(x)$	the logarithm to the base e.
$\lvert \bullet \rvert$	the cardinality.
$\sim$	distributed according to.
χ^2	Chi-square test statistic
A_n^2	Anderson darling test statistic
T_n^2	Cramer-Von Mises test statistic
D_n^2	Kolmogorov-Smirnov test statistic

Γ_{os}	Order test statistic
$F(T\|H_i, x)$	The commutative distribution of test statistic T under H_i hypothesis.
$\Gamma()$	Gamma distribution
χ_2^{2n}	Chi-square distribution with n degree of liberty
$\mathcal{N}(\mu, \sigma^2)$	Gaussian distribution with mean μ and variance σ^2.
$\mathcal{CN}(\mu, \sigma^2)$	circularly symmetric complex Gaussian distribution with mean μ and variance σ^2.
$\mathrm{Cov}\{X, Y\}$	the covariance of X and Y.
$\mathcal{G} = (V, E)$	Graph with Edge E and Vertex V.

Acronyms

AD	Anderson Darling
AGM	Arithmetic Geometric Means
AWGN	Additive White Gaussian Noise
BEE	Berkely Emulation Engine
BPF	Band Pass Filter
BPSK	Binary Phase Shift Keying
CAF	Cyclic Auto-correlation Function
CDF	Cumulative Distribution Function
CLT	Central Limit Theory
CM	Cramer-Von Mises
CPUP	Constant Primary User Protection
CR	Cognitive Radio
CROC	Complementary Receiver Operating Curve
CCS	Cooperative Spectrum Sensing
CSUSU	Constant Secondary User Spectrum Utilization
DSA	Dynamic Spectrum Access
DSS	Distributed Spectrum Sensing
ED	Energy Detection
EGC	Equal Gain Combining
EME	Energy Minimum Eigenvalue
ES	Exhaustive Search based solution
FC	Fusion Center
FCC	Federation Communication Commission
FFT	Fast Fourier Transform
GA	Genetic Algorithms
GG	Generalized Gaussian model
GM	Gaussian Mixture model
GoF	Goodness of Fit
HoS	High order Statistic
i.i.d.	independent and identically distributed
ISM	Industrial Scientific Medical
KS	Kolmogorov-smirnov
LLR	Log-Likelihood Ratio
MAP	Maximum APosteriori
MF	Matched Filter
MIMO	Multiple Input Multiple Output

ML	Maximum Likelihood
MME	Maximum Minimum Eigenvalue
MTM	Multi-Taper Method
OS	Order Statistic
OFDMA	Orthogonal Frequency Division Multiple Access
PDF	Probability Distribution Function
PSC	Public Safety Communication
PSD	Power Spectral Density
PU	Primary User
RF	Radio Frequency
ROC	Receiver Operating Curve
SCC	Standards Coordinating Committee
SCD	Spectral Correlation Density
SCF	Spectrum Correlation Function
SNR	Signal to Noise Ratio
SU	Secondary User
TDMA	Time Division Multiple Access
USRP	Universal Software Radio Peripheral

General Introduction

Due to the rapid development of wireless communications services, the requirement of spectrum is growing dramatically. The federation Communications Commission (FCC) has stated that some allocated frequency bands are largely unoccupied (under-utilized) most of the time. Cognitive Radio has emerged as a novel approach to enable dynamic spectrum access (DSA) by allowing unlicensed users to access the under-utilized licensed spectra when/where licensed primary users (PU) are absent and to vacate the spectrum immediately once a PU becomes active without causing harmful interference. This ability is dependent upon spectrum sensing (SS), which is one of the most critical functions to achieve such a dynamic spectrum access. Efficient signal detection is required to perform SS task. The detection performance in SS can be degraded due to many effects such as multipath fading, shadowing and the noise uncertainty problem. Hence, Cooperative Spectrum Sensing (CSS) has been introduced to alleviate these issues by taking advantage of cooperation among CR users. Cooperative spectrum sensing has attracted a lot of attention in the research community

In this thesis, we consider the issue of spectrum sensing and distributed spectrum sensing for cognitive radio networks. The purpose of this chapter is to introduce the problem addressed in the thesis. We will mention the motivation behind this work. Then, we show some challenging issues in spectrum sensing. Finally we provide a summary of the outline and contributions of the thesis.

0.1 Motivation

Spectrum sensing is an important task to find spectrum opportunities. Its main goal is spectrum opportunity discovery in a reliable manner. Important metrics to evaluate any spectrum method are the probability of detection P_d (defined as the probability that the signal is detected in the hypothesis H_1), and probability of false alarm, P_{fa} (defined as the probability that the signal is detected in the hypothesis H_0), where H_1 and H_0 are the hypothesis that the signal is present and the hypothesis that the signal is absent, respectively. Several spectrum sensing techniques have been proposed in literature, however claiming that a given method is the best is still a challenge. There are several important characteristics to be considered in order to decide on a specific sensing method:

- Prior knowledge: which can be defined as the quantity of information needed by the method to perform sensing. Methods that do not require any prior knowledge concerning primary signal, are known as blind detection methods.

- Sensing time: is proportional to the number of samples needed for the detection task. This characteristic is important for real-time applications. It can be used as a characteristic to compare sensing methods.

- Computational complexity: describes the degree of difficulty required to execute the technique.

- Noise rejection: shows the capacity of the method to be immune against noise variation.

- Sensing performance: In signal detection theory, Receiver Operating Characteristic (ROC) curve is a graphical plot of the sensitivity (Pd vs Pfa). ROC curve provides tools to select any possible optimal method. The best detector is the one which is situated more to the left upper corner, since with the same false alarm probability the detector gives better detection probability.

To make tradeoffs between these different characteristics, we propose in this thesis the study of a spectrum sensing method based on statistic test (Goodness of Fit (GoF) test). It will be shown that GoF based spectrum sensing has the nice feature that, it needs fewer samples (short sensing time) to perform sensing. Moreover,and compared to Energy Detection it will be shown that this method is less sensitive to noise uncertainty and it is independent of noise power. Therefore, this method will be used as a local sensing method (instead of energy detection) for the cooperative spectrum sensing scheme which has been widely considered for combating fading or shadowing as a single CR user cannot distinguish between a deep fade and a spectrum hole. Many studies have been carried out to develop blind spectrum sensing methods such as GoF based spectrum sensing, which is considered as a blind detection method. Moreover, to cope with several problems and impairments such fading, shadowing and uncertainties in the system parameters, GoF based spectrum sensing can be used as a local sensing method for cooperative spectrum sensing.

Cooperative spectrum sensing techniques (a group of CR nodes cooperate in order to perform the spectrum sensing) is one of the most effective ways to combat the above impairments. The cooperative spectrum sensing architecture can be either centralized or distributed. In a centralized cooperative sensing, a central unit called fusion center is required to collect sensing information from CR nodes, in order to identify the available spectrum, and broadcasts this information to other CR nodes. In the case of distributed cooperative sensing, CR nodes share their sensing information among each other in order to make their own decisions about spectrum availability. Compared to the centralized scheme, the distributed scheme does not need any central infrastructure resulting in a reduced cost. However its decision implementation is more complex.

0.2 Spectrum Sensing Challenges

Spectrum sensing challenges have been discussed in many studies such [9]. Any spectrum sensing method has to face some important challenges such:

1- Channel Uncertainty: If the primary transmitter suffers a deep fade due to various obstacles, the secondary user (CR user) may decide the presence of a spectrum hole and starts transmitting. This will cause interference leading to loss of data.

2- Noise Uncertainty: The noise power is a result of several sources which are not completely known to the CR user, therefore, one needs to estimate it. A wrong decision of the sensed signal may be resulted if we underestimate the noise level as the signal to noise ratio (SNR) falls below a threshold value (the case of energy detection method).

3- Aggregate-Interference problem: In cooperative spectrum sensing when multiple cognitive radios are deployed to detect the primary transmitter, the overall interference caused by the CR networks may be harmful to the primary receiver.

4- Hidden Terminal Problem: it occurs when the link from a primary transmitter to a secondary user is completely shadowed, due to certain obstacles or when they are separated with a very long distance. However, there may be a primary receiver in the vicinity of the secondary user. Hence, The secondary will detect a white space (a frequency band which is allocated to licensed users (primary users), but it is not utilized in some locations and at some times and could be accessed by unlicensed users) and then accesses the licensed channel, causing destructive interference to the primary receiver.

5- Quality of service degradation: The CR users have to detect if the primary user return to occupy the band and cease transmission immediately in order to switch to a new band. This transition involves a delay and a need to reset protocols to match the characteristics of the new frequency band, causing abrupt quality of service degradation.

0.3 Objectives and contributions

0.3.1 Objectives

The aim of this thesis is to study and analyze a method of local sensing based on statistic test (GoF test) according to the characteristics cited in section 0.1, evaluate and optimize the performance of cooperative spectrum sensing algorithms and apply the aforementioned local sensing method for distributed spectrum sensing techniques.

0.3.2 Key Contributions

The main contributions of this thesis are summarized as follows:

- We study the cooperative spectrum sensing by implementing some data fusion schemes in fusion center. We also analyze a quantized combination scheme based on a tree-bit quantization and compare its performance with some hard and soft combination schemes.
- For these combining schemes rules, the detection performance, with a Gaussian distribution assumption, is expressed in two different scenarios, CPUP (Constant Primary User Protection) and CSUSU (Constant Secondary User Spectrum Usability). A comparison is conducted between these proposed schemes in both scenarios, in terms of detection performance and throughput optimization of the CR network.
- The GoF sensing methods that compare the distribution of the energy of the received samples against the cumulative distribution function (CDF) of the noise energy is studied. Beside, a new GoF test statistic which takes into account the physical characteristic of spectrum sensing is presented and evaluated in terms of sensing performance.
- A consensus algorithm for distributed spectrum sensing (DSS) in cognitive radio networks (CRN) integrating a Goodness of Fit based spectrum sensing scheme is studied. Moreover, a weighted consensus based DSS scheme is proposed and its optimality is tested.

0.4 Thesis Outline

The remainder of this thesis is organized as follows:

Chapter 1

This chapter first provides an introduction to the concept of cognitive radio and the software defined radio (SDR). Then, the most common definitions of cognitive radio are presented as well as the cognitive cycle and it functionalities, beside, the concept of dynamic spectrum access and some of the standardization proposals are summarized. Finally, this chapter touches upon some of the possible applications of cognitive radio.

Chapter 2

This chapter gives an overview of spectrum sensing methods and their classifications. Then, it discusses the commonly detection methodologies used in spectrum sensing for CR. It presents and analyzes various spectrum sensing techniques regarding their advantages and drawbacks.

Chapter 3

This chapter proposed the study of the optimization in cooperative spectrum sensing. First, it studies cooperative spectrum sensing and signal detection in CRN by implementing some combining rules in the fusion center. For these combining rules, the detection performance, with a Gaussian distribution assumption, is expressed in two different scenarios, CPUP (Constant Primary User Protection) and CSUSU (Constant Secondary User Spectrum Usability). Finally, it analyzes the channel utilization (throughput vs sensing time relationship) for cooperative spectrum sensing under both mentioned scenarios and for different combining rules.

Chapter 4

Firstly, this chapter reviews the most popular GoF sensing methods for cognitive radio and present a comparative study in terms of detection performance. Secondly, it proposes two new GoF sensing methods and compare them against the conventional Anderson Darling (AD) sensing and energy based sensing. It proposes a new GoF test statistic by taking into account the physical characteristic of spectrum sensing. The derived GoF sensing method results in significant improvement in terms of sensing performance. Finally, it proposes how GoF based spectrum sensing can be integrated in a conventional wideband spectrum sensing scheme.

Chapter 5

The last chapter is devoted to the study of a consensus algorithm for distributed spectrum sensing (DSS) in CRN. Motivated by the fact that in GoF based spectrum sensing, the threshold for the binary test depends only on the desired false alarm probability and not on the local noise power as in energy detection, this chapter proposes to integrate a Goodness of Fit based spectrum sensing for DSS scheme instead of the existing work in this area which often applies energy detector as a local spectrum sensing method for DSS. Moreover, a weighted consensus based DSS scheme is proposed and its optimality is tested with some optimal schemes such as exhaustive search and genetic algorithms.

Chapter 6

This chapter provides some concluding remarks and delineates on directions in which the work in this thesis can be investigated.

Introduction to Cognitive Radio

1

In this chapter, we provide a detailed introduction to the concept of cognitive radio by introducing the software defined radio (SDR) as the key technology for cognitive radio. The known definitions of cognitive radio are presented as well as the cognitive cycle and it functionalities, the concept of dynamic spectrum access and some of the standardization proposals. Finally, we touch upon some of the possible applications of cognitive radio.

1.1 Wireless Communications

In recent decades, the wireless communication technology is advancing at a fast rate to provide network services anywhere and anytime. Consequently, the demand for radio spectrum is increasing and regulatory agencies in different countries thus allocate chunks of spectrum to different wireless services. As a natural resource, radio spectrum is scarce and limited. However, with steadily growing number of wireless subscribers and operators, the problem of spectrum scarcity is imposed. Many studies [10] [11] clearly suggest that currently spectrum scarcity is mainly due to the inefficient use of spectrum rather than the physical shortage of spectrum. As shown in Figure 1.1, some parts of spectrum remain largely underutilized, some parts are sparingly utilized, while the remaining parts of the spectrum are heavily occupied [1]. Thus, efficient use of spectrum is required. Many technologies intends to meet this requirement of effective utilization of radio spectrum such as [12]:

- Multiple-input multiple-output (MIMO) communications: MIMO systems share data among multiple antennas resulting in higher data throughput without additional spectrum usage, which improves spectral efficiency.

- Cooperative communications: by exploiting distributed spatial diversity in a multi-user environment, reliability and data rate are improved using Cooperative radio transmissions.

- Heterogeneous networks: spectral efficiency per unit area is enhanced by using a diverse set of base stations in different cells, which is necessary to support increasing node density and cell traffic in mobile networks.

- Other technologies: High modulation orders coupled to advances signal processing (joint detection, iterative techniques, ...).

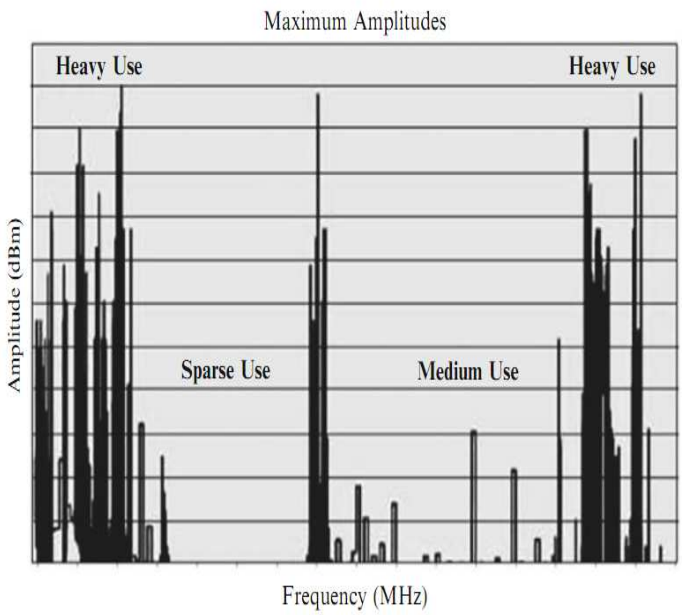

Figure 1.1 Spectrum utilization [1].

Despite these advanced technologies and in order to optimally manage available radio resources, Cognitive radio (CR) was suggested [13] [14].

1.2 Cognitive Radio

The Software Defined Radio is seen as a major factor on the road to CR. Its main goal is to give guidelines according to SDR methodologies that will be applied to the CR technology [15].

1.2.1 Software Defined Radio to Cognitive Radio

In 1999, J. Mitola III introduced the concept of Cognitive Radio (CR) who also coined the term Software Radio in 1991. The software radio aims at building multi-mode and multi-band platforms in order to provide flexible communications radios that can accommodate different standards within the same hardware. This is made possible by using software versatility. For those radios, 80% of the functionality is provided in software, compared to the 80% hardware in the 90s. By the end of the 90s, the software radio concept was on the verge of being ready for commercial applications, and Mitola thought about the ways of using the versatility brought by the software radios in order to optimize the performance of communication systems. This led to the Cognitive Radio concept. CR is envisioned as one key solution to solve spectrum congestion due to increasing number of systems / subscribers along with spectrum resource scarcity. This is one of the major topics under investigation and receives a particular attention both in civilian and military sides. Cognitive resource allocation means that spectrum can be shared between users. There is no need for comprehensive static allocation of frequencies to users or services in dedicated bands because the devices using this band organize the usage themselves. CR systems will embed software radio capabilities plus intelligence (evolution and optimization), awareness (sensing and modeling) and learning (building and retaining knowledge). A CR system is a system that is able to sense its operational environment and can dynamically and autonomously adjust its radio operating parameters accordingly to achieve or to be as close as possible to pre-defined target objectives. It learns from previous experiences. Cognitive radio techniques provide the capability to use or share the spectrum in an opportunistic manner.

1.2.2 Definitions of a cognitive radio

The following provides some of the more prominently offered definitions of cognitive radio.

Mitola *Wireless personal digital assistants and the related networks that are sufficiently computationally intelligent about radio resources, and related computer to computer communications, to detect user needs as a function of use context and to provide radio resources and wireless services most appropriate to those needs* [16].

FCC *has defined a cognitive radio as: A radio that can change its transmitter Parameters based on interaction with the environment in which it operates* [17]

Wikipedia *cognitive radio is a paradigm for wireless communication in which either a network or a wireless node changes its transmission or reception parameters to communicate efficiently, avoiding interference with licensed or unlicensed users. This alteration of parameters is based on the active monitoring of several factors in the external and internal radio environment, such as radio frequency spectrum, user behavior, and network state.*

IEEE 1900.1 *(a) A type of radio in which communication systems are aware of their environment and internal state and can make decisions about their radio operating behavior based on that information and predefined objectives; (b) cognitive radio (as defined in item a) that uses software-defined radio, adaptive radio, and other technologies to adjust automatically its behavior or operations to achieve desired objectives* [18].

Haykin: *cognitive radio is an intelligent wireless communication system that is aware of its environment and uses the methodology of understanding by building to learn from the environment and adapt to statistical variations in the input stimuli to achieve high reliability and efficient utilization of the radio spectrum* [14].

Scientific American: *Cognitive radio is an emerging smart wireless communications technology that will be able to find and connect with any nearby open radio frequency to best serve the user. Therefore, a cognitive radio should be able to switch from* *a band of the radio spectrum that is blocked by interference to a free one to complete a transmission link, a capability that is particularly important in an emergency* [19].

Definition of cognitive radio in this thesis: Intelligent Radio that autonomously changes its communication parameters (waveform) in response to user demands or to changes in the EM environment.

1.2.3 Functions and components of Cognitive Radio

The main goal of cognitive radio is to provide adaptability to wireless transmission through dynamic spectrum access so that the performance of wireless transmission can be optimized, as well as improving the utilization of the frequency spectrum. The main functionalities of cognitive radios are spectrum sensing, spectrum management, spectrum sharing and spectrum mobility, as it is depicted in figure 1.2.

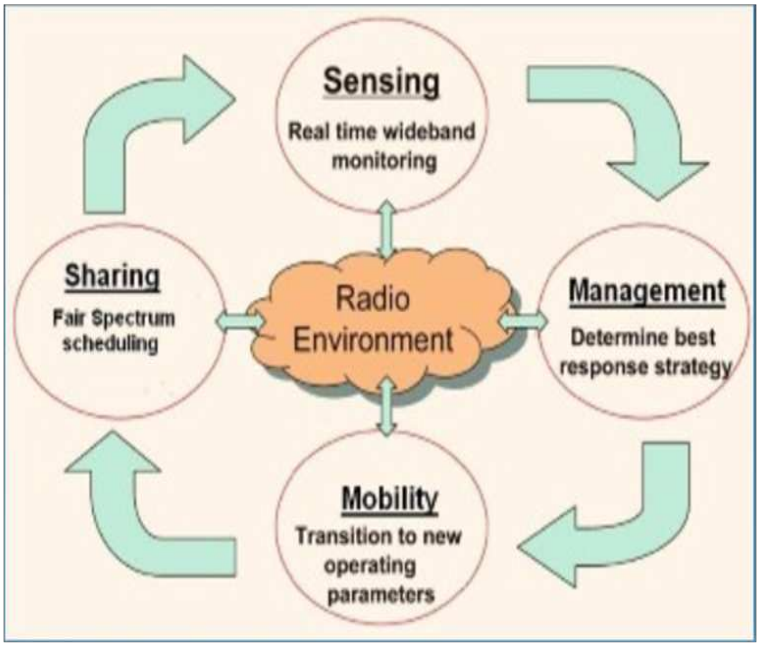

Figure 1.2 The main functionalities of cognitive radios

The aforementioned capabilities of CR, have given many ways of definition for CR and no globally adopted formal definition have been adopted yet.

Through spectrum sensing, the cognitive radio technology will enable the users to determine which portions of the spectrum is available (detecting spectrum holes) with the requirement of no harmful interference with other users.

The spectrum sensing information is exploited by the spectrum management function to select the best available channel and make optimal decisions on spectrum access. Whereas, spectrum sharing coordinates access to this channel with other users with fair spectrum scheduling method, which is one of the major challenges in open spectrum usage. If the status of the target spectrum changes (licensed user is detected), the spectrum mobility function will allow to vacate the channel and avoid communication loss on jammed channels. As a reference for how a cognitive radio could achieve these levels of functionality, in [2] Mitola introduces the cognition cycle, discussed in the next section.

1.2.4 Cognition Cycle

In [2], Mitola introduces the cognition cycle, shown in figure 1.3. In the cognition cycle, information about the operating environment (Outside world) is received by a radio based on a direct observation or signaling. After evaluation, the importance of this information is determined (Orient). Based on the previous task, the radio determines its alternative (Plan) and selects an alternative (Decide) in a manner that is likely to improve recovery. If a change in waveform was deemed necessary, the alternative (Act) is implemented by adjusting its resources and performing the appropriate signaling. These changes are then reflected in the interference profile presented by the cognitive radio in the Outside world. As part of this process, the radio uses these observations and decisions to improve the operation of the radio (Learn), perhaps by creating new modeling states or generating new alternatives.

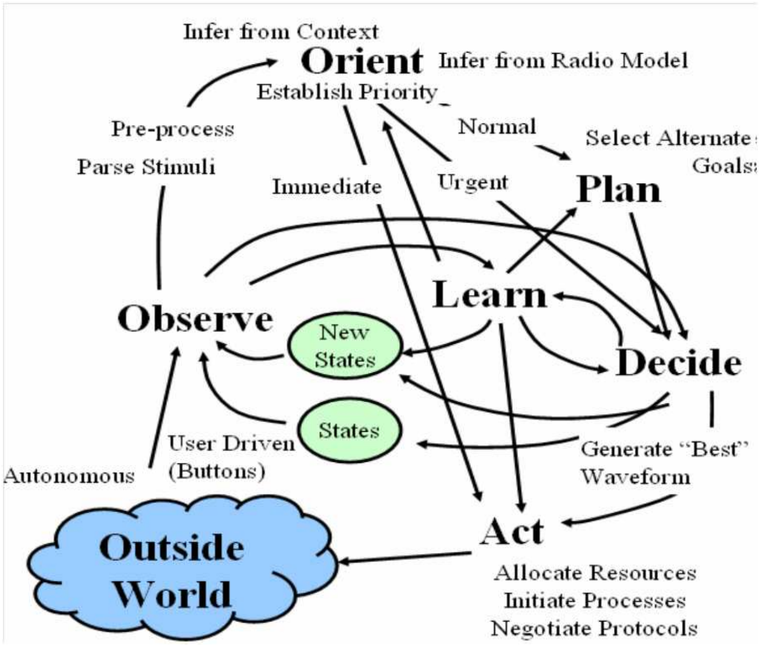

Figure 1.3 Cognitive cycle as introduced by Joseph Mitola [2].

1.2.5 Cognitive Radio Networks Architecture

In order to develop communication protocols, a clear description of Cognitive Radio Network architecture is necessary. In CR networks, and through its functionalities (spectrum sensing, spectrum management, spectrum sharing and spectrum mobility), a cognitive radio user should be able to detect spectrum holes so that it is able to release the frequency spectrum when the licensed users are detected. These functionalities must be located in the CR networks protocol stack. As shown in figure 1.4, the components of the Cognitive Radio network architecture, can be classified in two groups such as the primary network (licensed system) and the CR network (unlicensed system).

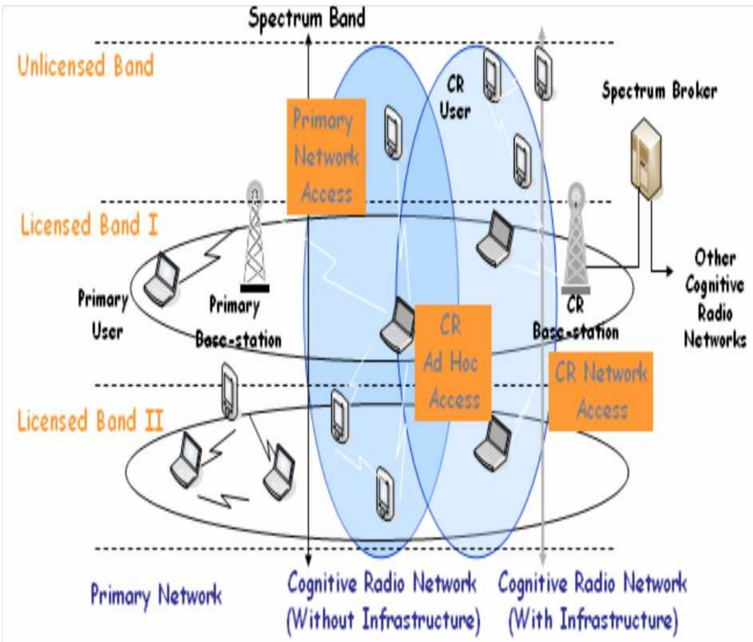

Figure 1.4 Cognitive Radio Networks Architecture [1].

1.3 Dynamic Spectrum Access and Management

The term dynamic spectrum access (DSA) is known as a technique adopted by a radio network to dynamically select the operating spectrum from the available spectrum. The DSA is an opposite approach to the current static spectrum management. The DSA consists to open licensed spectrum to secondary users without causing harmful interference to primary users [20].

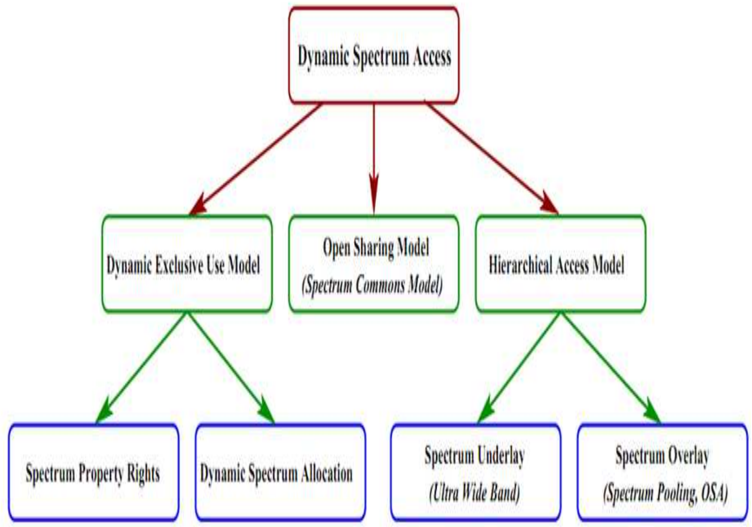

Figure 1.5 A taxonomy of dynamic spectrum access [3].

The diverse ideas presented at the first IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN) [3] suggest the extent of this term. As illustrated in Figure 1.5, dynamic spectrum access strategies can be classified in terms of access strategies under three models.

1.3.1 Dynamic Exclusive Use Model

This model maintains the basic structure of the current spectrum regulation policy: the spectrum is licensed to a user for exclusive use respecting some rules. The main idea is to introduce flexibility to improve spectrum efficiency. Two approaches have been proposed under this model: spectrum property rights [21] and dynamic spectrum allocation [22]. The first approach allows licensees to sell and trade spectrum and to freely choose technology. Note that even though licensees have the right to lease or share the spectrum for profit, such sharing is not mandated by the regulation policy. Whereas, the second approach aims to enhance spectrum efficiency through dynamic spectrum assignment by exploiting the spatial and temporal traffic statistics of different services. In other words, in a given region and at a given time, spectrum is allocated to services. This allocation, however, varies at a much faster scale than the current policy.

1.3.2 Open Sharing Model

This model referred to as spectrum commons, employs open sharing among peer users as the basis for managing a spectral region. Advocates of this model draw support from the phenomenal success of wireless services operating in the unlicensed ISM band (e.g., WiFi) [23].

1.3.3 Hierarchical Access Model

This model have been built upon a hierarchical access structure with primary and secondary users. It can be seen as can as a hybrid of the above two models. The basic idea is to opportunistically allow the secondary (unlicensed) users access the spectrum without interfering with the primary (licensed) users. This opportunistic access is done in two ways: spectrum underlay and spectrum overlay.

1. Spectrum overlay does not necessarily impose severe constraints on the transmit power of secondary users, but rather on their transmission time. Consequently, a secondary user accesses a spectrum hole assigned via DSA (this approach exploits spectrum white space).

2. Spectrum underlay requires strict constraints on the transmit power of secondary users. Their transmit power is thus low enough to be regarded as noise by primary users. Both primary and secondary users may thus transmit simultaneously in the same spectrum band.

This hierarchical model is perhaps the most compatible with the current spectrum management policy and legacy wireless systems.

Standards	Addressed issues
IEEE 1900.1	Identify and explain the concepts related to spectrum management and SDR.
IEEE 1900.2	Address the recommended practice for interference and coexistence analysis.
IEEE 1900.3	Develop and define testing methods for conformance evaluation of software components in SDR devices.
IEEE 1900.4	The coexistence support for the re-configurable heterogeneous air interface in next generation wireless systems .
1900.5-2011	Standard for Policy Language Requirements and System Architectures for Dynamic Spectrum Access Systems
1900.6a-2014	Spectrum Sensing Interfaces and Data Structures for Dynamic Spectrum Access and Other Advanced Radio Communication Systems.
1900.7	for Radio Interface for White Space Dynamic Spectrum Access Radio Systems Supporting Fixed and Mobile Operation.

Table 1.1 Components of the IEEE 1900 standards

1.4 Cognitive Radio Standardization

The standardization process aims to harmonize the ongoing research activities. The standardization is required for the development and implementation of a cognitive radio network due to the involving of many technical and economic aspects related to spectrum management using SDR.

From the most relevant standardization activities related to cognitive radio networks, we have:

- IEEE SCC 41 : The IEEE Standards Coordinating Committee (SCC) 41 standard had launched a series of related standards, namely, IEEE 1900 [24]. This standard addressed some issues related to Next Generation Radio and Spectrum Management (its development, implementation and deployment). The major components of the IEEE 1900 standards are summarized in table 1.1 :

  There are also other IEEE standards related to the cognitive radio, i.e. IEEE 802.11, 18, 19, 21, and 22.

- IEEE 802.22 wireless regional area networks (WRAN): which is introduced as the first worldwide effort to define a standardized air interface for fixed, point-to-multipoint WRANs that operate on unused channels in the VHF/UHF TV white spaces (TVWS) between 54 MHz-852 MHz [25]. In this IEEE 802.22 standards, the cognitive radio nodes and a base station determine the user radio terminals.

  Using cognitive radio techniques, the 802.22 working group developed a waveform in order to provide high bandwidth access in rural areas. IEEE 802.22 is composed of different standards:

- IEEE 802.22.1-2010 Standard for the Enhanced Interference Protection of the Licensed Devices.

- IEEE 802.22-2011 Standard for Cognitive Wireless Regional Area Networks (RAN) for Operation in TV Bands.

- IEEE 802.22.2 Standard for Recommended Practice for Installation and Deployment of 802.22 Systems.

- IEEE 802.11af: the international specifications for spectrum sharing among unlicensed white space devices and licensed services in the TV white space band are defined in this standard. A geolocation database is used to conduct spectrum sharing via the regulation of unlicensed white space devices [26]. A common operating mechanisms for white space devices is provided by the IEEE 802.11af standard satisfying multiple regulatory domains.

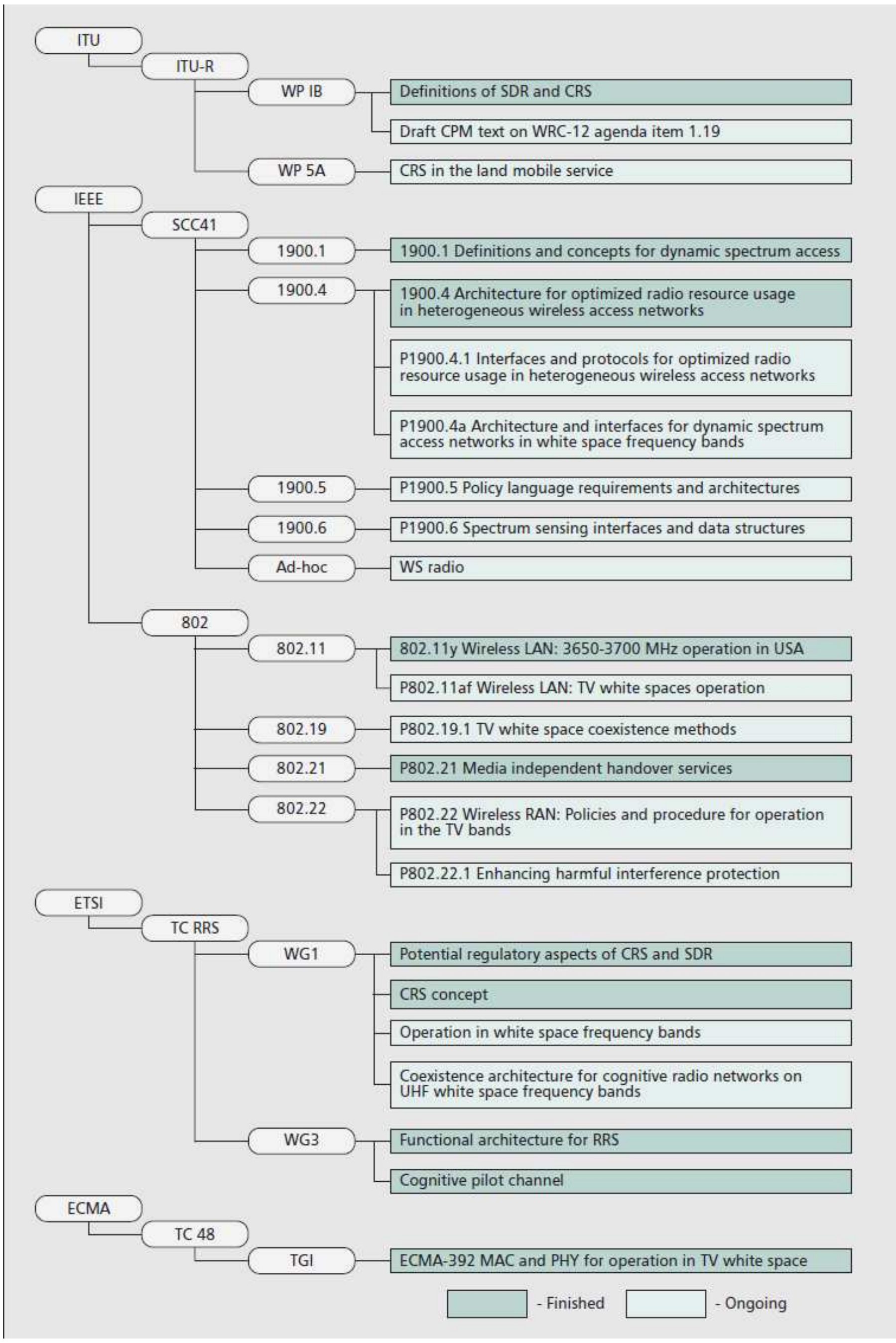

Figure 1.6 Summary of international standardization on CRN [4]

The authors in [27] mentioned that the 802.22 standard targets to achieve spectral efficiency of up to $3 bits/sec/Hz$ corresponding to peak download rates at coverage edge at $1.5 Mbps$. Simultaneously, the 802.22 system aims to achieve up to $100 km$ in coverage.

In figure 1.6, we give a summary of international standardization on CRN performed on all levels.

1.5 Cognitive Radio Applications

In this section, we provide some application where the concept of cognitive radio can be exploited [28].

- **Cellular Mobile Networks**: in this area, the cognitive radio technology can be brought via many challenging and open issues such as security and safety. The CR technology can also come up with solution to improve the cellular spectrum.
- **Energy efficiency**: To save energy in wireless networks, CR technology can be considered as strong candidate to improve energy efficiency. Based on its intelligence, CR can learn and adopt its parameters to enhance energy efficiency.
- **Public Safety Communication (PSC)**: In this context, the CR technology with its capabilities can address the issue of the presence of different technologies related to PSC, used by different national agencies, to improve the telecommunications systems in PSC.
- **Wireless Networks for Smart Grids**: In this domain, the CR technology is expected to play a key role. An efficient wireless networking is required by smart grid involving a large scale of metering data and covering different energy sources, these issues and others constitute a potential application for cognitive radio communications.
- **Vehicular Networks**: The CR technology can be introduced in this area by addressing issues such congestion avoidance in the spectrum, advanced power control, DSA and interoperability among existing communication devices.

- **Defense Application Systems**: The cognitive radio systems provide via its capability of interference mitigation, a promising key in defense communication scenarios such as battle fields.

1.6 Conclusion

In conclusion and through this chapter, we have introduced the topic of CR which is expected to improve the efficiency and flexibility of radio communications on the field through the dynamic use of the radio spectrum. This includes, improving adaptability to changing and unforeseen situations; improve the reliability and availability of communications in tactical radio networks, allow automatic and adaptive deployments, especially in unknown environments, that will be optimal for the considered field ; increase the capacity in a given portion of radio spectrum, allowing the introduction of more powerful features and services for a given radio type. Beside, we have discussed the functions and components of cognitive radio as well as the different standardization activities related to cognitive radio networks. The next section will be devoted to spectrum sensing, which is a crucial feature specific to CR and CR networks.

Spectrum Sensing for Cognitive Radio

2

In this chapter, firstly, we present an overview of spectrum sensing methods and their classifications. Then, we discuss the commonly detection methodologies used in spectrum sensing for CR. We present some spectrum sensing techniques such as energy based sensing, cyclostationary feature based sensing, matched filter based sensing and other sensing techniques. The energy based sensing is well detailed as it is considered as simplest detection method in a blind manner.

2.1 Introduction

Spectrum sensing is a key component of dynamic spectrum sensing paradigm to find spectrum opportunities. For practical dynamic spectrum sensing and access, power detectors are required.

Generally, in CR environments, sensing algorithms are expected to be able to detect the presence of signals at very low signal to noise ratio (SNR) levels within a limited observation time. Moreover, it is necessary that they are robust to practical impairments and parameter uncertainties. Therefore spectrum sensing is a difficult task in CR and to design detection algorithms that are capable to work under very harsh conditions is of fundamental importance. In this chapter, we provide an accurate analysis and study of different spectrum sensing algorithms: their advantages and drawbacks. Moreover, we study the detection performance of some spectrum sensing algorithms. Then, some other spectrum sensing methods are listed. Finally, we introduce the cooperative spectrum sensing as promising technique to improve detection performance. In the following, we define hypothesis H_1 as the probability of the presence of the signal and H_1 as the probability of the absence of the signal.

2.2 Overview of Spectrum Sensing Algorithms

In this section, we focus on the studies of the spectrum sensing algorithms proposed in literature to detect the presence of spectrum holes. Different techniques will be analyzed, with particular emphasis on the class of blind spectrum sensing techniques, which do not need a prior knowledge of the received signal.

The matched filter (MF) is considered as the optimum detector based on the classical detection theory. It provides the best detection performance, but has the disadvantage that it requires the knowledge of the signal to be detected, condition that in general in not satisfied.

The energy detector (ED) is the most used detector when the signal is unknown. This detector estimates the received energy in the band of interest and compares it to a threshold that is related to the noise power level and the required false alarm probability. The ED exhibits a low computational complexity and is widely used because it has a simple implementation. The main disadvantage of the ED is that it requires knowledge of the noise power to properly set the threshold. This requirement is often critical, in particular in low SNR environments, in which an imperfect knowledge of the noise power can cause severe performance losses. Moreover the ED cannot distinguish between interference and signal.

The ED detector can be considered as a blind spectrum sensing algorithm, in the sense that it does not require any knowledge of the signal to be detected. However it requires the knowledge of the noise power, which depends on the environment properties and can vary with the receiving node. Completely blind detection algorithms can be obtained by analyzing the auto-covariance properties of the received signal. These algorithms do not require any prior knowledge, but are based on the observation of some correlation properties in the received signal. Therefore specific solutions such as oversampling or the adoption of multiple antennas at the receiver are required. Typically these algorithms imply a high computational complexity (eg: algorithms based on eigenvalues of the auto-covariance matrix of the received signal samples).

When the signal to be detected has some known characteristics, the detection of such features is an effective method to identify such kind of signal. The cyclostationary method can be an appropriate sensing technique to recognize a particular transmission and/or extract its parameters. This technique enables separation between signal and noise components and it can be adopted for signal classification. This spectrum sensing method has high computational and implementation requirements. It is worth to mention that the cyclostationary method outperforms the ED method if the noise power is wrongly estimated.

To the above mentioned spectrum sensing algorithms, we can add also other algorithms derived from the spectral analysis such as: multi-taper spectral analysis, wavelet transforms and filter banks receivers based sensing methods. Generally, the spectrum sensing techniques can be classified as shown in figure 2.1

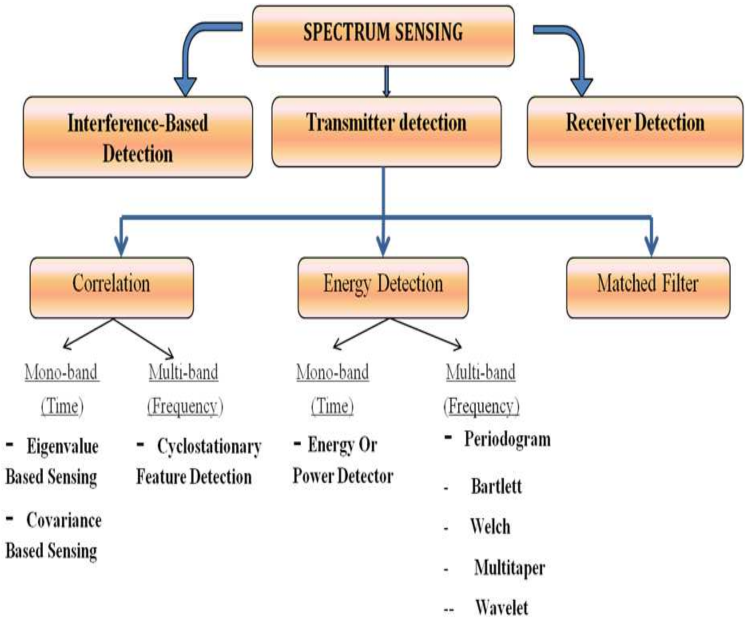

Figure 2.1 Classification of spectrum sensing techniques.

2.3 Statistical Detection Techniques

The concept of statistical detection is well studied in many fields such as: radar, communication engineering and statistical signal processing. Recently, this concept has been applied for spectrum sensing in cognitive radio networks. In this section, some detection techniques are presented in order to be used for spectrum sensing and detection in CRN. Generally, spectrum sensing can be modeled as a binary hypothesis testing problem, given as

$$y(n) = \begin{cases} w(n) & , H_0 \\ hx(n) + w(n) & , H_1 \end{cases} \tag{2.1}$$

where $y(n)$ are the complex samples of the sensed radio signal, $x(n)$ are samples of the transmitted primary user signal, h is the gain of the channel between the PU and the CR user, and $w(n)$ are samples of the noise over a bandwidth B.

2.3.1 Maximum A Posteriori Detection (MAP)

This detector uses the posteriori probabilities under hypothesis H_1 and H_0 for the received signal $y(t)$ to perform the hypothesis testing. The MAP detector is expressed as

$$ln\left[\frac{P(H_1|y)}{P(H_0|y)}\right] = \Lambda(y) \lessgtr_{H_1}^{H_0} 0 = \lambda \tag{2.2}$$

where, $P(H_1|y)$ and $P(H_0|y)$ are the posterior probabilities of $y(t)$ under hypothesis H_1 and H_0, respectively. Using the Bayes principal we can rewrite (2.2) as

$$\Lambda(y) = ln\left[\frac{P(y|H_1)}{P(y|H_0)}\right] \lessgtr_{H_1}^{H_0} ln\left(\frac{P(H_0)}{P(H_1)}\right) = \lambda^{'} \tag{2.3}$$

We can notice here that the computation of the detection threshold $\lambda^{'}$ needs the knowledge of the prior probabilities of the hypothesis H_0 and H_1.

2.3.2 Maximum Likelihood Detection (ML)

The ML detector is considered as the simplest detector which could be derived from the MAP detector when the prior probabilities of H_1 and H_0 are given such $P(H_1) = P(H_0) = 0.5$. The ML detector criteria is given by,

$$\Lambda(y) = ln\left[\frac{P(y|H_1)}{P(y|H_0)}\right] \lessgtr_{H_1}^{H_0} 0 = \xi \tag{2.4}$$

As one can see, the ML detector sets aside the prior probabilities of the hypothesis H_0 and H_1. Accordingly, this detector may not perform well when $P(H_1) \neq P(H_0)$.

2.3.3 The Neyman-Pearson Detection

The Neyman-Pearson Detector is quite useful in spectrum sensing. It consists on maximizing the probability of detection $P(\Omega(y) > \lambda|H_1)$ when the probability of false alarm $P(\Omega(y) > \lambda|H_0)$ is set to be constant, where $\Omega(y)$ is the likelihood ratio given by

$$\Omega(y) = \frac{P(y|H_1)}{P(y|H_0)}. \tag{2.5}$$

The Neyman-Pearson detection criteria is given by

$$\Omega(y) \lessgtr^{H_0}_{H_1} \lambda. \tag{2.6}$$

2.4 Detection performance

In spectrum sensing, the principal objective is to detect the presence of signals in the observed band, which consists in a binary decision between two hypothesis 'signal present' and 'signal absent'. For the evaluation of any detector, we need metrics to assess the effectiveness of the detection decision. The performance of the detection are expressed in terms of probability of false alarm P_{fa}, that is the probability that the decision metric Y exceeds the threshold λ in the hypothesis H_0 such:

$$P_{fa} = Pr(Y > \lambda|H_0), \tag{2.7}$$

and in terms of probability of detection P_d, that is the probability that the decision metric Y exceeds the threshold λ in the hypothesis H_1 such

$$P_d = Pr(Y > \lambda|H_1). \tag{2.8}$$

The probabilities P_{fa} and P_d are considered as the standard metrics used to evaluate the performance of a detector. These metrics are severely related to the right setting threshold value. The key problem in this regard is illustrated in Figure 2.2 which shows probability density functions of received signal under H_1 and H_0 Hypothesis.

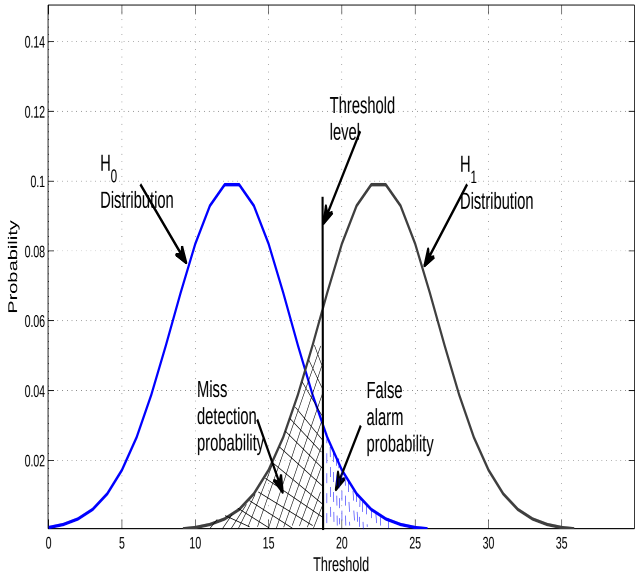

Figure 2.2 Threshold in ED: trade off between missed detection and false alarm.

2.5 Energy Detection Based Spectrum Sensing

The energy detector based spectrum sensing (ED) is the most popular method used to detect signals, known as *radiometry* in classical literature. After the pass band filter, with pass bandwidth W, the filtered signal then is amplified using a low noise amplifier and is down-converted to an intermediate frequency. Next, the received signal is sampled and quantized via an A/D converter. Finally, the resulting signal is squared and integrated over the sensing period T ($N = 2TW$, N: sample size), where $T = NT_s$ and T_s is the signal sampling period. The test statistic at the output of the integrator is compared with the threshold to make a final decision.

In the literature, we come across various algorithms indicating that energy detection can be implemented both in time and also frequency domain using Fast Fourier Transform (FFT) as shown in figure 2.3.

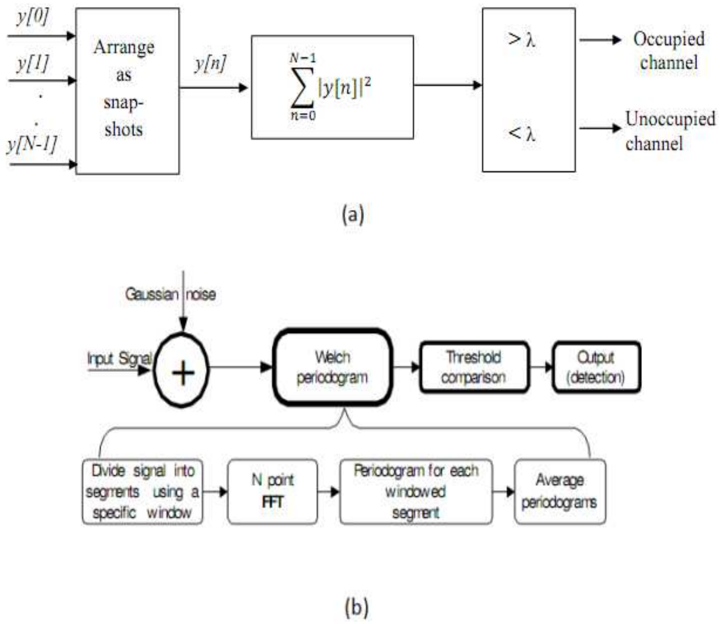

Figure 2.3 Energy detector: (a) time domain (b) frequency domain

2.5.1 Energy Detector

Generally, spectrum sensing can be modeled as a binary hypothesis testing problem as given in (2.1).

According to the Neyman-Pearson criterion, the likelihood ratio for the binary hypothesis test in (2.1) can be formulated as

$$\Omega_{LR} = \frac{f_{y|H_1}(x)}{f_{y|H_0}(x)} \tag{2.9}$$

where $f_{y|H}(x)$ is the probability density functions (PDF) of the received signal y under hypothesis H, where $H \in \{H_1, H_0\}$. Then, the log-likelihood ratio (LLR) can be written as the form $a + b \sum_{n=1}^{N} |y(n)|^2$ where N is the total number of samples and a and b are parameters. As we see, the LLR is proportional to $\sum_{n=1}^{N} |y(n)|^2$ which is the test statistic of energy detector when $x(t)$ is zero mean complex Gaussian [33].

The ideal ED test that we consider is

$$\Lambda(y) = \frac{1}{\sigma^2}\frac{1}{N}\sum_{n=1}^{N} |y(n)|^2 \lessgtr_{H_1}^{H_0} \lambda \tag{2.10}$$

where λ is the detection threshold. Assume that the signal samples are $x(n) \sim CN(0, 2S)$ and the signal to noise ratio is $\rho = S/\sigma^2$. For the Gaussian channel, the test statistic $\Lambda(y)$ follows a non central and a central chi-squared distribution under H_1 and H_0 respectively with $2N$ degrees of freedom [34]. Accordingly, the detection probability and the false alarm probability can be derived as [35],

$$P_d = P[\Lambda(y) > \lambda | H_1] = Q_N(\sqrt{2N\rho}, \sqrt{\lambda}) \tag{2.11}$$

$$P_{fa} = P[\Lambda(y) > \lambda | H_0] = \Gamma(N, \lambda/2) \tag{2.12}$$

where, $\Gamma(a, b) = \frac{1}{\Gamma(N)} \int_b^\infty u^{a-1} exp(-u) du$ is the upper incomplete Gamma function and $\Gamma(.)$ is the Gamma function.$Q_N(a, b)$ defines the generalized Marcum Q-function, and it is formulated as, $Q_N(a, b) = \int_b^\infty u^N exp(-(u^2 + a^2)/2) I_{N-1}(au)/a^{N-1} du$

where $I_\nu(.)$ is the modified Bessel function of the first kind of order $N - 1$.

The expression of P_{fa} in equation (2.12) is independent of ρ (SNR). Under fading, the value of ρ may vary. In this case, the probability of detection in equation (2.11) is given for the instantaneous SNR. Meaning that the resulting probability of detection may be derived by averaging equation (2.11) over the fading statistics.

$$P_d = \int_x Q_N(\sqrt{2N\rho}, \sqrt{\lambda}) f_\rho(\rho) d\rho \tag{2.13}$$

where $f_\rho(\rho)$ is the probability density function (pdf) of SNR under fading. Under Rayleigh fading, ρ has an exponential distribution given as $f_\rho(\rho) = \frac{1}{\bar{\rho}} exp(-\frac{\rho}{\bar{\rho}})$ for $\rho \geq 0$ and $\bar{\rho} = E[\rho]$ is the mean SNR. The authors in [35], derive a closed form expression for the detection probability, given by

$$P_d = exp\left(\frac{-\lambda}{2}\right)\sum_{n=0}^{N-2}\frac{1}{n!}\left(\frac{\lambda}{2}\right)^n + \left(\frac{1+\bar{\rho}}{\bar{\rho}}\right)^{N-1}\left[exp\left(\frac{-\lambda}{2+2\bar{\rho}}\right) - exp\left(\frac{-\lambda}{2}\right)\sum_{n=0}^{N-2}\frac{\left(\frac{\lambda\bar{\rho}}{2(1+\bar{\rho})}\right)^n}{n!}\right] \tag{2.14}$$

where $\bar{\rho}$ is the average SNR as determined by path-loss and the transmitted power of the primary user.

Figures 2.4 and 2.5 provide plots of CROC (Complementary Receiver Operating Characteristic) and ROC (Receiver Operating Characteristic) curves respectively, under AWGN and Rayleigh fading scenarios. $\bar{\rho}$ and N are assumed to be -5 dB and 60, respectively. It is shown that Rayleigh fading degrades the performance of energy detector significantly.

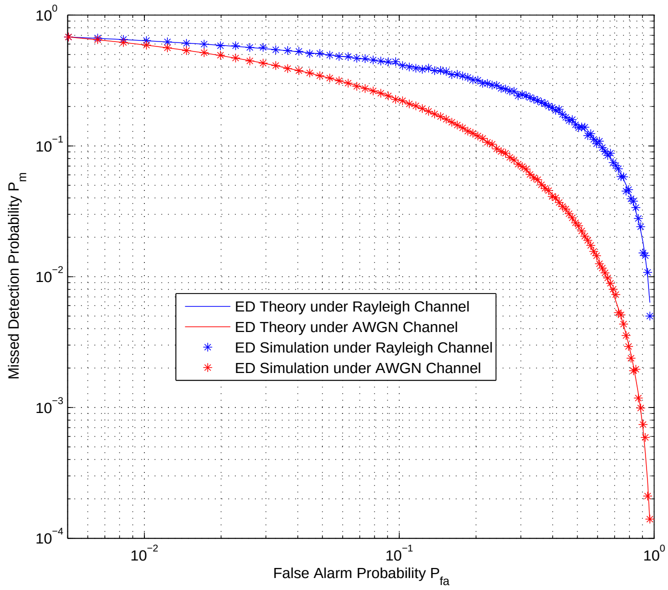

Figure 2.4 Complementary ROC curves for the energy detection under AWGN and Rayleigh fading channels

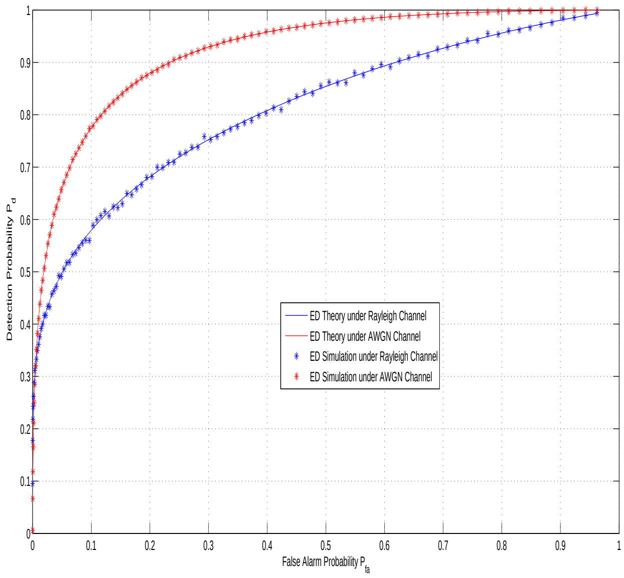

Figure 2.5 ROC curves for the energy detection under AWGN and Rayleigh fading channels

Using the Central Limit Theorem (CLT), the distribution of the test statistic (2.10) can be approximated with a Gaussian distribution for a sufficiently large number of samples. The Probability of False Alarm and the Probability of Detection, can be approximated, respectively, as [36]:

$$P_{fa} = Q\left(\frac{\lambda - N\sigma_w^2}{\sqrt{2N\sigma_w^4}}\right) \tag{2.15}$$

$$P_d = Q\left(\frac{\lambda - N(\sigma_w^2 + \sigma_x^2)}{\sqrt{2N(\sigma_w^2 + \sigma_x^2)^2}}\right) \tag{2.16}$$

where $SNR = \frac{\sigma_x^2}{\sigma_w^2}$.

Figure 2.6 shows the Gaussian approximation of P_{fa} and P_d. The exact curves (using Chi2 distribution) match well with the Gaussian approximation (using CLT) when SNR and N are assumed to be -5 dB and 100. This confirms the validity of the CLT approximation for the distribution of the test statistic for a sufficiently large number of samples.

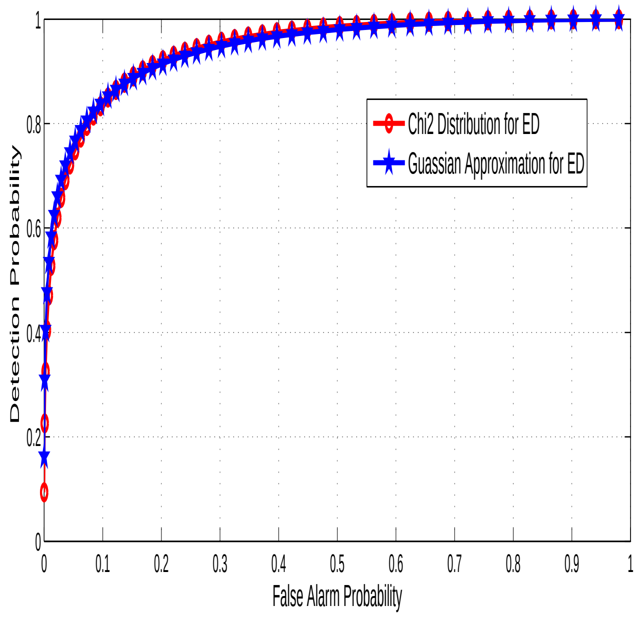

Figure 2.6 ROC curves for the energy detection with Gaussian approximation

2.5.2 Noise Power Uncertainty in Energy Detection

The energy detection (ED) that is adopted when the signal to be detected is completely unknown and no feature detection is therefore possible and this due to the simplicity of its implementation. The performance of the ED has been studied in previous section, where a perfect knowledge of the noise power at the receiver was assumed, allowing thus a proper threshold design. In that case, the ED can work with arbitrarily small values of probability of false alarm and arbitrarily high probability of detection, by using a sufficiently long observation interval, even in low signal-to-noise ratio (SNR) regimes. However, in real systems the detector does not have a perfect knowledge of the noise power level. The noise level is unknown and varies in time, causing critical implications for energy detection. The main problem derived by noise uncertainty is the problem of the so called SNR wall. This SNR wall is defined to as the value of SNR under which the detection may not be possible even for infinitely long observation samples [37].

Setting the threshold too high based on the wrong noise variance, would never allow the signal to be detected. If there is a x dB noise uncertainty, a lower bound for the detectable SNR can be expressed as $SNRwall = 10log_{10}[(x/10) - 1]dB$. This expression is only valid when the signal is not affected by fading. For example, if there is a $0.03dB$ uncertainty in the noise variance, then the signal in $-21dB$ SNR cannot be detected using the energy detector.

2.6 Matched Filter Based Spectrum Sensing

The matched filter based spectrum sensing (MF) is known to be the optimum detector of the transmitted signal, in the sense that it maximizes the SNR at the output of a linear filter used to compute the detection metric [38] [39]. The output of MF is compared with a threshold to decide about the presence or absence of a signal. MF assume a perfect knowledge of the signal structure such as the operating bandwidth, frequency, modulation type, pulse shape, packet format, etc. to demodulate the received signals. A wrong information about the PU s signal will result in a remarkable degradation in the detection performance of MF based spectrum sensing. On the other hand, most wireless communication systems exhibit certain patterns, such as pilot tones, such pilots that primary users embed in their transmission in order to perform synchronization and to allow channel estimation. If the pilot signals are perfectly known to cognitive radio sensor, they will allow a coherent detection which achieves the best possible robustness with respect to noise [40].

The detection is the test of the same binary hypotheses problem in (2.1).

In this binary hypotheses $x(n)$ is the known pilot data, $w(n)$ is a Gaussian noise with variance σ_w^2.

The decision statistic for MF based spectrum sensing can be stated as:

$$T = \sum_{n=1}^{N} y(n)x(n)^{\dagger} \tag{2.17}$$

where $x(n)^{\dagger}$ is the transpose conjugate of the pilot sequence.

the binary decision rule can be expressed as :

$$Decide\ for \begin{cases} H_0 & if\ T < \lambda \\ H_1 & if\ T \geq \lambda \end{cases} \tag{2.18}$$

where λ is the threshold to be compared with the decision statistic T, which is set to meet a desired Pfa.

In [41], it was shown that the decision statistic T follows a Gaussian distribution such:

$$T \sim \begin{cases} N(0, \sigma_n^2 \gamma) & , H_0 \\ N(\gamma, \sigma_n^2 \gamma) & , H_1 \end{cases} \tag{2.19}$$

where $\gamma = \sum_{n=1}^{N} y_p(n)^2$ Therefore, the P_d and the P_{fa} metrics for MF based spectrum sensing can be evaluated as:

$$P_{fa} = Pr(T > \lambda | H_0) = Q(\frac{\lambda}{\sqrt{\sigma_n^2 \gamma}}) \tag{2.20}$$

and

$$P_d = Pr(T > \lambda | H_1) = Q(\frac{\lambda - \gamma}{\sqrt{\sigma_n^2 \gamma}}) \tag{2.21}$$

The MF based spectrum sensing method requires short observation intervals to achieve a good detection performance. Although it is an ideal detection method, it cannot be adopted in a CR scenario if the cognitive user has not the knowledge of the primary interfere waveform. However, as being the optimal detector, its performance can be adopted as reference.

2.7 Cyclostationary Based Spectrum Sensing

In wireless communication, communication signals possess periodicity properties resulting in cyclostationary features. This periodicity may result from modulation, transmitted pilot or preambles. Such statistical periodicity is exploited in cyclostationary detection for cognitive radio by examining cyclic autocorrelation function (CAF) [42] or, equivalently in frequency domain by spectrum correlation function (SCF) [43]. This method is used to determine whether the PU is present or not having the knowledge that the noise does not have any cyclostationary or periodicity properties. Authors in [44] have treated the cyclostationry feature analysis in the general context of signal processing. In the context of CR, such analysis is used for spectrum sensing [45], [46]. The cyclostationary feature detection method can perform better than the energy detection method when cyclostationary features are properly identified. However, this method requires a higher sampling rate to get a sufficient number of samples, which increases the computational complexity. Moreover, the detection performance is largely affected when the spectral correlation density is weak which may be caused by a frequency offset and sample timing error.

2.7.1 Cyclostationary Analysis

We consider a random process $x(t)$. $x(t)$ is defined as a wide sense cyclostationary process if the following equations hold for the mean, E_x, and the auto-correlation function, R_x, of $x(t)$ such

$$E_x(t) = E_x(t + kT) = E[x(t)] \tag{2.22}$$

$$R_x(t, \tau) = R_x(t + kT, \tau) = E[x(t)x^\dagger(t + \tau)] \tag{2.23}$$

where t is the time variable, τ is the lag associated with the auto-correlation function, $x^\dagger(t)$ is the complex conjugate of $x(t)$, and k is an integer. The expression in (2.23) (periodic auto-correlation function) can be expressed in terms of the Fourier series given by

$$R_x(t,\tau) = \sum_{\alpha=-\infty}^{\infty} R_x^{\alpha}(\tau) exp(2\pi j\alpha t) \tag{2.24}$$

where

$$R_x^{\alpha}(\tau) = \lim_{T\to\infty} \int_T x(t+\frac{\tau}{2})x^{\dagger}(t-\frac{\tau}{2})exp(-2\pi j\alpha t)dt \tag{2.25}$$

The expression in (2.25) defines the cyclic auto-correlation function (CAF), and α is called the cyclic frequency parameter.

The spectral correlation density (SCD) function measures the spectral correlation present in a cyclostationary signal. The SCD of a process $x(t)$ is defined as the Fourier transform of the CAF such

$$S_x^{\alpha}(f) = \int_{-\infty}^{\infty} R_x^{\alpha}(\tau) exp(-2\pi j\alpha f\tau)d\tau \tag{2.26}$$

The expression (2.26) is used to detect an cyclostationary features in the cyclic frequency domain meaning that cyclostationary based spectrum sensing for CR exploit this property to decide about the existence of the PU. To compute the expression in (2.26), the cyclic periodogram method is proposed which is given by

$$S_x^{\alpha}(f) = \lim_{T_0\to\infty}\lim_{T\to\infty} \frac{1}{T_0 T} \int_{\frac{-T_0}{2}}^{\frac{T_0}{2}} X_T(t, f+\frac{1}{\alpha})X_T^{\dagger}(t, f-\frac{1}{\alpha})dt \tag{2.27}$$

where $X_T^{\dagger}(t,\theta)$ is the complex conjugate of $X_T(t,\theta)$, and $X_T(t,\theta)$ is the Fourier transform of $x(t)$ given by

$$X_T(t,\theta) = \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} x(u)exp(-2j\pi\theta u)du \tag{2.28}$$

This easy way of computing the SCD such in (2.27) approximates the theoretical SCD if it is computed over a sufficient number of samples.

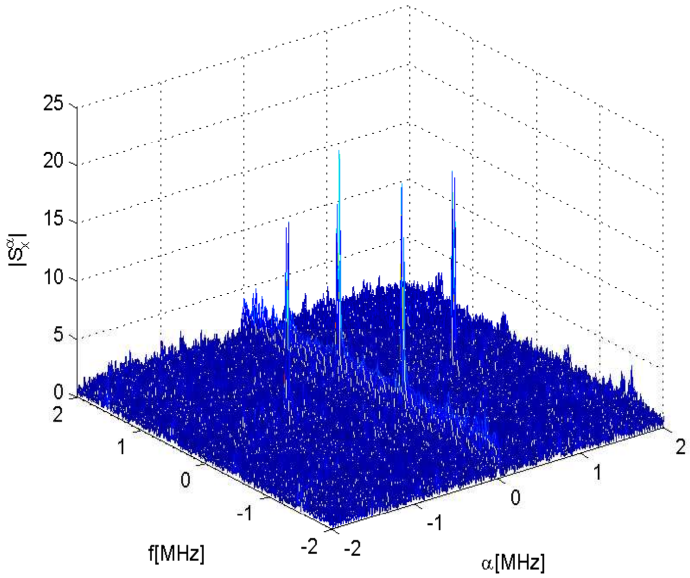

Figure 2.7 Spectral correlation density for BPSK with a signal to noise ratio of $2dB$ estimated over 50 BPSK symbols

Figure 2.7 shows an example of the SCD for BPSK modulated signal estimated over 50 symbols with $SNR = 2dB$. The cyclic frequency components can be identified in figure 2.7 independently of the noise component appearing at $\alpha = 0$. Accordingly, A better estimate of SCD can be used to detect the presence of the PU properly.

2.7.2 Cyclostationary Feature Detection for CR

In this section, we show the use of the SCD to perform spectrum sensing for CR. The hypothesis testing of spectrum sensing can be rewritten considering the SCD as

$$\begin{aligned} H_0 : S_y^{\alpha}(f) = &\quad S_w^{\alpha}(f) \\ H_1 : S_y^{\alpha}(f) = &\quad S_x^{\alpha}(f) \quad + S_w^{\alpha}(f), \end{aligned} \tag{2.29}$$

where, $S_w^{\alpha}(f)$ is the SCD of the additive noise $w(t)$, and $S_x^{\alpha}(f)$ is the SCD of the PU signal $s(t)$. It is known that the noise is not a cyclostationary process, therefore, the SCD of the noise is zero for $\alpha \neq 0$. Meaning that if a cyclic components (for $\alpha \neq 0$) are detected, then a signal is present. Based on this, the statistical test can be derived for this method of the cyclostationary detector as

$$T_{sc} = \sum_{\alpha, \alpha \neq 0} \sum_{f} S_x^{\alpha}(f) S^{\dagger\alpha}_{\ x}(f) \tag{2.30}$$

where $S^{\dagger\alpha}_{\ y}(f)$ is the conjugate of $S_y^{\alpha}(f)$. The statistical decision is then given by

$$d = \begin{cases} H_0 & ; T_{sc} < \lambda \\ H_1 & ; T_{sc} \geq \lambda \end{cases} \tag{2.31}$$

2.7.3 Cyclostationary based spectrum sensing limitations

Practical implementations of cyclostationary based detection algorithms are typically affected by two kinds of limitations: the knowledge of the cycle-frequencies of the signal to be detected and the presence of frequency offsets. In addition, cyclostationary based spectrum sensing methods are generally presenting high complexity, such as, the resolution on the cycle-frequency axis that depends on the oversampling adopted. In order to improve the cycle-frequency resolution a faster ADC and long observation times are required. The adoption of oversampling implies that a certain degree of spectral redundancy is always required for cyclostationary detectors.

2.8 Eigenvalue based Spectrum Sensing

The eigenvalue based detection algorithms are based on the eigenvalues of the covariance matrix of the received signal. The properties of these eigenvalues for the covariance matrix are exploited to decide about the presence of the PU signal. If observed samples are noise-only samples, then all eigenvalues will be equal to the noise power. Otherwise if the signal is present, it will introduce some degree of correlation in the covariance matrix. Meaning that if the primary signal appears as white noise, the eigenvalue based detection may fail.

Like ED detector, eigenvalues based detection is considered a a general-purpose detector: they can be applied to any kind of transmissions and do not require knowledge of any signal parameter or the propagation channel conditions. The main drawback is the complexity of covariance matrix computation as well as the eigenvalue decomposition.

In literature, three algorithms are studied based on the eigenvalues of the covariance matrix of the received signal

- Energy Minimum Eigenvalue ratio detector (EME) is based on the ratio of the received energy in the observed band and the minimum eigenvalue of the covariance matrix of the received signal [47]. The EME test statistic is given as

$$\Lambda_{EME} = \frac{T(N)}{\lambda_{min}} \lessgtr_{H_1}^{H_0} \xi_{EME} \tag{2.32}$$

  where $T(N)$ is the estimated received energy that can be computed as maximum likelihood estimate, as for the ED, or as average on the eigenvalues and λ_{min} is the smallest eigenvalue of the sample covariance matrix.

- Maximum Minimum Eigenvalues ratio detector (MME)is based on the ratio of the maximum and the minimum eigenvalues of the covariance matrix [48] [47]. Hence the test (2.32) can now turn in

$$\Lambda_{EME} = \frac{\lambda_{max}}{\lambda_{min}} \lessgtr_{H_1}^{H_0} \xi_{MME} \tag{2.33}$$

- Arithmetic Geometric Means detector (AGM) is based on the ratio of the arithmetic and geometric mean of the eigenvalues of the covariance matrix [49]. For the first time, this method has been proposed to count the number of the primary users which are transmitting, and then extended to perform also the sensing task.

2.8.1 Computation of the sample covariance matrix

The CR user receives a vector of samples $y(n)$ with length N. For the multiple antennas case, with K receiving antennas, The covariance matrix, can be simply estimated as

$$R_y(N) = \frac{1}{N}\sum_{n=0}^{N-1} y(n)y^{\dagger}(n) \tag{2.34}$$

2.8.2 Implementation of Maximum-Minimum Eigenvalues ratio detector (MME)

Let $x(n)$, $n = 0, 1, ..., MN-1$ be the received signal samples, which is over-sampled with oversampling factor . Let define: $x_i(n) = x(nM + i - 1)$, $i = 1, 2, ..., M$ and $n = 0, 1, ..., N-1$ We note $x(n) = [x_1(n)x_2(n).....x_M(n)]^T$, $n = 0, 1, .$ Choose a smoothing factor L and we compose y such

$$y(n) = [x(n)^T x(n-1)^T....x(n-L-1)^T]^T \tag{2.35}$$

To implement MME we follow this steps

Step1: Compute the sample covariance matrix using 2.34.

Step2: Compute the threshold ξ_{MME}

$$\xi_{MME} = \frac{(\sqrt{N}+\sqrt{ML})^2}{(\sqrt{N}-\sqrt{ML})^2}\left(1 + \frac{(\sqrt{N}+\sqrt{ML})^{-2/3}}{(NML)^{1/6}}F_1^{-1}(1-P_{fa})\right) \tag{2.36}$$

Where F_1 is the Tracy-Wisdom distribution of order 1 [8] and P_{fa} is the required probability of false alarm. The values of the Tracy-Wisdom distribution are given in Table 2.1.

t	-3.90	-3.18	-2.78	-1.91	-1.27	-0.59	0.45	0.98	2.02
$F_1(t)$	0.01	0.05	0.10	0.30	0.50	0.70	0.90	0.95	0.99

Table 2.1 Numerical table for the Tracy-Wisdom distribution of order 1 [8]

Step3: Compute the maximum eigenvalue and minimum eigenvalue of the matrix $R_y(N)$ and denote them as γ_{max} and γ_{min}, respectively.

Step4: Determine the presence of the signal based on the eigenvalues and the threshold: if $T = \frac{\gamma_{max}}{\gamma_{min}} > \xi_{MME}$, then, the signal is present; otherwise, the signal does not exist.

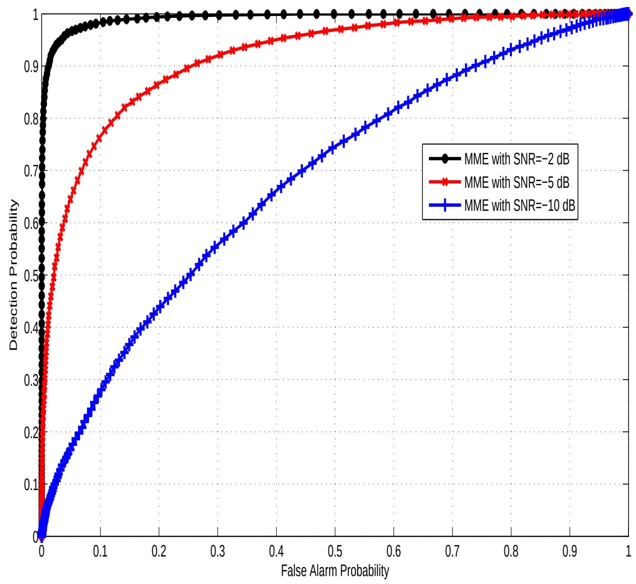

Figure 2.8 ROC curves for MME method under different SNR for 10000 simulation Monte Carlo

In figure 2.8, we depict the performance detection (ROC curve) of the MME ratio based sensing method under different SNR. It can be seen that the detection performance improves when the SNR increase.

2.9 Spectrum Sensing Methods between strength and weakness

The ED method is based on measuring the energy in a given frequency band and decide if it is / is not greater than a threshold. This method is simple and it is chosen for the simplicity of its implementation. However, it presents some limitation such: unknown noise level which is varying in time, causing threshold mismatching and the problem of the the SNR wall. The problem of unknown noise level can be solved by Maximum Minimum Eigen values (MME) Detection method which can estimate the noise and set properly the threshold.

Cyclostationary detection method identifies features of signals using the cyclic autocorrelation function. At low SNR, cyclostationary detection performs better than ED. However, it requires more data to be processed in order to get detailed information about the spectrum which result in a very complex computation.

When the transmitted signal is completely known to the CR receiver, it is seen that the optimum spectrum sensing technique is the matched filter detector and it can be adopted as reference. Matched filter detection needs a prior knowledge of the received signal, such as frequency, bandwidth, modulation type, pulse shaping. Therefore, MF requires a shorter sensing time to achieve a good detection performance compared to cyclostationary detection and energy detection. The main disadvantage is that MF method is able to detect the presence or absence of one specific signal. However ED and cyclostationary method are able to detect several signals in a large spectrum range.

2.10 Other Spectrum Sensing Methods

The spectrum sensing techniques mentioned above are the most important ones proposed in the literature for CR applications. To this list, we can add also other techniques.

2.10.1 Covariance Based Spectrum Sensing

Generally, the covariance of PU signals and the additive noise are different. This difference is exploited to decide about the presence of a PU signal. The authors in [50] have proposed a test statistics based on the sample covariance matrix of the received signal for spectrum sensing. The sample covariance of the received signal $y(n)$ is expressed as

$$\hat{R}_L(\nu, v) = \begin{pmatrix} R(0) & R(1) & ... & R(L-1) \\ R(1) & R(2) & ... & R(L-2) \\ . & . & ... & .. \\ R(L-1) & R(L-2) & ... & R(0) \end{pmatrix} \tag{2.37}$$

For a limited sample size N, the elements of $\hat{R}_L$ are given by

$$R(l) = \frac{1}{N} \sum_{n=0}^{N-1} y(n) y(n-l)^{\dagger} \quad for \quad l = 0, 1, ..., L-1 \tag{2.38}$$

As it was mentioned in the eigenvalue based spectrum sensing, under hypothesis H_0 , the elements of $\hat{R}_L$ are all equal to zero except the diagonal elements which are equal to the noise power. However, under hypothesis H_1, the non diagonal elements would become nonzero. Based on this finding, one could detect the presence of PU signal. In this context, a statistical test have been proposed as follow

$$T = \frac{T_1}{T_2} \tag{2.39}$$

where $T_1 = \frac{1}{L} \sum\limits_{\nu=1}^{L} \sum\limits_{v=1}^{L} |\hat{R}_L(\nu, v)|$ and $T_2 = \frac{1}{L} \sum\limits_{\nu=1}^{L} |\hat{R}_L(\nu, \nu)|$

To decide about the presence of the PU signal, the ratio T is compared to a predefined threshold such

$$decision = \begin{cases} H_0 & ; T < \lambda \\ H_1 & ; T \geq \lambda \end{cases} \quad (2.40)$$

As eigenvalue based spectrum sensing, covariance based sensing assume the existence of correlation in the sensed PU signals. Meaning that if the PU signal appears as white noise, the covariance based detection may also fail.

2.10.2 Wavelet Based Spectrum Sensing

The Wavelet transform is a way of decomposing a signal of interest into a set of basis waveforms, called wavelets, in order to detect singularities or changes in the power spectral density (PSD). Thus, Wavelets are proposed for spectrum sensing by detecting edges in the PSD of a wideband signal [5]. This is done under the assumption that the irregularities in the power spectral density represent the spectral boundaries. This boundaries correspond to transitions from an occupied band to an empty band or vice versa. Once the powers within bands between two edges are estimated and using edge positions, the detection is performed by characterizing the frequency spectrum as occupied or empty. The edge detection of a wideband signal is illustrated in figure 2.9.

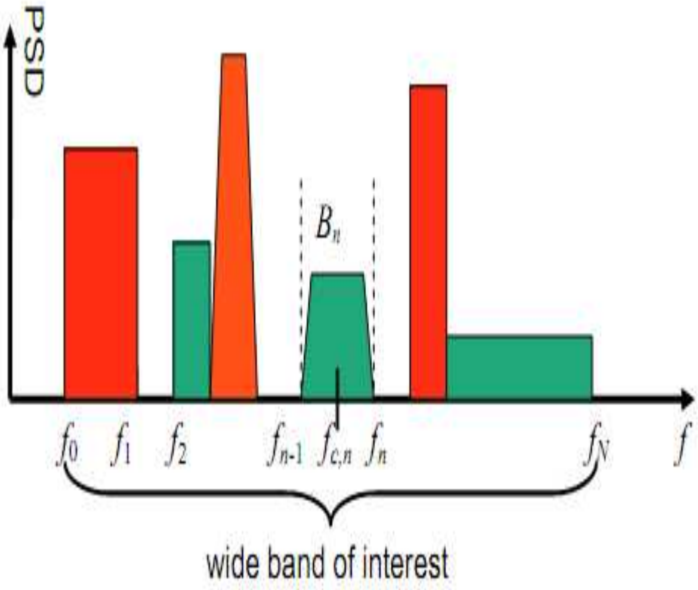

Figure 2.9 The PSD structure of a wideband signal with N bands [5]

For signal detection, the wavelet approach avoids the use of multiple narrowband bandpass filters (BPF) and offers the advantage of simple receiver architecture. However, it requires high sampling rate under the discrete domain.

2.10.3 Filter Bank Based Spectrum Sensing

The application of filter bank for spectrum sensing in CR is proposed in [51]. When a set of bandpass filters are used to estimate the signal spectra for multicarrier communications in CRN. To perform wide spectrum sensing using such filters, the signal power at the outputs of each subcarrier channel is measured. This method presents the inconvenient of the requirement of many bandpass filters in the receiver. Besides, the implementation of the filter bank approach needs a large number of RF components for wideband sensing. However, the filter bank based spectrum sensing resuls in a lower variance when the PSD is low (due to its better response of the bandpass filter).

2.10.4 Multitaper Method Based Spectrum Sensing (MTM)

This method was first proposed by Thomson [52] in order to analyze climate data. The MTM refers to methods for estimating power spectral using the set of orthogonal sequences such as the popular discrete prolate spheroidal (also called Slepian) sequences, as the windows applied to particular periodogram [53]. In his overview paper [54], Haykin had presented a possible application of MTM to CR detection.

The spectrum estimate is given as the average of all particular periodograms using Slepian sequence. These sequences present a property that most of the energy of its Fourier transforms is confined within a limited frequency band for a finite sample size. This nice feature allows to reduce variance of the spectral estimate without energy leakage into adjacent bands. The corresponding power spectrum estimate is given as

$$P_{MTM}(f) = \frac{1}{n} \sum_{i=0}^{n-1} \frac{1}{\lambda_i} | \sum_{k=0}^{N-1} w_i[k] x[k] exp(-2j\pi f k|^2 \tag{2.41}$$

Where n is the used windowing sequences (Slepian sequence), w_i are the i^{th} sequence and λ_i are their corresponding eigenvalues. Therefore, as a Non-parametric method, MTM is considered to be a well suited method for multi-Band spectrum sensing in CRN.

2.10.5 High-order Statistics Based Spectrum Sensing

Higher-order statistics (HoS) based spectrum sensing algorithms have been recently proposed for CR [55] and [56]. In the most of the CR applications, the first-order and second-order statistics have been used to detect the PU signals, whereas HoS algorithms are based on the third and higher order statistics, representing by some basic statistics such as moment and cumulant. HoS based spectrum sensing methods have been used also to make classification of certain kinds of PU waveforms. HoS methods have been considered as alternative solutions to obtain better detection performance compared to the traditional detection methods based on the first and second-order statistics.

2.11 Cooperative Spectrum Sensing

The cooperative spectrum sensing technique can be applied to sense the environment and detect active transmissions if the detection is hard using one user (single detection) due to the low SNR condition and hidden terminal problem. Cooperative sensing can be implemented in two fashions: centralized or distributed. In Centralized cooperative spectrum sensing strategies, a central unit (fusion center) collects sensing information from CR users, as illustrated in figure 2.10, resulting in detection performance improvement [57], in terms of detection probability, reduction of the sensing time, and contrast of the hidden node problem. The sensing data that must be exchanged among the CR nodes is the main cost related to cooperation.

According to the kind of information shared among cognitive nodes, we distinguish two group of cooperative algorithms: hard fusion and soft fusion schemes. In hard fusion scheme cognitive radio users share their local decision, However in soft fusion scheme, they report their measurement (such their received energy) to make a better decision. It has been prouved that, soft fusion can achieve a higher detection probability than hard fusion in detriment of an increase of the data to be transmitted to the fusion center [57].

Since hard fusion requires the transmission of one bit, it fits very well with energy consumption which is a crucial constraint to be minimized. For these reasons, the next section will be focused on the optimization of cooperative spectrum sensing. It is worth to mention that within the next section, we will always refer to centralized cooperative schemes, in which the sensing information is reported to a fusion center, which has a role to merge all the measurements and perform the final decision.

In many scenarios such as in ad hoc cognitive radio networks, deploying a central fusion may not be feasible. Therefore, in order to perform detection, distributed spectrum sensing would be required in such cognitive radio networks as illustrated in figure 2.11. In this scheme, CR users make a local sensing and establish communication links with their own neighbors to locally exchange sensing information among them in order to make their own decisions.

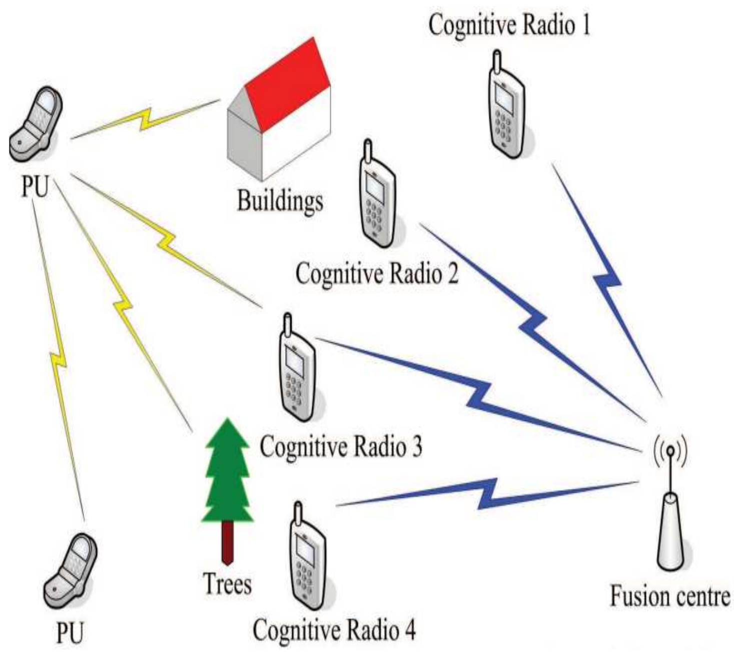

Figure 2.10 Schematic illustration of centralized cooperative spectrum sensing scheme

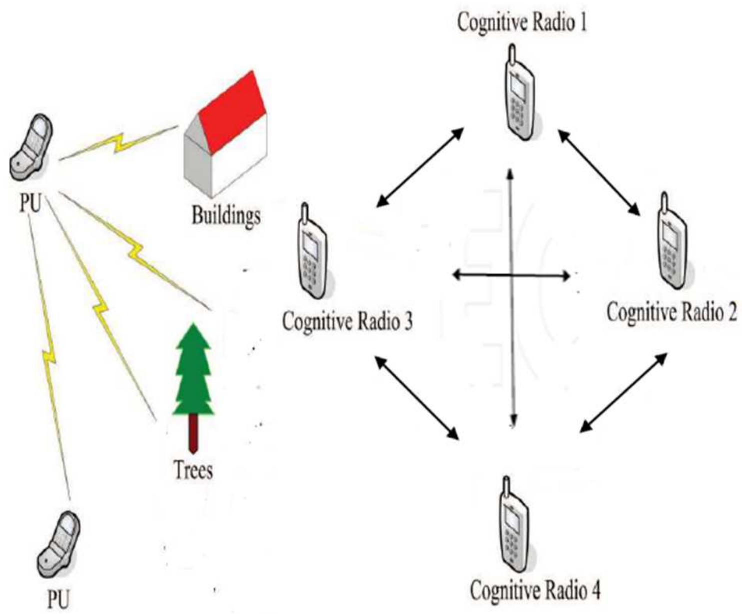

Figure 2.11 Schematic illustration of distributed cooperative spectrum sensing scheme

2.12 Conclusion

This chapter presented the topic of spectrum sensing for CR and explained how spectrum sensing algorithms can be classified. In addition, the chapter provides some comparisons between this algorithms and advantages and drawbacks of each one. Some common used algorithms for sensing are explained and studied. Beside, the conventional energy detector and system model has been discussed. The following chapter will be focused on the study of the cooperative spectrum sensing and throughput optimization problem. Finally, we give a short introduction to cooperative spectrum sensing which will be our concern in the next chapter.

Optimization of Centralized Cooperative Spectrum Sensing for Cognitive Radio Networks

3

One of the most important challenges for a CR system is to perform spectrum sensing in a fading and shadowing environment. Cooperation among multiple CRs helps to enhance the reliability of detection of the primary user (PU) when a single CR performs unreliable decision. In this chapter, we study cooperative spectrum sensing (CSS) in its centralized scheme with different combining rules implemented in the fusion center (FC). The performance of CSS is analyzed under two different operational modes, namely, CPUP (Constant Primary User Protection) and CSUSU (Constant Secondary User Spectrum). Moreover, the relationship between CR users throughput and sensing time is studied for both scenarios and under different combining rules.

3.1 Introduction

As a key technique of spectrum sensing for Cognitive Radio (CR), cooperative sensing was proposed to combat some sensing problems as fading, shadowing, and receiver uncertainty problems [58]. As shown in figure 3.1, CR3 suffers from the receiver uncertainty problem because it is located outside the transmission range of primary transmitter and it is unaware about the existence of primary receiver. So, transmission from CR3 can interfere with the reception at primary receiver. CR2 suffers from multipath and shadow fading causing by building and trees. The main idea of cooperation is to improve the detection performance by taking advantage of the spatial diversity, in order to increase the detection probability to better protect a primary user, and reduce false alarm to utilize the idle spectrum more efficiently.

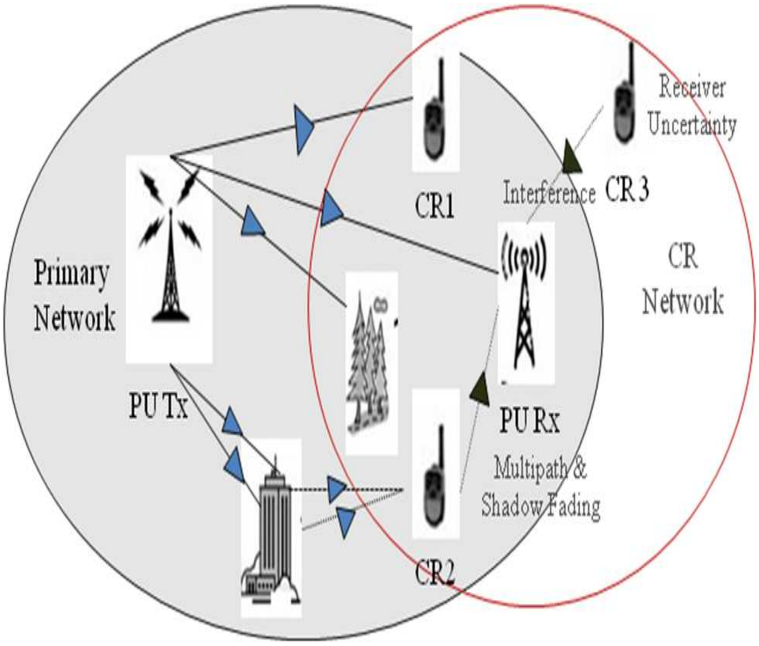

Figure 3.1 Sensing problems (receiver uncertainty, multipath and shadowing).

The three step process of cooperative sensing [59]:

- The fusion center selects a channel or a frequency band of interest for sensing and requests all cooperating CR users to individually perform local sensing.
- All cooperating CR users report their sensing results via the control channel.
- Then the FC fuses the received local sensing information to decide about the presence or absence of signal

To implement these processes seven elements of cooperative sensing are presented from [6] as illustrated in figure 3.2.

- Cooperation models: is concerned with how CR users cooperate to perform sensing.

- Sensing techniques: this element is crucial in cooperative spectrum sensing to sense primary signals by using signal processing techniques.

- Hypothesis testing: in order to decide about the presence or absence of a PU, a statistical test is performed to get binary decision on the presence of PU.

- Control channel and reporting: is used by CR users to report sensing result of each CR users to the FC.

- Data fusion: is a process of combining local sensing data to make cooperation decision.

- User selection: in order to maximize the cooperative gain, this element provides us the way to optimally select the cooperating CR users.

- Knowledge base: means a prior knowledge included PU and CR user location, PU activity, and models or other information in the aim to facilitate PU detection.

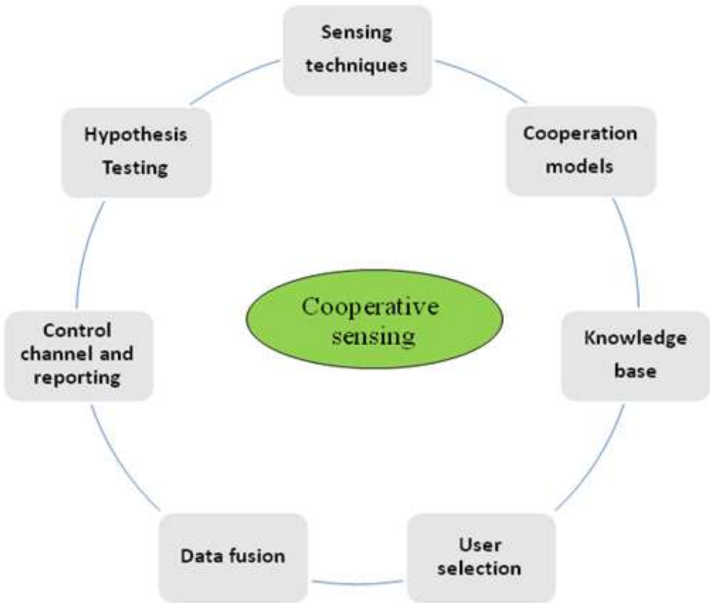

Figure 3.2 Elements of cooperative spectrum sensing [6].

3.2 Related Works

The decision on the presence of PU is achieved by combining all individual decisions of local SUs at a central Fusion Center (FC) using various fusion schemes. These schemes can be classified as hard decision fusion, soft decision fusion, or quantized (softened hard) decision. In [60], a logic OR fusion rule for hard-decision combining was presented for cooperative spectrum sensing. In [61], two simple schemes of hard decision combining are studied: the OR rule and the AND rule. In [62]- [63], another sub-optimal hard decision scheme is used called Counting Rule. In [64] that half-voting rule is shown as the optimal decision fusion rule in cooperative sensing based on energy detection. In [65] a soft decision scheme is described by taking linear combination of the measurements of the various cognitive users to decide between the two hypotheses. However, in [66] collaborative detection of TV transmissions is studied while using soft decision using the likelihood ratio test. It is shown that soft decision combining for spectrum sensing achieves more precise detection than hard decision combining. And this was confirmed in [67] when performing Soft decision combination for cooperative sensing based on energy detection. Some soft combining technologies are discussed in [68], [69] and [70] as square-law combining (SLC), equal gain combining (EGC) and square-law selection (SLS) over AWGN, Rayleigh and Nakagami-m channel.

3.3 Issues in Cooperative Spectrum Sensing

Cooperative spectrum sensing scheme involves many important issues that need to be addressed which are summarized as

- Cooperation Overhead and the Reporting Channel: in designing a cooperative spectrum sensing technique, one must be aware about the overhead associated with the cooperation protocol. The cooperation overhead needs to be minimized to maximize the spectral efficiency. For that, we need to design a reporting channel such that the overhead associated with the cooperation is minimized

- Unreliable received measurement: when using non optimum detector, the received measurement from CR user to FC may be unreliable leading to significant errors in the decisions made by the fusion center.

- Security Issues : In cooperative sensing networks, jammers, and intruders try to disturb the sensing process. Therefore, a cooperative detection needs to resist to different attacks.

- Spatial Limitation: One needs to consider a the geographic range or the spatial limitation for cooperative spectrum sensing. Otherwise, it is possible the cell range covered by the network is large and certain cognitive radio users could use the spectrum without causing harmful interference due to sufficient spatial separation, however, these CR users may not be allowed to transmit since the fusion center had reported that a PU is present in the environment.

3.4 System Model

Consider a cognitive radio network, with K cognitive users (indexed by $i \in \{1, 2, ..., K\}$), and a fusion center to sense the spectrum in order to detect the existence of the PU, suppose that each CR performs local spectrum sensing independently by using N samples, and makes its own observation based on the received signal. Hence, the spectrum sensing problem can be formulated as a binary hypothesis testing problem with two possible hypothesis H_0 and H_1. It is worth to mention that the channels between different CR users and the PU user are considered as independent, meaning that no channel correlation is considered in the system model.

$$\begin{aligned} H_0 : y_i(n) = & \quad w_i(n) \\ H_1 : y_i(n) = & \quad h_i x(n) + w_i(n), \end{aligned} \tag{3.1}$$

Where $x(n)$ is samples of transmitted signal (PU signal), $w_i(n)$ is the receiver noise for the i^{th} SU which is assumed to be an i.i.d. random process with zero mean and variance $\sigma_{w_i}^2$ and h_i is the complex gain of the channel between the PU and the i^{th} SU (AWGN channel); H_0 and H_1 represent whether the signal is absent or present respectively. Using energy detector, i^{th} SU will compare the collected energy E_i with a predefined threshold λ_i to get the decision Δ_i whether the PU channel is occupied or idle [71].

$$E_i = \sum_{n=1}^{N} y_i^2(n) \tag{3.2}$$

$$\Delta_i = \begin{cases} 1 & E_i > \lambda_i \\ 0 & otherwise \end{cases} \tag{3.3}$$

Detection probability $P_{d,i}$ and false alarm probability $P_{f,i}$ of the CR user i are defined as:

$$P_{d,i} = Pr(\Delta_i = 1|H_1) = Pr(E_i > \lambda_i|H_1) \tag{3.4}$$

$$P_{f,i} = Pr(\Delta_i = 1|H_0) = Pr(E_i > \lambda_i|H_0) \tag{3.5}$$

Assuming that $\lambda_i = \lambda$ for all SU, the detection probability, false alarm probability and miss detection probability over AWGN channels can be expressed as follows respectively [72]

$$P_{d,i} = Q_m(\sqrt{2\gamma_i}, \sqrt{\lambda}) \tag{3.6}$$

$$P_{f,i} = \frac{\Gamma(m, \frac{\lambda}{2})}{\Gamma(m)} \tag{3.7}$$

$$P_{m,i} = 1 - P_{d,i} \tag{3.8}$$

Where γ_i is the signal to noise ratio (SNR) for CR node i, $m = TW$ is time bandwidth product, $Q_N(.,.)$ is the generalized Marcum Q-function, $\Gamma(.)$ and $\Gamma(.,.)$ are complete and incomplete gamma function respectively.

According to the central limit theorem, E_i is asymptotically normally distributed if N is large enough. In this case, we can model the statistics of E_i as a Gaussian distribution with mean ($N\sigma_{w_i}^2$) and variance ($2N\sigma_{w_i}^4$) under hypothesis H_0, and as Gaussian distribution with mean ($N[\sigma_{w_i}^2 + \sigma_s^2]$) and variance ($2N[\sigma_{w_i}^2 + \sigma_s^2]2$) under hypothesis H_1.

In this way, for large N (long sensing time), the Probability of False Alarm and the Probability of Detection, can be approximated, respectively, as

$$P_{f,i} = Q(\frac{\lambda - N\sigma_{w_i}^2}{\sqrt{2N\sigma_{w_i}^4}}) \tag{3.9}$$

$$P_{d,i} = Q(\frac{\lambda - N(\sigma_{w_i}^2 + \sigma_s^2)}{\sqrt{2N(\sigma_{w_i}^2 + \sigma_s^2)^2}}) \tag{3.10}$$

3.5 Fusion Rules

This section gives a summary of some fusion rules that are being compared in the study.

3.5.1 Hard fusion rules

In this scheme, each user locally decides on the presence or absence of the primary user and sends a one bit decision to the data fusion center. One advantage of this method is the easiness and that it needs less bandwidth [67]. When binary decisions are reported to the common node, three rules of decision can be used such as **AND**, **OR**, and **majority rule**. Assume that the individual statistics $\Delta(i)$ are quantized to one bit with $\Delta(i) = 0, 1$; is the hard decision from the i^{th} user , 1 means that a signal is present and 0 means that the signal is absent. The **AND** rule decides that a signal is present if all users have detected a signal. The cooperative test using the AND rule can be formulated as fellow

$$\begin{aligned} H_1 : &\quad \sum_{i=1}^{K} \Delta(i) = K \\ H_0 : &\quad \textit{otherwise}, \end{aligned} \tag{3.11}$$

The **OR** rule decides that a signal is present if any of the users detects a signal. Hence, the cooperative test using the OR rule can be formulated as fellow:

$$\begin{aligned} H_1 : &\quad \sum_{i=1}^{K} \Delta(i) \geqslant 1 \\ H_0 : &\quad \textit{otherwise}, \end{aligned} \tag{3.12}$$

The third rule is the **voting rule** that decides on the signal presence if at least M of the K users have detected a signal with $1 \leqslant M \leqslant K$. The test is formulated as:

$$\begin{aligned} H_1 : &\quad \sum_{i=1}^{K} \Delta(i) \geqslant M \\ H_0 : &\quad \textit{otherwise}, \end{aligned} \tag{3.13}$$

A majority decision is a special case of the voting rule when $M = K/2$, the same as the **AND** and the **OR** rule which are also special cases of the voting rule for $M = K$ and $M = 1$ respectively. Cooperative detection probability Q_d and cooperative false alarm probability Q_f are defined as:

$$\begin{aligned} Q_d: \quad & Pr\{\Delta = 1|H_1\} = Pr\{\sum_{i=1}^{K} \Delta(i) \geqslant M|H_1\} \\ Q_f: \quad & Pr\{\Delta = 1|H_0\} = Pr\{\sum_{i=1}^{K} \Delta(i) \geqslant M|H_0\}, \end{aligned} \tag{3.14}$$

Where Δ is the final decision. Note that **OR** rule corresponds to the case of $M = 1$, so

$$Q_d = 1 - \prod_{i=1}^{K}(1 - P_{d,i}) \tag{3.15}$$

$$Q_f = 1 - \prod_{i=1}^{K}(1 - P_{f,i}) \tag{3.16}$$

And the AND rule can be evaluated by setting $M = K$.

$$Q_d = \prod_{i=1}^{K} P_{d,i} \tag{3.17}$$

$$Q_f = \prod_{i=1}^{K} P_{f,i} \tag{3.18}$$

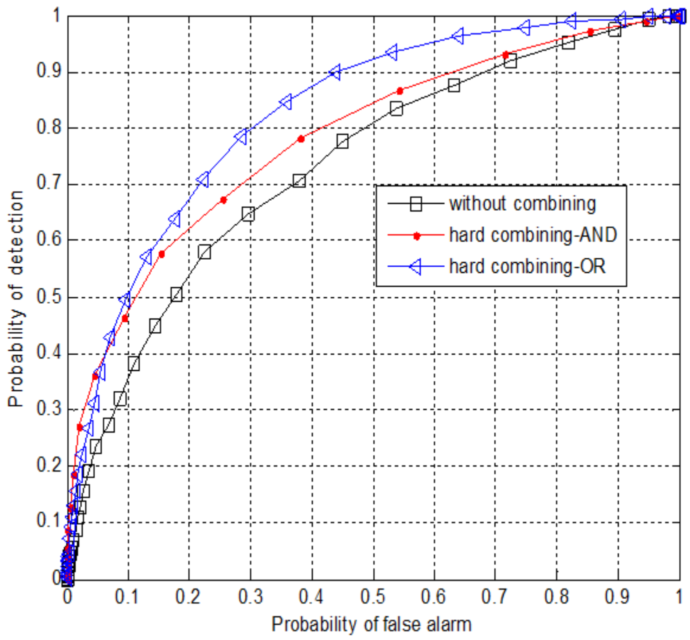

Figure 3.3 ROC for the hard fusion rules under AWGN channel, $SNR = -2dB$, $K = 3$ users, and energy detection over 1000 samples.

As shown in figure 3.3, the OR rule has better detection performance than AND rule which provides slightly better performance at low Pfa than the OR, because the data fusion center decide in favor of H_1 when at least one CR user detects PU signal, however in AND rule, to decide the presence of primary user, all CR users must detect the PU signal. The result shows that increasing number of users improves the detection performance comparing with the non-cooperative case.

3.5.2 Soft data fusion

In soft data fusion, CR users forward the entire sensing data result to the center fusion without performed any local decision and the decision is made by combining these results at the fusion center by using appropriate combining rules such as square law combining (SLC), maximal ratio combining (MRC) and square law selection (SLS). Soft combination provides better performance than hard combination, but it requires a wider bandwidth for the control channel [73]. It also requires more overhead than the hard combination scheme [67].

3.5.2.1 Square Law Combining (SLC)

It is one of the simplest soft methods, a linear soft combining scheme [74], in this method the estimated energy in each node is sent to the center fusion and there they will be added together. Then this summation is compared to a threshold to decide on the existence or absence of the primary user and a decision statistic is given by

$$E_{SLC} = \sum_{i=1}^{K} E_i \tag{3.19}$$

Where E_i designs the statistic from the i^{th} SU. The detection probability and false alarm probability are formulated as follow

$$Q_{d,SLC} = Q_{mK}(\sqrt{2\gamma_{SLC}}, \sqrt{\lambda}) \tag{3.20}$$

$$Q_{f,SLC} = \frac{\Gamma(mK, \lambda/2)}{\Gamma(mK)} \tag{3.21}$$

where $\gamma_{SLC} = \sum_{i=1}^{K} \gamma_i$ and γ_i is the received SNR at cognitive radio i.

3.5.2.2 Maximum Ratio Combining (MRC)

The difference between this method and the SLC is that in this method the energy received in the center fusion from each user is multiplied to a weight and then added. This weight depends on the distance from the SU and PU and the SNR of the channel that separates them.

$$\gamma_{MRC} = \sum_{i=1}^{K} w_i \gamma_i \tag{3.22}$$

Over AWGN channels, the probabilities of false alarm and detection under the MRC diversity scheme can be given by

$$Q_{d,MRC} = Q_m(\sqrt{2\gamma_{MRC}}, \sqrt{\lambda}) \tag{3.23}$$

$$Q_{f,MRC} = \frac{\Gamma(m, \lambda/2)}{\Gamma(m)} \tag{3.24}$$

3.5.2.3 Selection Combining (SC)

In the SC scheme, the fusion center selects the branch with highest SNR, and the decision statistic is given by

$$\gamma_{SC} = max(\gamma_1, \gamma_2, ..., \gamma_K) \tag{3.25}$$

Over AWGN channels, the probabilities of false alarm and detection under the SC diversity scheme can be given by

$$Q_{d,SC} = Q_m(\sqrt{2\gamma_{SC}}, \sqrt{\lambda}) \tag{3.26}$$

$$Q_{f,SC} = \frac{\Gamma(m, \lambda/2)}{\Gamma(m)} \tag{3.27}$$

Figure 3.4 shows the ROC curves of different soft combination schemes under AWGN channel; we observe from this figure that the MRC scheme exhibits the best detection performance but it requires channel state information. The SLC scheme does not require any channel state information and still present better performance than SC, the optimal scheme is SLC when any information of channel is given.

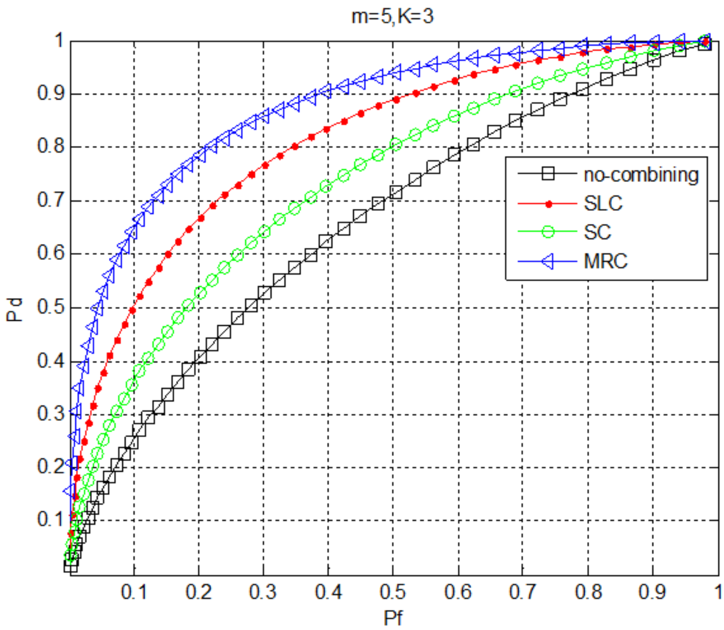

Figure 3.4 ROC for soft fusion rules under AWGN channel with K=3 users, and energy detection with m=5.

3.5.3 Quantized data fusion

In this scheme, we try to realize a tradeoff between the complexity and overhead, instead of one bit hard combining where there is only one threshold dividing the whole range of the detected energy into two regions, a better detection performance can be obtained if we increase a number of threshold to get more regions of observed energy.

In [67], the two-bit hard combining scheme is proposed when dividing the whole range of the detected energy into four regions, in the following, we propose a three-bit combining scheme.

In the three-bit scheme, seven threshold $\lambda_1, \lambda_2 ..., \lambda_7$, divide the whole range of statistics into 8 regions as it is depicted in figure 3.5. Each CR user forwards 3 bit information to point out the region of the observed energy. We decide about the presence of the signal if any one of the observed energies falls in region 7, and for all regions we define some weights as a decision criterion ($w_7, w_6..., w_0$), so nodes that observe higher energies in upper regions have greater weights than nodes that observe lower energies in lower regions.

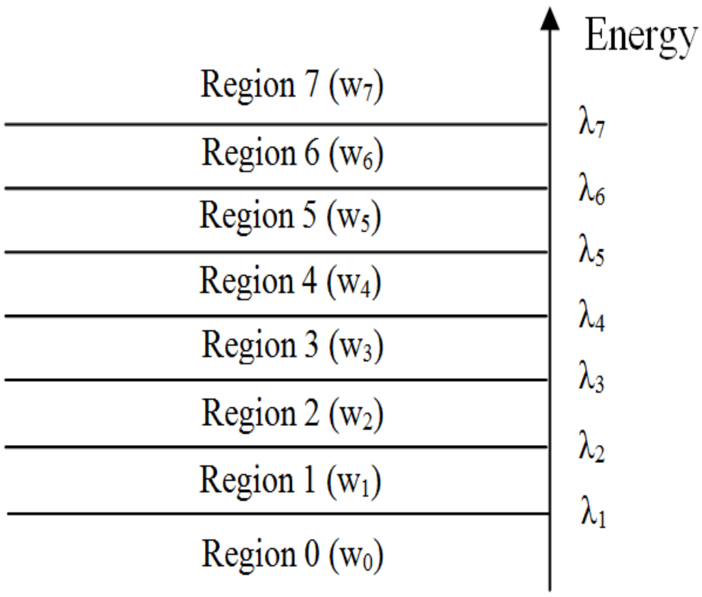

Figure 3.5 Principle of three-bit hard combination scheme.

The presence of the signal of interest is decided at the decision center by using the following equation

$$\sum_{i=1}^{7} w_i n_i \geq K \tag{3.28}$$

where K is the total number of nodes in the network, n_i is the number of observed energies falling in region i and w_i is the weight value of region i. Figure 3.6 shows a ROC curves for quantized data fusion with 2-bit and 3-bit hard combination, this figure indicates that the proposed 3-bit hard combination scheme presents much better performance that the 2-bit hard combination scheme at the cost of one more bit of overhead for each CR user, this scheme can achieve a good trade-off between detection performance and complexity.

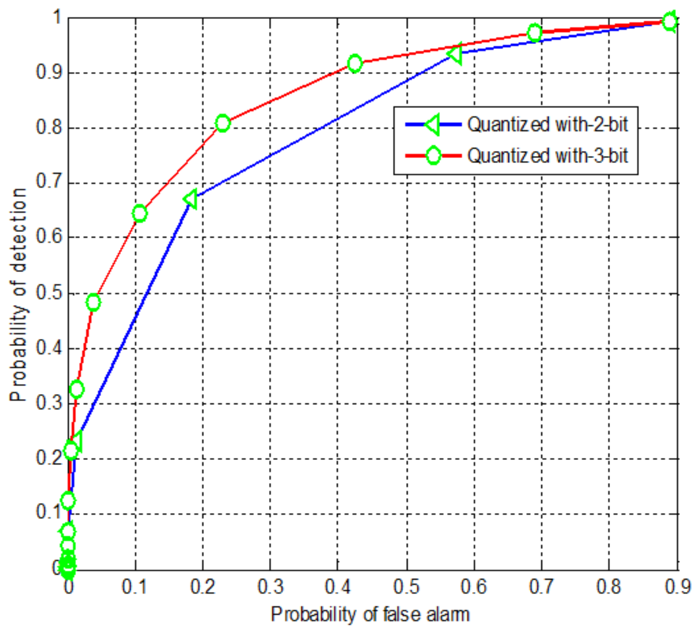

Figure 3.6 ROC curves for quantized data fusion under AWGN channel with $SNR = -2dB$, $K = 3$ CR users and $N = 1000$ samples.

Figure 3.7 shows a ROC curves for the fusion rules under AWGN channel. As the figure indicates , all fusion methods outperform single node sensing, the soft combining scheme representing here with the SLC rule outperforms the hard and quantized combination at the cost of control channel overhead, the 3-bit quantized combination scheme shows a comparable detection performance to the SLC with less complexity and overhead.

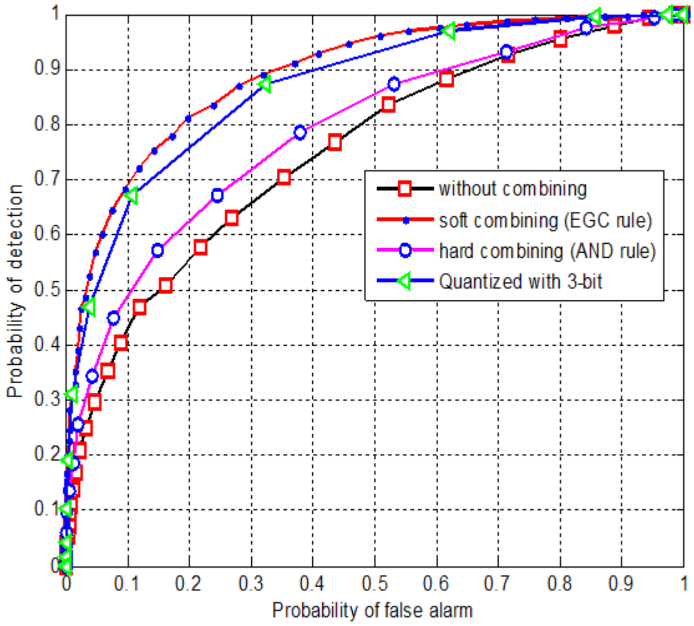

Figure 3.7 ROC for combining fusion rules under AWGN channel with $K = 3$ users, $SNR = -2dB$ using energy detection with $N = 1000$ samples.

3.6 Cognitive Radio Transmission Scenarios

In this section, the sensing performance of a CR and a CR network is evaluated under two different operational modes, CPUP (Constant Primary User Protection) and CSUSU (Constant Secondary User Spectrum Usability) transmission modes. The CPUP mode guarantees a minimum level of interference to the PU (we fix the probability of detection at the required level) and try to find a trade-off between the probability of false alarm and the sensing time at a particular SNR. The CSUSU scenario is taken from the CR perspective; by keeping fixed the usability of unoccupied bands at a certain level (we fix the Probability of false alarm at lower values) and try to find the trade-off between the probability of detection and the sensing time at a particular SNR.

For this study, the energy detector (ED) is used as a method for spectrum sensing. It has been seen that the statistics of the energy is asymptotically normally distributed if N (sample size) is large enough (the central limit theorem

(CLT)). In this case, we can model the statistics of the energy as a Gaussian distribution. In this section, all derived probabilities are based on CLT.

Under CPUP, we can express P_f in terms of $\bar{P_d}$ and N as

$$P_f = Q(Q^{-1}(\bar{P_d})(1 + SNR) + SNR\sqrt{\frac{N}{2}}) \tag{3.29}$$

where $\bar{P_d}$ is the required probability of detection under CPUP and $SNR = \sigma_s^2/\sigma_w^2$ is the signal to noise ratio of the PU signal at the CR.

Under CSUSU, we can express P_d in terms of $\bar{P_f}$ and N as

$$P_d = Q(\frac{Q^{-1}(\bar{P_f}) - SNR\sqrt{\frac{N}{2}}}{1 + SNR}) \tag{3.30}$$

where $\bar{P_f}$ is the required probability of false alarm under CSUSU.

3.6.1 Combining Rules for CSS under CR Transmission Scenarios

The CSS aims to improve detection sensitivity, especially when working under low signal to noise ratio (such as the SNR level proposed by 802.22 working group, which is $-22dB$ [75]). In the following subsections, we will study three different combining rules for CSS: hard combining rule (OR and AND rule), soft combining rule (Equal Gain combining rule) and quantized combining rule (two-bit quantized combining rule). For each combining rule, we will express the CR network probability of false alarm Q_f in terms of the required overall probability of detection $\bar{Q_d}$ and N under CPUP scenario. We will also formulate the CR network probability of detection Q_d in terms of the required overall probability of false alarm $\bar{Q_f}$ and N under CSUSU scenario.

3.6.1.1 Hard fusion rule under CPUP and CSUSU scenarios

The CR users network probabilities can be stated under CPUP and CSUSU scenarios. The overall probabilities under CPUP scenario where the probability of detection is fixed at a satisfactory level, can be expressed as

- **Under OR rule**.

$$Q_f = 1 - \prod_{i=1}^{K}(1 - Q(Q^{-1}(1-(1-\bar{Q_d})^{\frac{1}{K}})(1+SNR_i) + SNR_i\sqrt{\frac{N}{2}}) \tag{3.31}$$

- **Under AND rule**.

$$Q_f = \prod_{i=1}^{K} Q((Q^{-1}(\bar{Q_d})^{\frac{1}{K}})(1+SNR_i) + SNR_i\sqrt{\frac{N}{2}}) \tag{3.32}$$

Similarly for the CSUSU scenario, the overall false alarm probability of the CR users network is set constant at $\bar{Q_f}$, and the overall probability of detection can be expressed as

- **Under OR rule**.

$$Q_d = 1 - \prod_{i=1}^{K}(1 - Q(\frac{Q^{-1}(1-(1-\bar{Q_f})^{\frac{1}{K}}) - SNR_i\sqrt{\frac{N}{2}}}{1+SNR_k})) \tag{3.33}$$

- **Under AND rule**.

$$Q_d = \prod_{i=1}^{K} Q(\frac{Q^{-1}(\bar{Q_f}^{\frac{1}{K}}) - SNR_i\sqrt{\frac{N}{2}}}{1+SNR_i}) \tag{3.34}$$

3.6.1.2 Soft fusion rule under CPUP and CSUSU scenarios

Equal Gain Combining (EGC) or Square Law Combining (SLC) (as described in section 3.5.2.1), is one of the simplest linear soft combining rules. In this method the estimated energy in each node is sent to the fusion center in which they will be added together. The summation is compared to a threshold to decide on the existence or absence of the PU. The decision statistic is given by

$$E_{EGC} = \sum_{i=1}^{K} E_i \tag{3.35}$$

where E_i denotes the statistic from the i^{th} CR user. It was proved that E_{EGC} has a chi-square distribution with $N * K$ degree of freedom. According to the central limit theorem, the distribution of E_{EGC} can be approximated to a Gaussian distribution if the product $N * K$ is large enough. In this case, the overall detection probability and false alarm probability for CR users network can be written as follows

$$Q_d = Q(\frac{\lambda - N(\sum_{i=1}^{K} \sigma_{w_i}^2 + \sigma_s^2)}{\sqrt{2N(\sum_{i=1}^{K} \sigma_{w_i}^2 + \sigma_s^2)^2}}) \tag{3.36}$$

$$Q_f = Q(\frac{\lambda - N\sum_{i=1}^{K} \sigma_{w_i}^2}{\sqrt{2N\sum_{k=1}^{K} \sigma_{w_i}^2}}) \tag{3.37}$$

Therefore, we can derive the CR network probabilities under CPUP and CSUSU scenarios based on EGC combining rule.

In CPUP, we fix the probability of detection at $\bar{Q_d}$, and the Q_f is expressed as:

$$Q_f = Q(Q^{-1}(\bar{Q_d})(1 + SNR) + SNR\sqrt{\frac{NK}{2}}) \tag{3.38}$$

Similarly, Q_d under CSUSU when fixing the probability of false alarm at $\bar{Q_f}$ can be expressed as:

$$Q_d = Q(\frac{Q^{-1}(\bar{Q_f}) - SNR\sqrt{\frac{NK}{2}}}{1 + SNR}) \tag{3.39}$$

3.6.1.3 Quantized fusion rule under CPUP and CSUSU scenarios

In this section, we consider the two-bit combining rule to be studied under CPUP and CSUSU scenarios. The two-bit combining rule is proposed in [67] when dividing the energy region into four sub-regions and assigns different weights to each sub-region. Instead of one bit hard combining, two bits are used to indicate the decision. The presence of the signal of interest is decided at the FC when $\sum_{i=0}^{3} w_i n_i \geqslant L^2$, where n_i is the number of observed energies falling in region i. Different weights are allocated for the four sub-regions, $w0 = 0, w1 = 1, w2 = L, andw3 = L^2$. In this case, the PU is declared present if any one of the observed energies falls in region 3, or L ones fall in region 2, or L^2 ones fall in region 1, (L is a parameter to be optimized). The scheme is shown in figure 3.8, where $\lambda_1, \lambda_2, and\ \lambda_3$ are the thresholds for the energy detector.

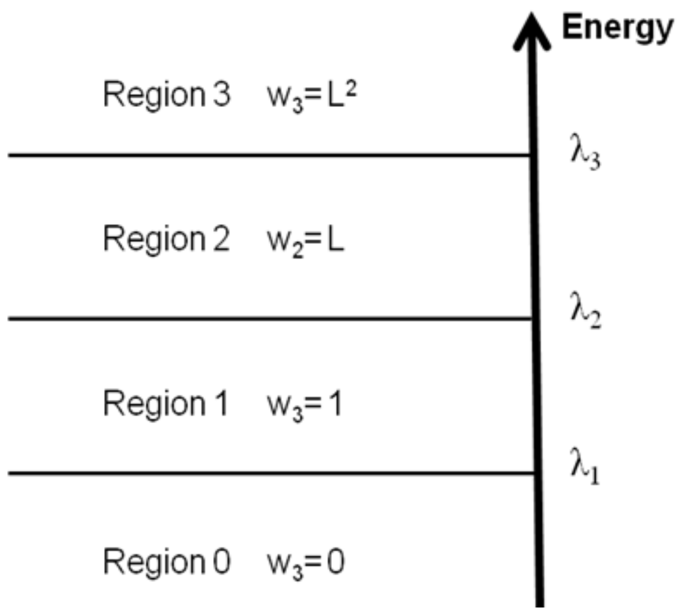

Figure 3.8 The 4 energies regions for the two-bit combination scheme.

For the two-bit combining rule with K cooperative users, the Q_f is given as

$$(1-Q_f)(1+\rho)^K = \sum_{i=0}^{L^2-1} \begin{pmatrix} K \\ i \end{pmatrix} \left\{ \sum_{j=0}^{J_i} \begin{pmatrix} i \\ j \end{pmatrix} (1-\beta_{f1})^{i-j} (\beta_{f2} - \beta_{f1}\beta_{f2})^j \right\} \rho^i \tag{3.40}$$

with $J_i = min\left\{\lfloor \frac{L^2-1-iw_1}{w_2-w_1} \rfloor, i\right\}$; $\beta_{f1} = \frac{P_{f2}}{P_{f1}}$; $\beta_{f2} = \frac{P_{f3}}{P_{f2}}$; and $\rho = \frac{P_{f1}}{1-P_{f1}}$

P_{fi} is the false alarm probability in region i and β_{f1}, β_{f2} are parameters to be optimized. The optimal values of β_{f1}, β_{f2} can be found numerically by maximizing the overall detection probability of the CR network Q_d given by

$$Q_d = 1 - \sum_{i=0}^{L^2-1} \left(\begin{pmatrix} K \\ i \end{pmatrix} (1-P_{d1})^{K-1} \left\{ \sum_{j=0}^{J_i} \begin{pmatrix} i \\ j \end{pmatrix} (P_{d1} - P_{d2})^{i-j} (P_{d2} - P_{d3})^j \right\} \right) \tag{3.41}$$

where P_{di} is the detection probability in region i. Under CPUP scenario, we fix the probability of detection at $\bar{Q_d}$, and we can rewrite (3.41) as:

$$(1-\bar{Q_d})(1+\rho)^K = \sum_{i=0}^{L^2-1} \begin{pmatrix} K \\ i \end{pmatrix} \left\{ \sum_{j=0}^{J_i} \begin{pmatrix} i \\ j \end{pmatrix} (1-\beta_{d1})^{i-j} (\beta_{d2} - \beta_{d1}\beta_{d2})^j \right\} \rho^i \tag{3.42}$$

with $\beta_{d1} = \frac{P_{d2}}{P_{d1}}$; $\beta_{d2} = \frac{P_{d3}}{P_{d2}}$; and $\rho = \frac{P_{d1}}{1-P_{d1}}$

In (3.42), β_{d1}, β_{d2}, and L are parameters to be optimized. Similarly to [67], these parameters can be found by minimizing the overall false alarm probability given in (3.44) under CSUSU scenario. For our simulations, we fix the values of β_{d1}, β_{d2}, K, and L. The parameter ρ can be found numerically by solving the equation (3.42). Then we can find P_{d1}, P_{d2} and P_{d3} based on the values of ρ ,β_{d1} and β_{d2}. Finally, the false alarm probability in each region can be computed as:

$$P_{fi} = Q(Q^{-1}(P_{di})(1+SNR) + SNR\sqrt{\frac{N}{2}}) \tag{3.43}$$

The overall false alarm probability of networks can be written as:

$$Q_f = 1 - \sum_{i=0}^{L^2-1}\left(\begin{pmatrix} K \\ i \end{pmatrix}(1-P_{f1})^{K-1}\left\{\sum_{j=0}^{J_i}\begin{pmatrix} i \\ j \end{pmatrix}(P_{f1}-P_{f2})^{i-j}(P_{f2}-P_{f3})^{j}\right\}\right) \tag{3.44}$$

Similarly, under CSUSU and for a fixed false alarm probability $\bar{Q}_f$ and optimized values of β_{f1}, β_{f2} and L, we can use equation (3.40) to search ρ numerically. Then we find P_{f1}, P_{f2} and P_{f3} based on ρ, β_{f1}, β_{f2} given in (3.40). After that we compute the detection probability P_{di} in each region based on the following expression:

$$P_{di} = Q\left(\frac{Q^{-1}(P_{fi}) - SNR\sqrt{\frac{N}{2}}}{1+SNR}\right) \tag{3.45}$$

Finally, we can conclude the overall detection probability of networks by using the expression (3.41).

3.6.2 Performances detection of CSS under CPUP and CSUSU Transmission mode

In this section, we have performed MATLAB simulations to study the performances detection of CSS under CPUP and CSUSU Transmission mode. It should be noted that all selected simulation parameters are based on the IEEE 802.22 WRAN. The frame duration (T) is set to 100 ms and the bandwidth channel of the PU is fixed to be $6MHz$. The signal to noise ratio SNR is put to $-18dB$ for all K CR users. In a first step we will evaluate the detection performances of the different schemes under the CPUP and CSUSU scenarios as a function of the sensing time. Figure 3.9 shows the overall false alarm probability curves of the OR hard combining rule, the AND hard combining rule, the two-bit quantized combining rule and EGC soft combining rule over AWGN channel under CPUP scenario. For the two-bit quantized combining rule, we set $L = 2$, $\beta_{d1} = 0.6$ and $\beta_{d2} = 0.3$. Under CPUP, we fix the network detection probability to 0.95 with $K = 10$ CR users.

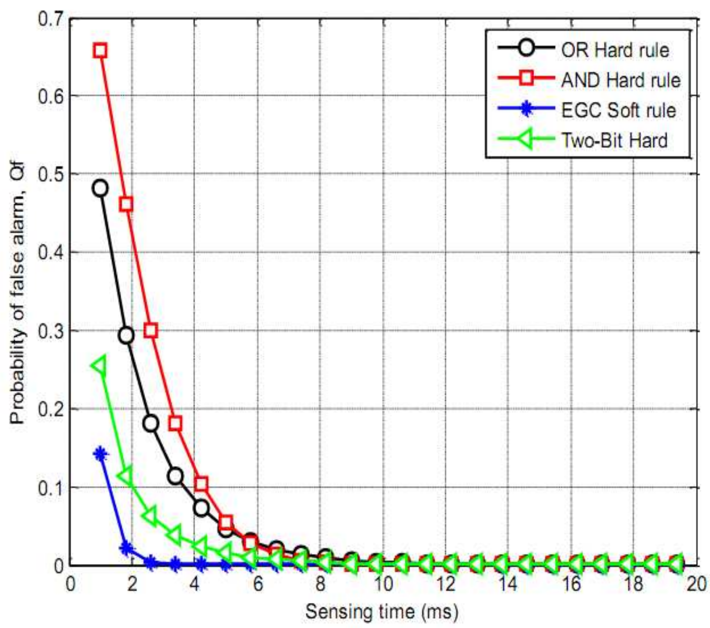

Figure 3.9 Probability of false alarm versus sensing time under CPUP scenario using different combining rules (K=10, $\bar{Q}_d = 0.95$)

Figure 3.9 indicates that the two-bit quantized combining rule exhibits much better performance than the one-bit quantized combining rule in terms of probability of false alarm to the detriment of one bit of overhead, the EGC soft combining rule has better performance comparing to other schemes at the expense of bandwidth overhead. Therefore, the two-bit quantized combining rule achieves a good trade-off between performance detection and overhead.

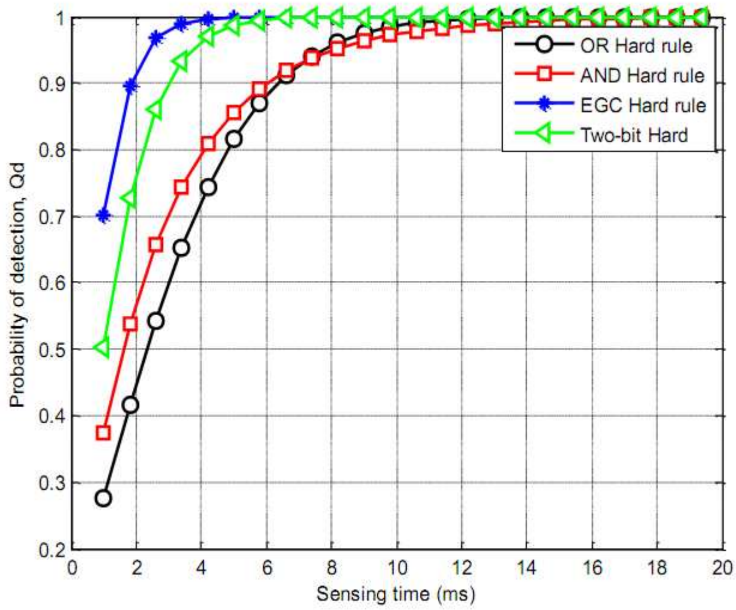

Figure 3.10 Probability of detection versus sensing time under CSUSU scenario using different combining rules (K=10, $\bar{Q_f} = 0.05$)

In figure 3.10, we plot the overall detection probability curves of the OR hard combining rule, AND hard combining rule, the quantized two-bit combining rule, and EGC soft combining rule over AWGN channel under CSUSU scenario. For the two-bit quantized combining rule, we set $L = 2$, $\beta_{f1} = 0.25$ and $\beta_{f2} = 0.1$. Under CSUSU, we fix the network false alarm probability to 0.05.

As it was shown previously under CPUP, the two-bit quantized combining rule exhibits much better performance that the one-bit quantized combining rule in terms of probability of detection at the expense of one bit of overhead. The EGC soft combining rule outperforms the other rules however it requires more bandwidth overhead of reporting channel. In this case, the two-bit quantized combining rule achieves a good trade-off between performance detection and overhead.

3.7 Throughput Optimization for Cooperative Spectrum Sensing in CRN

Through the mechanism of spectrum sensing, we aim to get the optimal sensing time, in order to maximize the user data throughput of the CR network. The optimum capacity throughput of the CR users according with the requirements about the sensing accuracy must be searched.

In [76], the CR users network throughput is maximized subject to adequate protection provided to PUs by determining the optimal k-out-of-N combining rule. The sensing-throughput relationship is also analyzed. In [77], optimal multi-channel cooperative sensing algorithms are considered to maximize the CR users network throughput subject to per channel detection probability constraints. The problem is solved by an iterative algorithm. In [78] the optimal sensing duration is studied to maximize the achievable throughput for the secondary networks. The motivation behind throughput Optimization, is to provide solutions to realize a tradeoff between the performance of CSS in terms of detection and throughput and overhead in terms of the reporting channel bandwidth and complexity.

3.7.1 Throughput Optimization under CR Transmission Scenarios

In this section, we analyze the relationship between the CR users capacity (throughput) and sensing capabilities for CSS under the CPUP and CSUSU scenarios. For this study, we consider a TDM based system in which each frame consists of one sensing slot of duration (t) plus one data transmission slot of (T-t), with T is the total frame duration. The CR users network might operate at the PU licensed band if the fusion center decides that the channel is idle, this occurs in two cases:

1- When the PU is inactive and the channel is declared idle, the probability of that state can be written as: $P(H_0|H_0) = P(H_0)(1 - P_f)$.

2- When the PU is active and the channel is declared idle, the probability of that state can be written as: $P(H_0|H_1) = P(H_1)(1 - P_d)$.

The channel utilization or the normalized capacity of the system can be expressed as [76]

$$C = \left(1 - \frac{t}{T}\right)\left[(1 - P_f)P(H_0) + (1 - P_d)P(H_1)\right] \tag{3.46}$$

The objective is to determine the optimal sensing time (t) such that the CR users network throughput is maximized. In the case of CSS, this objective can be formulated as follows:

$$maxC = \left(1 - \frac{t}{T}\right)\left[(1 - Q_f)P(H_0) + (1 - Q_d)P(H_1)\right] \tag{3.47}$$

Subject to:

$$\begin{aligned} &0 < t < T \\ &Q_d \geqslant \bar{Q}_d \\ &Q_f \leqslant \bar{Q}_d \end{aligned} \tag{3.48}$$

Referring to [79], the optimization problem presented in (3.47) is a convex optimization problem if it satisfies the constraint $Q_f(t) \leqslant \frac{1}{2}$, which is the case for practical CR systems.

Thereafter, we can find the optimal $t^* = argmax(C)$ numerically for K number of CR users and respecting the constraints given in (3.48) under the two scenarios CPUP and CSUSU for different combining rules presented in section 3.6.1.

3.7.2 Capacity Optimization detection for CSS under CPUP and CSUSU Transmission mode

In this section, we have performed MATLAB simulations to evaluate the optimization problem (3.47). It should be noted that all selected simulation parameters are based on the IEEE 802.22 WRAN. The frame duration (T) is set to 100 ms and the bandwidth channel of the PU is fixed to be $6MHz$. The signal to noise ratio SNR is put to $-18dB$ for all K CR users. In this section, we present simulations results to show the relationship between CR users network throughput and the sensing time for cooperative spectrum sensing. The PU absent probability on the channel is $P(H_0) = 0.8$, and The PU present probability on the channel is $P(H_1) = 0.2$.

Figure 3.11 shows the normalized capacity of the CR user network under CPUP scenario using different combining rules. In figure 3.9, it was observed that that false alarm probability decreases with increasing the sensing time which suppose to increase the CR users capacity. However, figure 3.11 points out that increasing the sensing time does not result in a monotonic increasing of the throughput of the CR users networks. There is an optimal sensing time at which the CR users network throughput is maximized. It is seen that the EGC soft combining rule exhibits the shortest sensing time with the highest value of capacity comparing to the other combining rules. The two-bit quantized combining rule outperforms the one-bit quantized combining rule in terms of optimal sensing time and the corresponding maximum capacity.

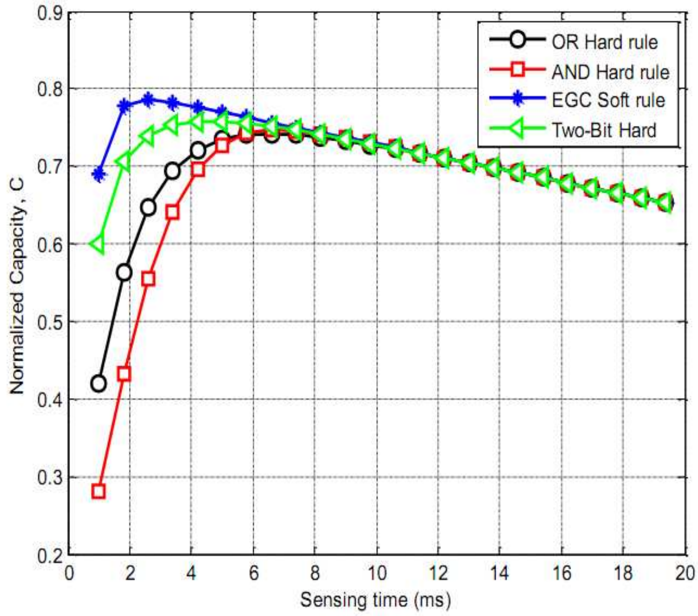

Figure 3.11 Normalized capacity versus sensing time under CPUP scenario using different combining rules (K=10, $\bar{Q}_d = 0.95$)

Figure 3.12 shows the normalized capacity of the CR network under CSUSU scenario using different combining rules. Therefore, there is no optimal sensing time as it was found under CPUP scenario, this result is trivial in the sense that the expression of the capacity is more dominated by the first term $(1 - Q_f)$ in (3.47) which is fixed under CSUSU scenario.

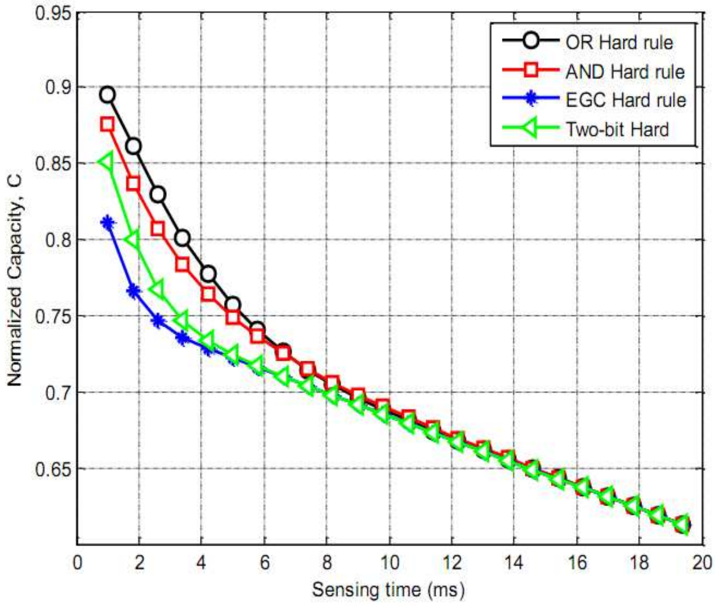

Figure 3.12 Normalized capacity versus sensing time under CSUSU scenario using different combining rules (K=10, $\bar{Q}_f = 0.05$)

3.8 Conclusion

In this chapter, we have presented the cooperative spectrum sensing as an effective method to combat many effects such as multipath fading and shadowing and hidden node problem. Firstly, the effect of fusion rules for cooperative spectrum sensing (CSS) has been studied and compared. It was shown via simulations that the EGC soft combining rule outperforms the hard and the two-bit quantized combining rules and the quantized two-bit combining rule exhibits better performance detection than the hard combining rule. We have extended the two-bit quantized scheme to three-bit quantized scheme, allowing to get comparable detection performance as EGC soft combining rule with less overhead. Then, the performance of CSS has been investigated under two operational scenarios, namely, CPUP and CSUSU using different combining rules (OR, AND, EGC and the quantized two-bit). Through this study, we have confirmed the effectiveness of the combining rules. Further, the relationship between CR users throughput and sensing time has been studied for both scenarios and under different combining rules. The simulation results showed that under CPUP, there is an optimal sensing time for which the CR users network throughput is maximized. The optimal sensing time and the corresponding maximized value of the CR users throughput depend on the combining rule used. The highest value of the throughput can be obtained by the EGC soft combining rule. The two-bit quantized combining rule which has been derived in this paper could be an appropriate combining rule to realize a trade-off between performances (in terms of detection and throughput) and overhead (in terms of complexity and reporting channel bandwidth). In this chapter, we have considered Gaussian approximation of different probabilities, one could extend it by considering the exact distribution (chi-square distribution) and derived all expression of overall probabilities. In the next chapter, we will study a blind method of local sensing based on statistic test (Goodness of Fit test).

Blind Spectrum Sensing Based on Statistic test (GoF test)

4

4.1 Introduction

Recently, the Goodness of-Fit Test (GoF) has been applied for hypothesis testing in the case of spectrum sensing for cognitive radio (CR). GoF sensing has the nice feature that it only needs a few samples to perform sensing. In this chapter, we first review the most popular GoF sensing methods for cognitive radio. We propose then a new spectrum sensing method based on GoF test of the energy of the received samples with a chi-square distribution. Based on the energy of the received samples, we compare the existing GoF sensing methods in the literature. If needed, the GoF spectrum sensing methods are adapted and modified to cope with complex samples at the input. Secondly, we propose the LLR-GoF sensing method in which a chi-square distribution is used for GoF testing, and also study some typical impairment for spectrum sensing, i.e. the effect of a non Gaussian noise and noise uncertainty on the performance of GoF based sensing. As a model for the non Gaussian noise, we used the Gaussian mixture (GM). Thirdly, we propose two GoF sensing methods and compare them against the conventional Anderson Darling (AD) sensing. The first proposed method consists in splitting the received samples in blocks, and applying the GoF sensing among the blocks. In the second method, we propose a new GoF test statistic by taking into account the physical characteristic of spectrum sensing. The derived GoF sensing method results in significant improvement in terms of sensing performance. Finally, we present a wideband spectrum sensing scheme using GoF based sensing.

4.2 Goodness of Fit Tests

GoF tests were proposed in mathematical statistics by measuring a distance between the empirical distribution of the observation made and the assumption distribution. In CRNs, GoF sensing is used to solve a binary detection problem and to decide whether the received samples are drawn from a distribution with a Cumulative Distribution Function (CDF) F_0, representing the noise distribution, or there are drawn from some distribution different from the noise distribution. The hypothesis to be tested can be formulated as follows:

$$\begin{aligned} H_0 &: F_n(x) = F_0(x) \\ H_1 &: F_n(x) \neq F_0(x), \end{aligned} \tag{4.1}$$

for a random set of n independent and identically distributed observations and where $F_n(x)$ is the empirical CDF of the received sample and can be calculated by:

$$F_n(x) = |\{i : x_i \leq x, 1 \leq i \leq n\}/n|, \tag{4.2}$$

where $| \bullet |$ indicates cardinality, $x_1 \leq x_2 \leq .... \leq x_n$ are the samples under test and n represents the total number of samples.

Many goodness of fit test are proposed in literature. The most important ones are the Kolmogorov- Smirnov test, the Cramer-von Mises test, the Shapiro-Wilk test and the Anderson-Darling test. In the following, we recall briefly these GoF tests.

A. Kolmogorov- Smirnov test (KS test): In this test the distance between $F_n(x)$ and $F_0(x)$ is given by:

$$D_n = max|F_n(x) - F_0(x)|, \tag{4.3}$$

where $F_n(x)$ is the empirical distribution which is defined in (4.2). If the samples under test are coming from $F_0(x)$, then, D_n converges to 0.

The distribution density function of the KS test is independent of the distribution of noise under H_0. The distribution of D_n under H_0 can be formulated as [80]

$$F(D_n|H_0;x) = \sum_{j=-\infty}^{+\infty} (-1)^j exp(-2j^2x^2) \tag{4.4}$$

B. Cramer-Von Mises (CM test): In this test, the distance between $F_n(x)$ and $F_0(x)$ is defined as:

$$T_n^2 = \int_{-\infty}^{\infty} [F_n(x) - F_0(x)]^2 \, dF_0(x). \tag{4.5}$$

By breaking the integral in (5) into n parts, T_n^2 can be writen as:

$$T_n^2 = \sum_{i=1}^{n} [z_i - (2i-1)/2n]^2 + (1/12n), \tag{4.6}$$

with $z_i = F_0(x_i)$

C. Anderson-Darling test (AD test): This test can be considered as a weighted Cramer-Von Mises test where the distance between $F_n(x)$ and $F_0(x)$ is given by:

$$A_n^2 = \int_{-\infty}^{\infty} [F_n(x) - F_0(x)]^2 \, \frac{dF_0(x)}{F_0(x)(1 - F_0(x))}. \tag{4.7}$$

The expression of A_n^2 can be also simplified to:

$$A_n^2 = -n - \frac{\sum_{i=1}^{n} (2i-1)(\ln z_i + \ln(1 - z_{(n+1-i)}))}{n}, \tag{4.8}$$

with $z_i = F_0(x_i)$.

The distribution of A_n^2 under H_0 can be written as [81]

$$\begin{aligned} F(A_n^2|H_0;x) = {} & \frac{\sqrt{2\pi}}{x} \sum_{j=0}^{+\infty} a_j (4j+1) exp(-\frac{(4j+1)^2\pi^2}{8x}) \\ & \int_0^{+\infty} exp((\frac{x}{8(w^2+1)} - \frac{(4j+1)^2\pi^2w^2}{8x}))dw \end{aligned} \tag{4.9}$$

where $a_j = (-1)^j \Gamma(j + \frac{1}{2}) / (\Gamma(\frac{1}{2}) j!)$

4.3 Spectrum Sensing method based on GoF test using chi-square distribution

As a starting point, we recall the model in [82] in which the authors consider an AWGN channel.

$$\begin{array}{rcl} H_0 : y(i) & = & w(i) \\ H_1 : y(i) & = & \sqrt{\rho}x + w(i), \end{array} \tag{4.10}$$

where H_0 and H_1 represent the hypothesis of absence and presence of a primary signal, respectively. x represents the transmitted signal, ρ is the signal to noise ratio (SNR), $w(n)$ is the real Gaussian noise with zero mean and unit variance and $y(n)$ are real valued. In [82], the sensing method is based on testing the GoF of the received samples compared to the Gaussian distribution.

The authors in [82] assumed that the transmitted signal $x = 1$, in other words, the data is represented as $y(i) = \sqrt{\rho} + w(i)$. The model in 4.10 does not reflect a realistic scenario, as normally the received signal is complex and can vary in time.

We have proposed to start from the more general hypothesis test:

$$\begin{array}{rcl} H_0 : y(i) & = & w(i) \\ H_1 : y(i) & = & \sqrt{\rho}x(i) + w(i), \end{array} \tag{4.11}$$

where $x(i)$ are the received complex samples of the transmitted signal and $w(i)$ is the complex Gaussian noise. We now consider the random variable $Y(i) = |y(i)|^2$ which corresponds to the received energy. It is proven that the variable $Y(n)$ is chi-squared distributed with 2 degree of freedom under H_0 hypothesis.

Proof:

Let $Z(1), Z(2) \cdots Z(n)$ be real independent random variable with $Z(n) \backsim N(0,1)$. If $Y = \sum\limits_{i=1}^{n} Z(i)^2$ then Y follows the chi-square distribution with n degrees of freedom, and denoted as $Y \backsim \chi_i^2$. In our case, we consider $Z(i)$ complex normal distributed variable and $Y(i) = |Z(i)|^2 = \alpha(i)^2 + \beta(i)^2$, where

$\alpha(i)$ and $\beta(i)$ are real and imaginary part of $Z(i)$ which are normal distributed variable. Therefore $Y(i)$ is chi-square distributed variable with 2 degree of freedom under hypothesis H_0.

We will consider a normal noise in order to be able to compare the different GoF sensing methods, this assumption is not limiting. The performance of the GoF sensing is independent of the noise distribution, as the distribution of GoF test statistic (A_n^2, T_n^2, D_n, ..) under H_0 is independent of the $F_0(y)$ [81] [80] [83] [82].

The spectrum sensing problem can be reformulated as a test hypothesis represented in (4.11) where we test whether the received energy $Y(i) = |y(i)|^2$ samples are drawn from a chi-square distribution with 2 degrees of freedom or not. The CDF of the chi-square distribution is given by:

$$F_0(y) = 1 - e^{-y/2\sigma_n^2} \sum_{k=0}^{m-1} \frac{1}{k!} (\frac{y}{2\sigma_n^2})^k, y > 0, \tag{4.12}$$

with $2m$ is the degree of freedom (in our case $m = 1$).

In summary, the proposed GoF sensing method follow these steps:

Step1 From the complex received samples $y(i)$, calculate the energy samples $Y(i) = |y(i)|^2$

Step2 Sort the sequence $\{Y(i)\}$ in increasing order such as $Y(1) \leq Y(2) \leq \cdots \leq Y(n)$

Step3 Calculate the GoF test statistic T^* , with F_0 given in (4.12).
use (4.3) for KS GoF sensing
use (4.6) for CM GoF sensing
use (4.8) for AD GoF sensing

Step4 Find the threshold λ for a given probability of false alarm such that:

$$Pfa = P\{T^* > \lambda | H_0\}. \tag{4.13}$$

Step5 Accept the null hypothesis H_0 if $T^* \leq \lambda$, where T^* is the GoF test statistic (KS, CM or AD) . Otherwise, reject H_0 in favour of the presence of the signal.

The value of λ is determined for a specific value of P_{fa}. Tables listing values of λ corresponding to different false alarm probabilities P_{fa} are given according to the test considered. Otherwise, these values can be computed by Monte Carlo approach.

4.3.1 Performance comparison of existing GoF sensing methods

In this subsection, we will analyze and compare the performance of existing GoF sensing methods.

Thereafter, simulation results are presented to show the sensing performance of various GoF sensing methods compared to the conventional ED sensing. Figure 4.1, shows the ROC curves of GoF sensing methods (AD, CM and KS) and ED sensing for a fixed number of 80 samples and a given SNR equal to $-6dB$. It is clear that ED sensing outperforms the considered GoF sensing methods. Likewise, AD sensing is the best among the considered GoF sensing methods. This is indeed confirmed in the simulation results as shown in Figure 4.2, where the detection probability versus SNR is plotted for a fixed number of 80 samples and at given false alarm probability $Pfa = 0.05$. ED sensing has better performance than the three GoF sensing methods. To achieve 90% of detection probability, ED sensing outperforms AD sensing of about $1dB$, and AD sensing presents a slight difference in gain compared to CM sensing and KS sensing of about $0.2dB$ and $0.5dB$ respectively.

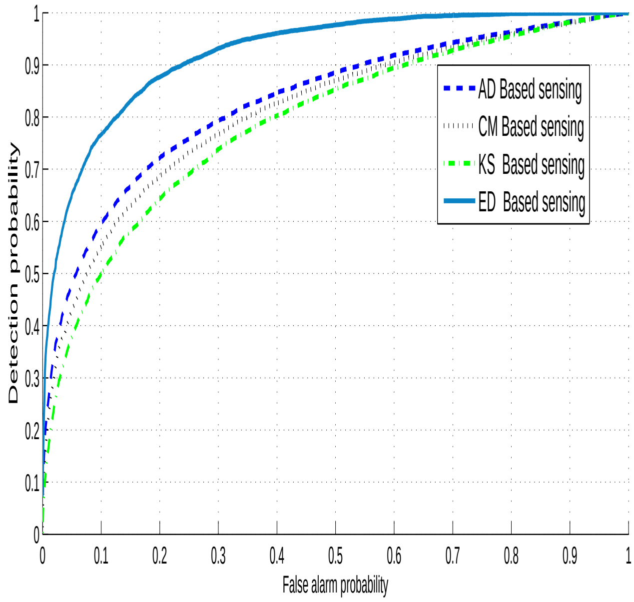

Figure 4.1 Detection probability versus false alarm probability of various GOF test based sensing at $SNR = -6dB$ and $n = 80$ samples

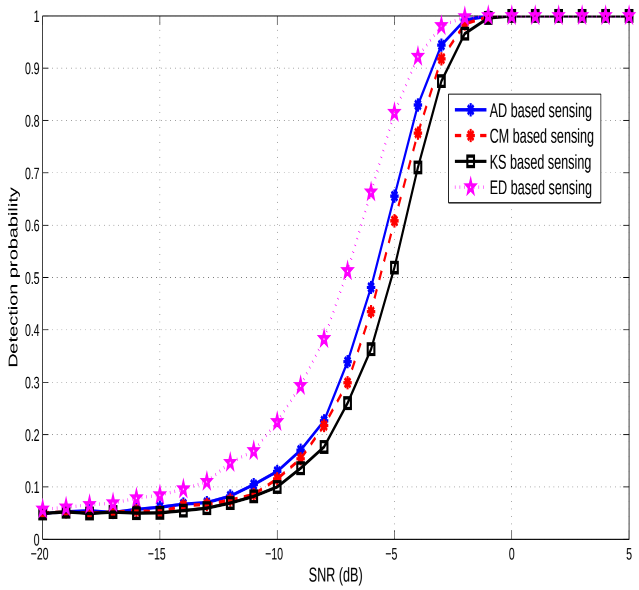

Figure 4.2 Detection probability versus SNR for different GOF tests based sensing with $Pfa = 0.05$ and $n = 80$ samples

4.4 Adaptation of existing GoF tests for spectrum sensing

In this section, we apply some existing GoF statistic tests for spectrum sensing. We adapt the GoF sensing algorithms to be used for complex input samples. The performance of the methods will then be evaluated.

4.4.1 Modified AD GoF sensing

The AD test assigns weights to both tails of the distribution. In [84], authors proposed a modified form of the AD test using the weight function that emphasizes the upper tail deviation. The weight function is $\psi(x) = [1 - F(x)]^{-1}$. By introducing this weight in the generalized following test statistic.

$$A_c^2 = n \int_{-\infty}^{+\infty} (F(y) - F_0(y))^2 \psi(F_0(y)) dF_0(y) \tag{4.14}$$

we get a modified Anderson Darling statistic which can be calculated as:

$$M_A D = \frac{n}{2} - 2\sum_{i=0}^{n} z_{(i)} - \sum_{i=0}^{n} (2 - \frac{(2i-1)}{n}) log(1 - z_{(i)}) \tag{4.15}$$

with $z_{(i)} = F_0(y)$

The resulting test can be applied to spectrum sensing by following the same steps as in Section 4.3. Through Monte-Carlo simulation, we can derive the threshold corresponding to some critical values of the probability of false alarm. It was found that to target a Pfa value of 0.01, the decision threshold must be set to 2.062.

to test the enhancement of the modified AD GoF sensing, Monte-Carlo simulations were performed. In Figure 4.3, we show detection performance as a function of SNR for a fixed value of $Pfa = 0.05$ and limited number of samples $n = 80$. It can be seen that the modified AD sensing outperforms the AD sensing of about $0.1dB$ gain.

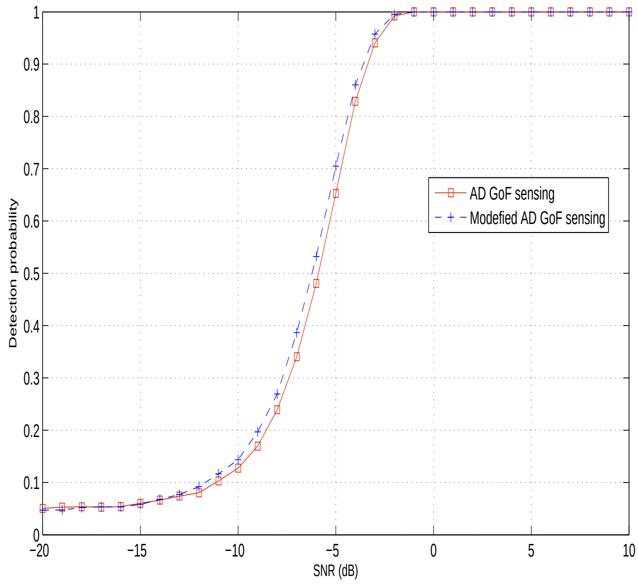

Figure 4.3 Detection probability versus SNR for modified AD GoF sensing with $Pfa = 0.05$ and $n = 80$ samples

4.4.2 Chi-square GoF test for spectrum sensing

The Chi-square test is a GoF test commonly used for testing whether observed data are representative of a particular distribution. The chi-square test is an alternative to the AD, CM and KS GoF tests. While AD, CM and KS GoF tests are restricted to continuous distributions, Chi-square GoF test can be applied to discrete distribution such as the binomial and the Poisson distribution.

In general, the chi-square test statistic is given as

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \tag{4.16}$$

where O_i is the observed frequency for bin i and E_i is the expected frequency for bin i. The expected frequency is calculated by:

$$E_i = n(F(Y_u) - F(Y_l)) \tag{4.17}$$

where F is the CDF for the distribution being tested, Y_u is the upper limit for class i, Y_l is the lower limit for class i, and n is the sample size. The test is sensitive to the choice of the bins. Although there is no optimal choice for the number of bins k, there are several formulas which can be used to calculate this number based on the total sample size n. For example, the following empirical formula:

$$k = 1 + log_2 n \tag{4.18}$$

To apply chi-square test for spectrum sensing, we propose the following method

Step1 From the complex received samples $y(i)$, calculate the energy samples $Y(i) = |y(i)|^2$

Step2 Sort the sequence $\{Y(i)\}$ in increasing order such as $Y(1) \leq Y(2) \leq \cdots \leq Y(n)$

Step3 Calculate k based on 4.18.

Step4 Break down the sorting sequences into k bins.

Step5 calculate the chi-square test statistic given in 4.16, taking in account that the distribution being tested is F_0, given in 4.12.

Step6 Find the threshold λ for a given probability of false alarm through Monte-Carlo simulation, otherwise, the chi-square GoF test approaches the chi-square distribution with degrees of freedom equals to $k - 1 \ as \ n \longrightarrow \infty$.

The performance of the chi-square GoF sensing method is numerically evaluated through Monte-Carlo simulations. In Figure 4.4, we show the detection probability versus SNR for a given false alarm probability $Pfa = 0.05$ and for total received samples $n = 80$. It is clear that the proposed method performs less than the AD based sensing, however, it presents a slight good performance compared to AD based sensing at very low SNR.

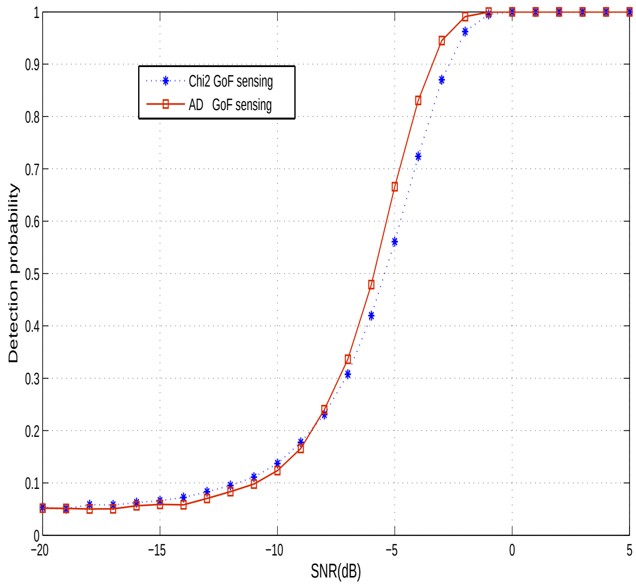

Figure 4.4 Detection probability versus SNR for chi-square GoF sensing over AWGN channels with $Pfa = 0.05$ and n=80 samples

4.4.3 Order Statistic (OS) GoF sensing method

The order statistic is a GoF test based on ρ-vector which provides a direct judgment of fit with the considered distribution. The elements of ρ-vector are the quantiles of order statistics [85]. OS GoF test can be used to assess the distribution under hypothesis H_0 (F_0), by deriving the ρ-vector.

To perform OS GoF sensing, we propose the following steps.

Step1 From the complex received samples $y(i)$, we calculate the energy samples $Y(i) = |y(i)|^2$

Step2 Calculate z_i such as:

$$z_i = F_0(y_i) \tag{4.19}$$

with F_0 given in 4.12.

Step3 Sort the element z_i in ascending order such as $z_{(1)} \leq z_{(2)} \leq \cdots \leq z_{(n)}$

Step4 Perform the β CDF transformation of the ordered z_i to obtain the ρ - vector

$$\rho_i = \beta(z_{(i)}; i, n-i+1) \tag{4.20}$$

where $\beta(x; \alpha, \beta)$ denotes beta CDF with α and β are shape parameters of the distribution. ρ_i can be simplified (by applying integration by part) to the following expression:

$$\rho_i = \sum_{j=i}^{n} \frac{n!}{j!(n-j)!} z_{(i)}^{j} (1 - z_{(i)})^{(n-j)} \tag{4.21}$$

Step5 Arrange ρ_i in an ascending order such as $\rho_{(1)} \leq \rho_{(2)} \leq \cdots \leq \rho_{(n)}$

Step6 Calculate the test statistic Γ_{os} [86]

$$\Gamma_{os} = \sum_{i=1}^{n} |\rho_{(i)} - \frac{i}{(n+1)^2}| \tag{4.22}$$

Once the test Γ_{os} is computed, it will be compared to a predefined threshold λ and the statistical test reduces to:

$$\begin{aligned} H_0 &: \Gamma_{os} \leq \lambda_{os} \\ H_1 &: \Gamma_{os} > \lambda_{os}, \end{aligned} \tag{4.23}$$

with λ_{os} is the threshold that is dependent on the required probability of false alarm.

Likewise, the performance of the OS GoF sensing is evaluated and compared to AD sensing. In Figure 4.5, we show the detection performance of OS sensing when the SNR was varied from $-20dB$ to $5dB$ (keeping the samples number n=80 and for fixed $Pfa = 0.05$). It is shown that the performance of the proposed OS sensing is superior to the performance of AD sensing.

The table 4.1 gives some critical value of Pfa and the corresponding decision threshold, theses values are derived by Monte-Carlo simulations.

Pfa	0.1	0.05	0.01
Threshold	31.082	32.774	36.526

Table 4.1 Threshold values for some given Pfa with $n = 80$ samples (OS Sensing)

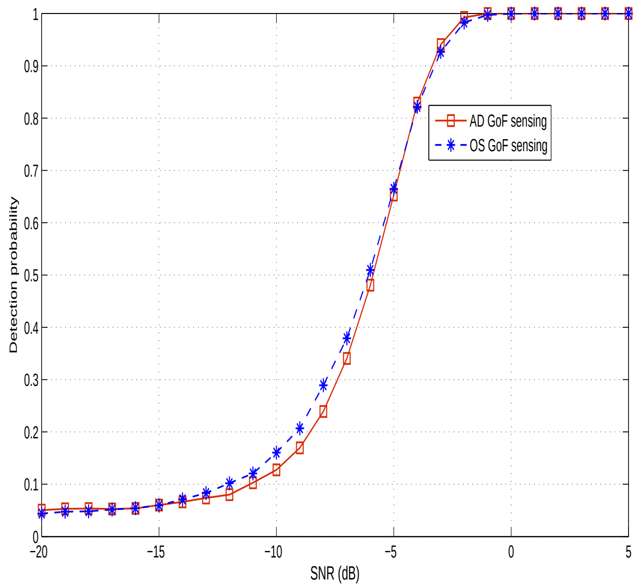

Figure 4.5 Detection probability versus SNR for OS sensing with $Pfa = 0.05$ and n=80 samples

4.5 Spectrum Sensing Based on The Likelihood Ratio Goodness of Fit test

In this section, a blind spectrum sensing method based on goodness-of-fit (GoF) test using likelihood ratio (LLR) is studied. In the proposed method, a chi-square distribution is used for GoF testing. The performance of the method is evaluated through Monte Carlo simulations.

4.5.1 Likelihood based Goodness of fit test

In [87], the author proposes a new, more general approach of parametrization to construct a general GoF test. With this approach, they could generate the traditional GoF tests including KS, CM and AD. Moreover, they provided also a new, more powerful GoF test, based on likelihood ratio. The author in [87] formulated the hypothesis test as follows:

$$\begin{aligned} H_0 &: \ H_0(t) : F_n(t) = F_0(t) \quad for\, all\, t \in (-\infty, \infty) \\ H_1 &: \ H_1(t) : F_n(t) \neq F_0(t) \quad for\, some\, t \in (-\infty, \infty) \end{aligned} \tag{4.24}$$

meaning that testing H_0 versus H_1 is equivalent to testing $H_0(t)$ versus $H_1(t)$ for every $t \in (-\infty, \infty)$.

Two types of statistic for testing H_0 versus H_1 were proposed :

$$Z = \int_{-\infty}^{\infty} Z_t \, dw(t), \ \ and \tag{4.25}$$

$$Z_{max} = \sup_{t \in (-\infty,\infty)} \{Z_t \, w(t)\} \tag{4.26}$$

with Z_t a statistic for testing $H_o(t)$ versus $H_1(t)$ and $w(t)$ some weight function. Large values of Z or Z_{max} will reject a null hypothesis H_0. In [87], authors presents two natural candidates for Z_t, the Pearson χ^2 test statistic and the likelihood ratio (LLR) test statistic. The LLR test statistic is given by:

$$G_t^2 = 2n[F_n(t) \log\{\frac{F_n(t)}{F_0(t)}\} + (1 - F_n(t)) \log\{\frac{1 - F_n(t)}{1 - F_0(t)}\}]. \tag{4.27}$$

where $F_n(t)$ is the empirical distribution function of the received samples.

Taking in (4.25) Z_t as G_t^2 and choosing an appropriate weight function $w(t)$, produces a powerful goodness of fit tests statistic Z_A, comparing to the traditional tests.

$$Z_A = -\sum_{i=1}^{n} [\frac{log\{F_0(X_{(i)})\}}{n - i + \frac{1}{2}} + \frac{log\{1 - F_0(X_{(i)})\}}{i - \frac{1}{2}}]. \tag{4.28}$$

For the proposed spectrum sensing method in this section, we will use the test statistic Z_A as LLR-GoF test. Once the test Z_A is computed, it will be compared to a predefined threshold λ with:

$$\begin{aligned} H_0 &: Z_A \leq \lambda \\ H_1 &: Z_A > \lambda, \end{aligned} \tag{4.29}$$

4.5.2 The proposed spectrum sensing (LLR-GoF sensing)

The proposed spectrum sensing method can be summarized in the following steps:

Step1 from the complex received samples $y(i)$, calculate the energy samples $Y(i) = |y(i)|^2$

Step2 Sort the sequence $\{Y(i)\}$ in increasing order such as $Y(1) \leq Y(2) \leq \cdots \leq Y(n)$

Step3 Calculate the test Z_A according to (equa:llr7), with F_0 given in (4.12).

Step4 Find the threshold λ for a given probability of false alarm such that:

$$Pfa = P\{Z_A > \lambda | H_0\}. \tag{4.30}$$

Step5 Accept the null hypothesis H_0 if $Z_A \leq \lambda$. Otherwise, reject H_0 in favour of the presence of the primary user signal.

To find λ, it is worth to mention that the distribution of Z_A under H_0 is independent of the $F_0(y)$. The value of λ is determined for a specific value of P_{fa}. A table listing values of λ corresponding to different false alarm probabilities P_{fa} is given in [87]. Otherwise, these values can be computed in advance by Monte Carlo approach.

Figure 4.6 presents the detection probability as a function of the false alarm probability (ROC curves) of the proposed LLR-GoF sensing method compared to the AD-GoF sensing and the energy detection (ED). The results are obtained by 10000 Monte-Carlo simulations. The simulations are performed using only 80 samples of the received signal with a signal to noise ratio (SNR) equal to $-6dB$. It can be seen in Figure 4.6 that the proposed LLR-GoF sensing outperforms the AD-GoF sensing and approaches the performances of the ED based sensing. For example, for $Pfa = 0.2$, the probability of detection P_d for the ED sensing equals 0.885, for AD based sensing P_d equals 0.715. However, for the proposed LLR-GoF sensing, P_d equals 0.862 .

In figure 4.7, the values of the detection probability versus SNR are plotted for the three sensing methods. The $P_f a$ is set to 0.05 and the SNR varies from $-20dB$ to $10dB$, keeping the number of samples n to 80 samples. It can be seen that the proposed LLR-GoF sensing has almost $1dB$ gain over AD GoF sensing, however the ED sensing outperforms the proposed LLR-GoF sensing with almost $0.2dB$ of gain when $Pd = 0.8$ and $Pfa = 0.05$, hence the performance of the ED sensing is indeed better than that of the proposed LLR-GoF based sensing and AD based sensing.

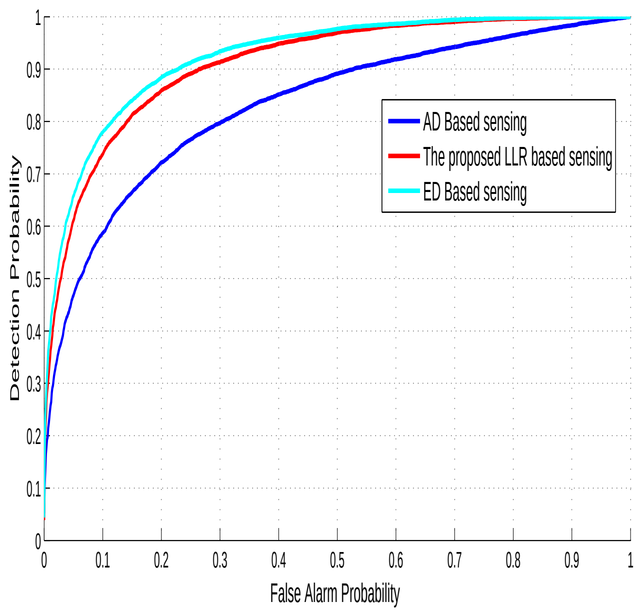

Figure 4.6 Detection probability versus false alarm probability over AWGN channels with $SNR = -6$ dB and $n = 80$ samples

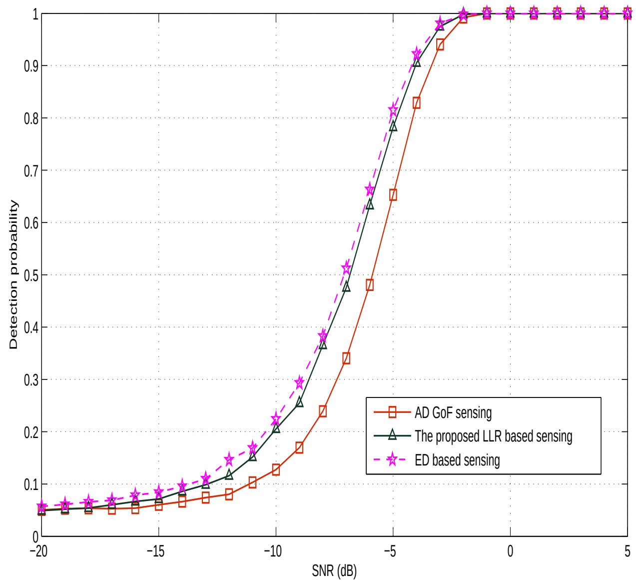

Figure 4.7 Detection probability versus SNR over AWGN channels with $Pfa = 0.05$ and n=80 samples

4.6 GoF Sensing Under Non Gaussian Noise and Noise Uncertainty

4.6.1 Non Gaussian noise (GM Model)

It is worth to mention that the existing works on GoF for spectrum sensing [82] [88] [86] [89] and [90] is focusing on detecting a signal in white Gaussian noise. In our work, we will also focus on detecting signals in white non-Gaussian noise. In literature, a lot of models are proposed to pattern a non Gaussian noise. The most used models are the Gaussian Mixture model (GM) and the generalized Gaussian model (GG). For our spectrum sensing model, we will work with the GM model [91], as it has been used in practical applications in [92] and in radio signal detection applications in [93]. To apply the GoF test for spectrum sensing, we need to know the Cumulative distributed function (CDF) of the non Gaussian noise (GM CDF). The pdf of GM noise has three parameters α ,β , and σ and is defined as [93]:

$$f_w(w) = \frac{c}{\sigma\sqrt{2\Pi}}[\alpha exp(-\frac{c^2w^2}{2\sigma^2}) + \frac{1-\alpha}{\beta}exp(-\frac{c^2w^2}{2\sigma^2\beta^2})] \quad (4.31)$$

where $c = \sqrt{\alpha + (1-\alpha)\beta^2}$

Figure 4.8 depicts a probability distribution function (pdf) of a white non Gaussian noise (GM) with the following selected parameters $\alpha = 0.9$, $\beta = 5$ and $\sigma = 1$.

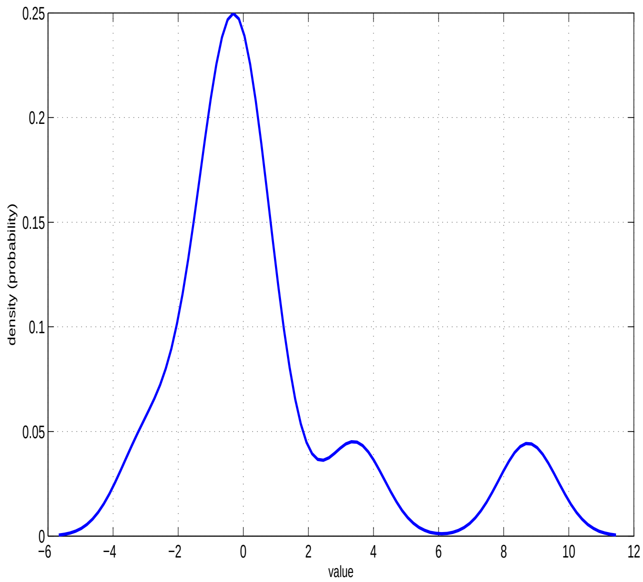

Figure 4.8 probability distribution function (pdf) of GM noise $\alpha = 0.9$, $\beta = 5$ and $\sigma = 1$

The CDF F_0 of the energy of the non-Gaussian noise samples under H_0 hypothesis can be derived from the GM's pdf. For that we have: if $Y = X^2$

and X is GM noise with CDF $F_X(x)$

$$\begin{aligned} F_0(y) = & \quad P(Y \leq y) = P(-\sqrt{y} \leq X \leq \sqrt{y}) \\ = & \quad F_X(\sqrt{y}) - F_X(-\sqrt{y}) \end{aligned} \tag{4.32}$$

Once we get the CDF of the non Gaussian noise, we apply our proposed algorithm of subsection(4.3). Note that the knowledge of F_0 is required to apply the GoF test, therefore, if the parameters of the GM model are unknown, they must be estimated first.

To evaluate the effect of a non Gaussian noise on the sensing performance, we have performed simulations with the selected GM noise. We set the parameters of the non Gaussian noise as: $\alpha = 0.9$, $\beta = 5$ and $\sigma = 1$. Figure 4.9 presents results of the AD GoF sensing under Gaussian noise and non Gaussian noise. It is shown that the effect of considering a non Gaussian noise decrease slightly the performance of the AD GoF sensing.

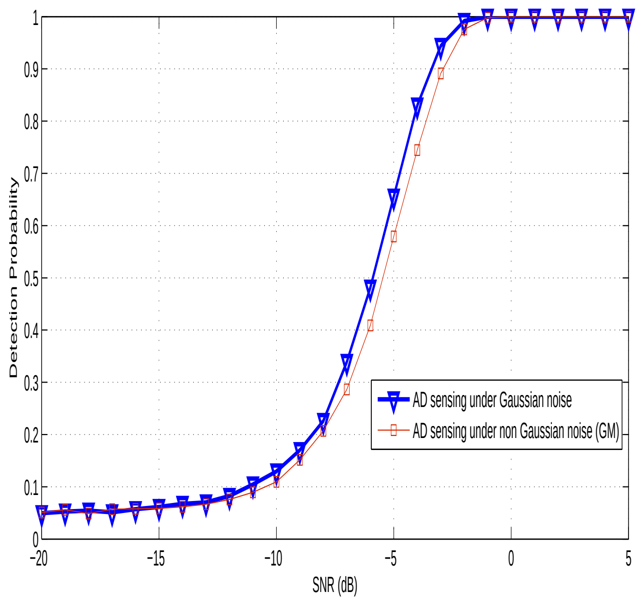

Figure 4.9 Detection probability versus SNR under Gaussian and non Gaussian noise for AD-GoF, with $Pfa = 0.05$ and $n = 80$ samples

Figure 4.10 shows the results of the LLR GoF sensing. Just as in the AD GoF sensing, our proposed method is slightly degraded under non Gaussian noise.

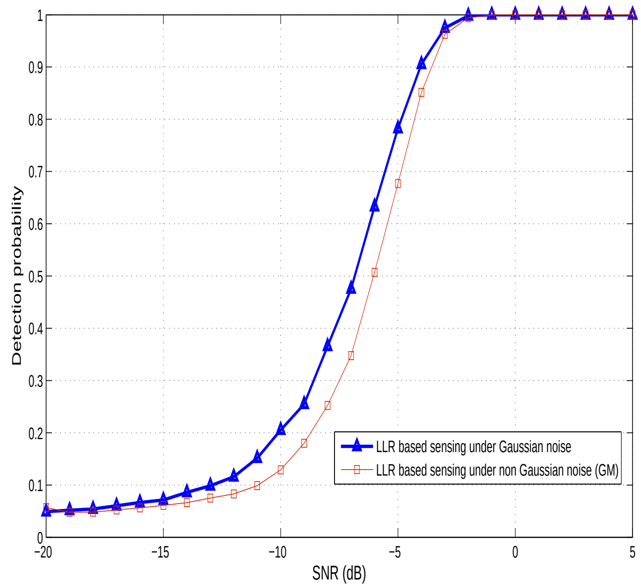

Figure 4.10 Detection probability versus SNR under Gaussian and non Gaussian noise for LLR-GoF, with $Pfa = 0.05$ and $n = 80$ samples

However, it can be seen in figure 4.11 that the performance of the ED is significantly influenced by the considered non Gaussian noise. It has to be noted that the considered non Gaussian noise ($\alpha = 0.9$, $\beta = 5$ and $\sigma = 1$) is very unfavorable for ED. In order to obtain a $P_{fa} = 0.05$, the threshold λ in the binary hypothesis test needs to be shifted rightly at certain level. Anyway, GoF sensing is less effected by the non Gaussian noise, as the test is performed on the mismatch between the measured CDF and the reference CDF F_0.

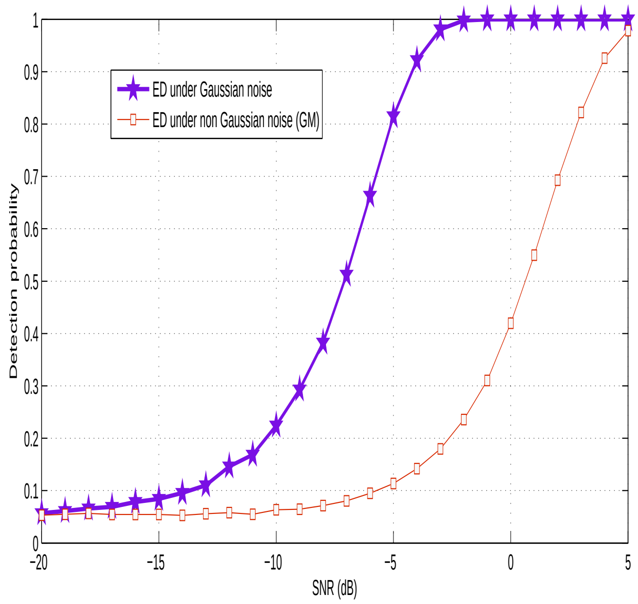

Figure 4.11 Detection probability versus SNR under Gaussian and non Gaussian noise for ED, with $Pfa = 0.05$ and $n = 80$ samples

4.6.2 Noise uncertainty

One of the main issues with ED, is the impact of noise uncertainty on the detection performance. It is shown in [37] and [94] that ED is very sensitive to noise uncertainty. The aim of this subsection it to study the effect of noise uncertainty on GoF sensing methods compared to ED.

Through simulation, we have compared the impact of noise uncertainty on both methods, ED based spectrum sensing and GoF sensing.

The noise uncertainty is modeled by letting the actual noise variance be limited within a set given by a nominal noise variance and an uncertainty parameter ρ such that $\sigma_n^2 \in [\frac{1}{\rho}\sigma^2, \rho\sigma^2]$.

There is a fundamental difference between ED and GoF sensing when it comes to noise uncertainty . The energy detector suffers under noise uncertainty because computing the threshold λ for the binary test requires knowledge of the underlying noise variance. In order to guarantee a given false alarm rate P_{fa}, the threshold λ will be calculated for the worst case, i.e. a noise variance of $\rho\sigma^2$, leading to higher values of λ and hence to a decrease in detection probability.

In GoF sensing, the distribution of the test statistic G_t^2 or A_n^2 under the H_0 hypothesis is independent of the noise distribution. As a consequence, the value of the threshold λ for the GOF binary test will not be influenced by the noise uncertainty. However, the calculation of the test statistic (G_t^2 or A_n^2) requires the exact knowledge of the underlying theoretical noise CDF F_0. In summary, for GOF sensing, noise uncertainty will, via F_0, indirectly affect the value of the test statistic, but not the detection threshold. For the simulation of the GoF sensing under noise uncertainty, we will also follow a worst case approach, by considering a reference noise CDF F_0 given in (4.12) based on the highest noise variance $\rho\sigma^2$, which will eventually lead to a reduction of the detection probability.

In figure 4.12, we have plotted the detection probability versus SNR for several values of noise uncertainty ($0dB, 0.5dB, 2dB, 4dB$) in the case of the ED spectrum sensing method. It is shown that the performance of the ED are significantly decreasing when the noise uncertainty level is increasing.

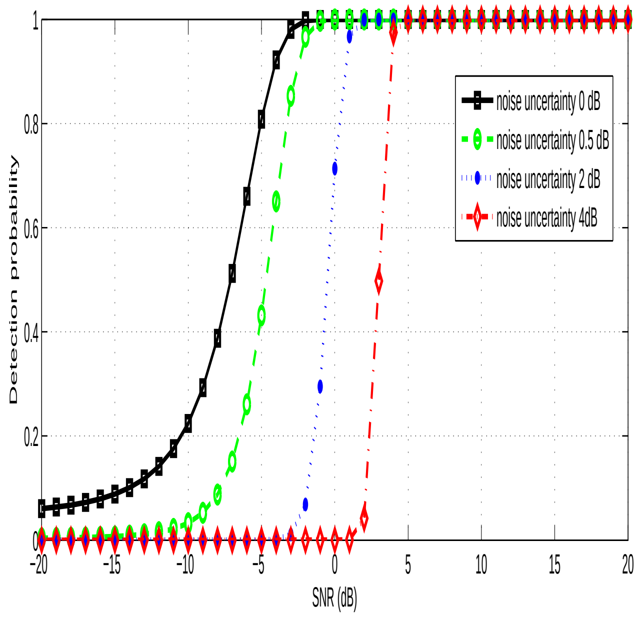

Figure 4.12 Impact of noise uncertainty on ED with $Pfa = 0.05$ and $n = 80$ samples

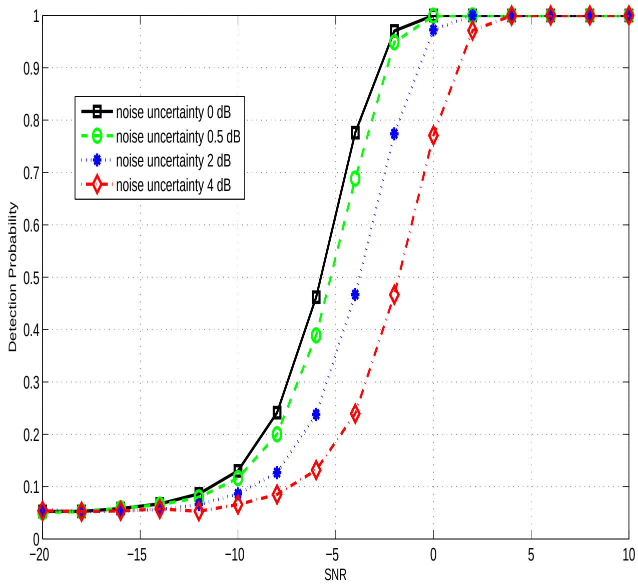

Figure 4.13 Impact of noise uncertainty on GoF test based sensing with $Pfa = 0.05$ and $n = 50$ samples

In similar way, in figure 4.13, we have plotted the detection probability as a function of SNR when considering a noise uncertainty for GoF based spectrum sensing. It can be seen that under uncertainty in the noise statistic of the CDF under hypothesis H_0 (F_0) , the impact on the performance of the the GoF based spectrum sensing is significantly less than the impact on energy detection. Intuitively, this can be explained by the fact that in ED, the value of P_{fa} and P_d are directly affected by the noise uncertainty. In case of GoF based sensing the test statistic Z_A (or A_n^2) is indirectly affected by the noise uncertainty via the CDF F_0 under hypothesis H_0.

Note also that, in figure 4.12, for high values of noise uncertainty the P_d drops to 0. This effect is known as the SNR wall [37]. This effect is not observed in GoF based spectrum sensing for the given simulation parameters.

4.7 New proposed GoF sensing method

4.7.1 AD sensing method based on sub-blocks

In this subsection, a new AD sensing method is proposed. The method consists of breaking down the received signal samples into sub-blocks as depicted in Figure 4.14. It is worth to mention that the proposed method is applied when the distribution of the noise is Gaussian, and when we are provided with a sufficient sample size. We can summarized the method in the following steps:

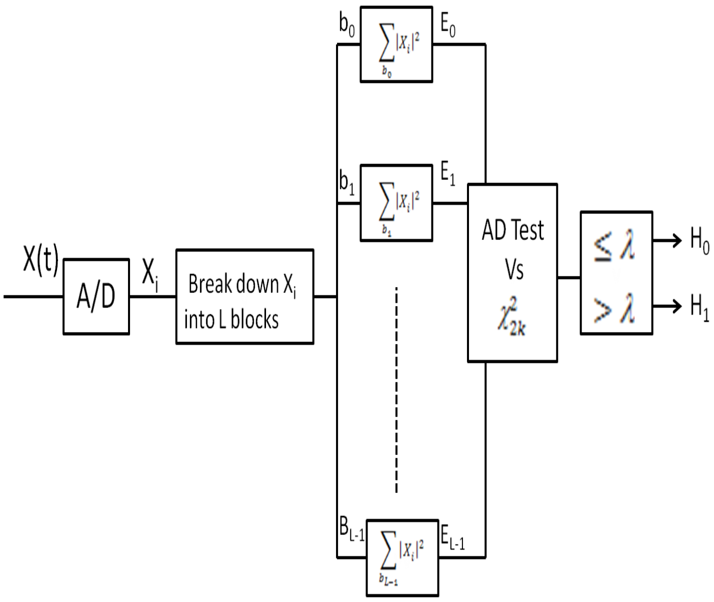

Figure 4.14 A new AD sensing method block diagram

Step1 Divide the complete received signal samples $y(i)$ in L blocks, each block has K samples with $n = K * L$

Step2 Calculate the energy of each block $Y(j) = \sum_K |y(i)|^2$ for $j = 1, ...L$

Step2 Sort the sequence $\{Y(j)\}$ in increasing order such as $Y(1) \leq Y(2) \leq \cdots \leq Y(L)$

Step3 Calculate the GoF test T^* using (4.7), with F_0 given in (4.12) by adapting the degree of freedom of the χ^2 by $m = 2 * K$.

Step4 Find the threshold λ for a given probability of false alarm (through Monte-Carlo simulations).

Step5 Accept the null hypothesis H_0 if $T^* \leq \lambda$. Otherwise, reject H_0 in favour of the presence of the primary user signal.

To evaluate the performances of the proposed method, Monte-Carlo simulations are carried out.

In Figure 4.15, the sensing performance of the new AD sensing method is shown with total number of samples $n = 1000$ and for $Pfa = 0.01$, when the SNR varies from $-20dB$ to $1dB$. It can be seen that when the number of block($L = 1000$ (AD GOF sensing), $100, 50$ *and* 20) decreases, the sensing performance is improved. This means that the GoF test is applied to a chi-squared distribution with degree of freedom $2K = 2n/L$ ($K = 1, 10, 20, 50$) respectively. The zoomed-in figure confirms the finding that increasing K results on improving detection performance.

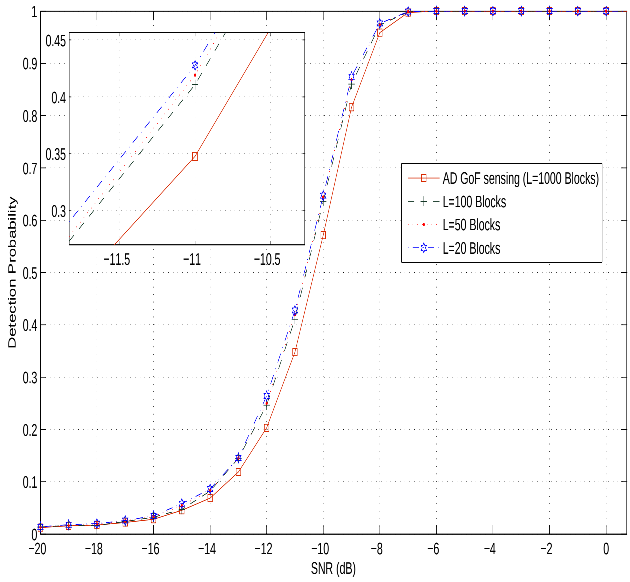

Figure 4.15 Detection probability versus SNR over AWGN channels with $Pfa = 0.01$ for the AD GoF sensing based on sub-blocks

4.7.2 Spectrum Sensing Method Based on The new GoF statistic test

The aforementioned GoF tests use the statistical hypothesis testing in eqation 4.1(which means testing the hypothesis H_0). However, in the H_1 hypothesis, it can be noted that the overall power of the received signal should always be larger than the noise power, as noise and signal are uncorrelated. Which result in having a cumulative distribution function under hypothesis H_1 on the right of the cumulative distribution function of the noise, meaning that the area above the expected continuous CDF of the random variable (energy of samples in our case) will also increase. The above finding is based on the property of the expected value of a non-negative random variable.

$$E[X] = \int_0^\infty (1 - F_X(x))dx \tag{4.33}$$

In our sensing model as in [95], the received energy $Y_i = |X_i|^2$ is a non negative random variable and equation (4.33) is applicable. As the received signal $\{X_i\}$ has zero means, $E[Y] = E[|X_i|^2] = \sigma_X^2$. Hence, we find

$$\sigma_X^2 = \int_0^\infty (1 - F_Y(x))dx \tag{4.34}$$

In other words, the received signal power equals the area of the region lying above the CDF $F_Y(x)$ and below the line at height 1 to the right of the origin. Under H_0 hypothesis, this means that the area above F_0 equals the noise power σ_w^2 as depicted in figure 4.16. Under H_1 hypothesis, the total power in the received signal will increase to $\sigma_s^2 + \sigma_w^2$, meaning that the area above the expected continuous CDF of the random variable Y_i will also increase, shifting this CDF to the right.

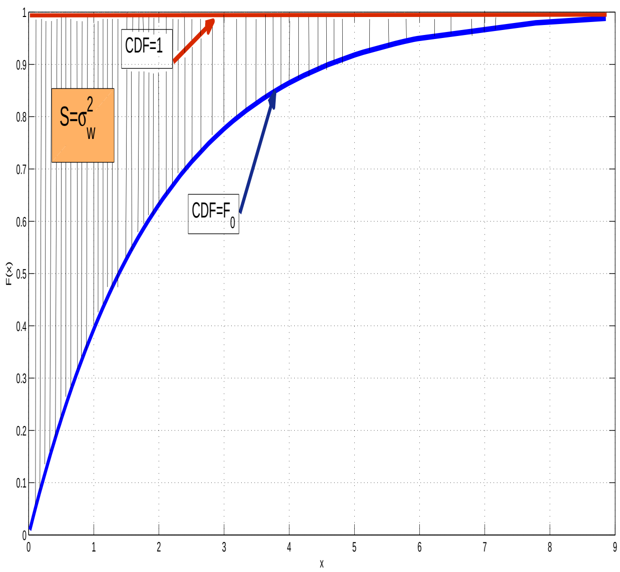

Figure 4.16 Noise power area

Therefore, the statistical hypothesis comes down to test one of these inequalities such as:

$$\begin{aligned} H_0 &: F_n(y) \geq F_o(y) \\ H_1 &: F_n(y) < F_o(y) \end{aligned} \tag{4.35}$$

The problem with the AD test (and also with the Von Mises test) is that the deviation of the empirical CDF $F_n(x)$ to the reference CDF $F_0(x)$ can be either to the left and to the right as the test is based on the square of the difference $[F_n(x) - F_0(x)]^2$. For spectrum sensing application, the sign of difference is significant for the raison cited above. Therefore, the associated statistical of the GoF test statistic can be given as:

$$S_n = n \int_{-\infty}^{+\infty} [F_0(y) - F_n(y)]\phi(F_0(y))dF_0(y). \tag{4.36}$$

According to the choice of the weight function $\phi(t)$, we can derive the corresponding test statistic of the statistical hypothesis in (4.35).

When $\phi(t) = 1$, the above equation(4.36) can be simplified as:

$$
\begin{aligned}
S_n &= n \int_{-\infty}^{+\infty} [F_0(y) - F_n(y)] dF_0(y) \\
&= n \int_{-\infty}^{y_1} F_0(y) dF_0(y) \\
&+ n \int_{y_1}^{y_2} (F_0(y) - \frac{1}{n}) dF_0(y) \\
&+ \ldots \\
&+ n \int_{y_(n-1)}^{y_n} (F_0(y) - \frac{n-1}{n}) dF_0(y) \\
&+ n \int_{y_(n)}^{+\infty} (F_0(y) - 1) dF_0(y) \\
&= -\frac{n}{2} + \sum_{i=1}^{n} ((F_0(y)) \\
&= -\frac{n}{2} + \sum_{i=1}^{n} (z_i)
\end{aligned}
\tag{4.37}
$$

When $\phi(t) = \dfrac{1}{t(1-t)}$, the above equation(4.36) can be simplified as

$$
\begin{aligned}
S_n &= n \int_{-\infty}^{+\infty} [F_0(y) - F_n(y)]\phi(F_0(y))dF_0(y) \\
&= n \int_{-\infty}^{y_1} \frac{F_0(y)}{F_0(y)(1-F_0(y))} dF_0(y) \\
&+ n \int_{y_1}^{y_2} \frac{F_0(y) - \frac{1}{n}}{F_0(y)(1-F_0(y))} dF_0(y) \\
&+ \ldots \\
&+ n \int_{y_(n-1)}^{y_n} \frac{F_0(y) - \frac{n-1}{n}}{F_0(y)(1-F_0(y))} dF_0(y) \\
&+ n \int_{y_(n)}^{+\infty} \frac{F_0(y) - 1}{F_0(y)(1-F_0(y))} dF_0(y) \\
&= -\sum_{i=1}^{n} (ln(1-F_0(y)) - ln(F_0(y))) \\
&= -\sum_{i=1}^{n} (ln(1-z_i) - ln(z_i))
\end{aligned}
\tag{4.38}
$$

When $\phi(t) = \dfrac{1}{(1-t)}$, the above equation(4.36) can be simplified as

$$
\begin{aligned}
S_n &= n \int_{-\infty}^{+\infty} [F_0(y) - F_n(y)]\phi(F_0(y))dF_0(y) \\
&= n \int_{-\infty}^{y_1} \frac{F_0(y)}{(1-F_0(y))} dF_0(y) \\
&+ n \int_{y_1}^{y_2} \frac{F_0(y) - \frac{1}{n}}{(1-F_0(y))} dF_0(y) \\
&+ ... \\
&+ n \int_{y_{(n-1)}}^{y_n} \frac{F_0(y) - \frac{n-1}{n}}{(1-F_0(y))} dF_0(y) \\
&+ n \int_{y_{(n)}}^{+\infty} \frac{F_0(y) - 1}{(1-F_0(y))} dF_0(y) \\
&= -n - \sum_{i=1}^{n} ln(1 - F_0(y)) \\
&= -n - \sum_{i=1}^{n} ln(1 - z_i)
\end{aligned}
\tag{4.39}
$$

Once the test S_n is calculated, it will be compared with a decision threshold λ to decide whether to accept H_1 or reject it (accept H_0). The threshold λ can be determined according to the given value of the false alarm probability. The decision threshold λ is computed through Monte Carlo simulation.

In Figure 4.17, the performance comparison between the new GoF sensing method, AD GoF sensing [95] and ED sensing is depicted. This figure shows detection performance in terms of detection probability as a function of SNR with $n = 80$ and $Pfa = 0.05$ for different weights. The new GoF sensing method outperforms the AD sensing method. The best performance is obtained with weight $\phi = \frac{1}{1-t}$ corresponding to (4.39) which has comparable detection performance with ED sensing.

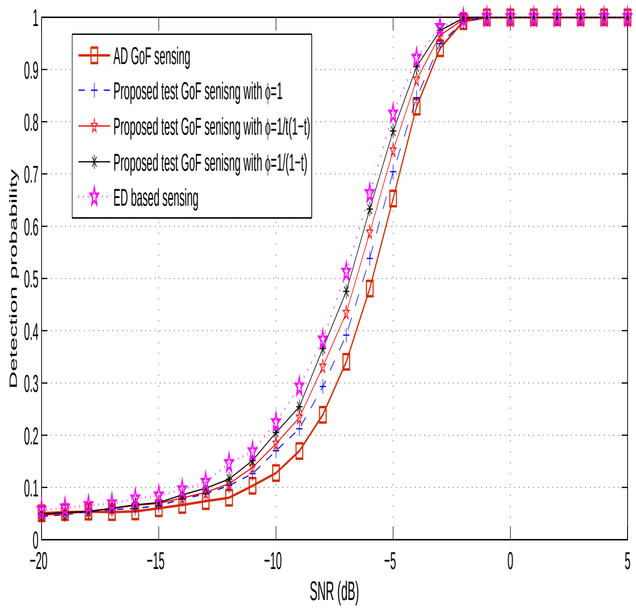

Figure 4.17 Detection probability versus SNR for the proposed GoF sensing under different weights, with $Pfa = 0.05$ and n=80 samples

The table 4.2 gives a corresponding λ for some critical values of Pfa.

$\phi = 1$	Pfa	0.1	0.05	0.01
	Threshold	3.536	4.480	6.295
$\phi = \frac{1}{t(1-t)}$	Pfa	0.1	0.05	0.01
	Threshold	21.875	28.165	39.484
$\phi = \frac{1}{1-t}$	Pfa	0.1	0.05	0.01
	Threshold	12.522	16.136	23.928

Table 4.2 Threshold values for some given Pfa and n=80 samples

The simulations results show that the new GoF sensing method has the best performance and the lowest computational complexity.

4.8 Wide-band Spectrum Sensing based on GoF testing

A wideband spectrum sensing structure is about searching multiple bands at a time. Wideband spectrum sensing has been studied before in the literature, such as in [96] and [97]. Wide-band spectrum sensing can be classified according to the sampling rate into [7] : Nyquist wide-band sensing when the sampling rate at which the signals are acquired is above the Nyquist rate, and sub-Nyquist wide-band sensing when it below the Nyquist rate.

In this section, motivated by its nice feature mentioned in section 4.3, the narrow-band spectrum sensing based on GoF is used for a Nyquist wide-band sensing known also as a conventional wide-band sensing. The detailed of this method can be found in [7].

The target of this scheme is the FFT power spectrum distribution under H_0 hypothesis. Considering X_k, the Fourier coefficient for frequency bin k of a complex Gaussian noise vector $\mathrm{x} = \{x_n\}$ of length N. It can be stated that the k^th power spectrum coefficient $|X_k|^2$, normalized by $var(X_k)/2$ follows a χ_2^2 distribution [7]. The wide-band sensing method is represented in figure 4.18.

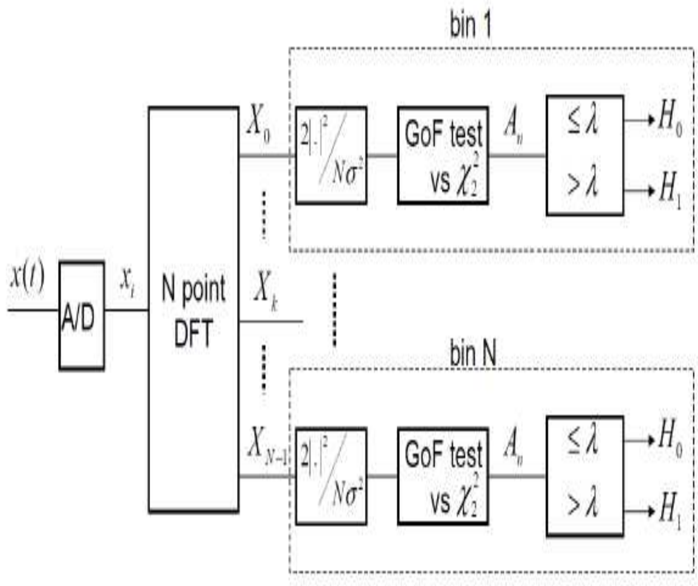

Figure 4.18 Wideband sensing method block diagram [7].

It is tested through the narrow-band GoF based Spectrum Sensing that, if the normalized power spectrum coefficient $\frac{2|X_k|^2}{N\sigma^2}$ follows a χ_2^2 distribution, the H_0 hypothesis is selected. Otherwise, the H_1 hypothesis is selected.

Next, the performance of the Wide-band Spectrum Sensing based on GoF testing is discussed based on synthetic data and Real Data [7].

4.8.1 Result on Synthetic Data

In order to test the performances of the proposed method, in [7], we have considered one narrow-band signal, with high SNR, occupying a frequency band of $10MHz$. The incoming signal, $\{x_i\}$ which is a complex base-band signal, is sampled at $10MHz$. The parameters for the wide-band sensing algorithm are listed below:

- The complex noise (AWGN) has a noise power density of $0dBm/Hz$.

- $K = 40$: is the number of consecutive segment,

- $N = 1024$: is the number of points for the DFT ,

- $K.N = 40960$ is the total number of samples,

- $10kHz$: equals approximately the width of the frequency bins,

- $Pfa = 0.01$ is the fixed false alarm probability corresponding to a threshold $\lambda = 3,89$ [83],

- The signal to detect is a BPSK modulated signal at $3MHz$ and a bandwidth of $25kHz$.

- The modulated symbols are shaped using a RRC pulse shape with $\alpha = 0.5$.

- The power of the modulated signal is set to obtain an SNR of $10dB$.

Figure 4.19 shows the empirical CDF for every frequency bin. The blue curve corresponds to the empirical CDFs of a bin under H_0 hypothesis. However, the red curve presents the empirical CDFs of a bin under H_1 hypothesis. The green curve is for the reference CDF F_0 for the GoF testing which is a χ^2_2. It can be observed the presence of 3 empirical CDFs corresponding to the BPSK signal and all the red curves (empirical CDFs) close to the F_0 CDF are false alarms.

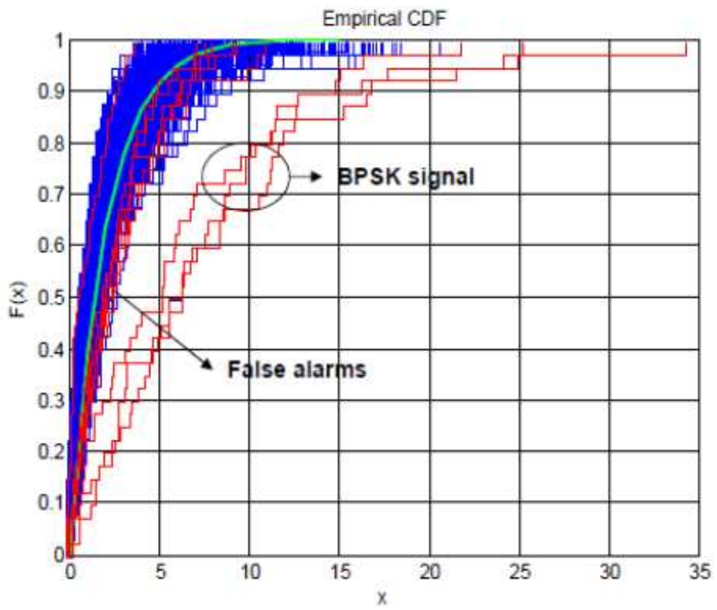

Figure 4.19 Empirical CDF for every frequency bin: in blue the CDFs in the H_0 hypothesis, in red the CDFs in the H_1 hypothesis. The CDF F_0 is represented in green [7].

In a second scenario, two modulated signals are considered keeping the same previous setup.

The first signal is a BPSK modulated signal, centered at $3MHz$, with $SNR = 0dB$.

The second signal is a DAB mode-I signal, centered around $7MHz$ with $SNR = -5dB$.

Figure 4.20 shows the result corresponding to the this scenario. It can be seen that most of the frequency bins where a modulated signal is present are tagged as occupied, with an 1 on the y-axis means that the frequency bin is found to be in the H_1 hypothesis. The rest of 1 in the H_0 hypothesis, corresponds to false alarms. Through this simulation, the strength of the wide-band GoF spectrum sensing is proved.

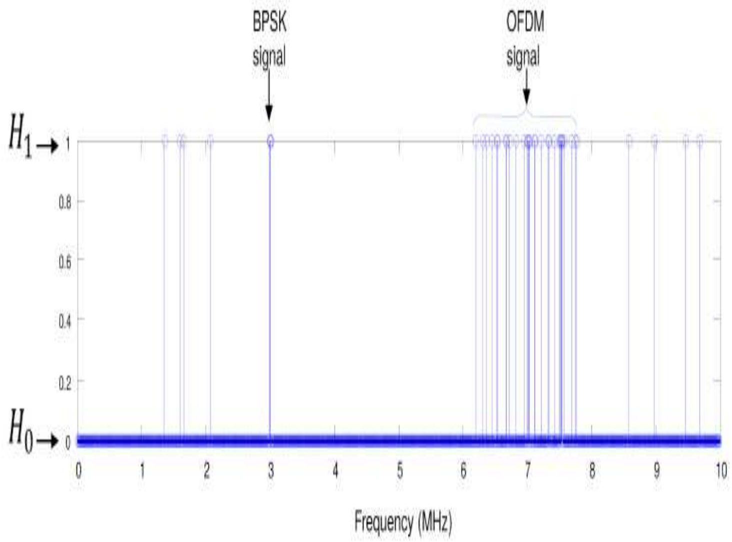

Figure 4.20 Wide-band sensing result on the 2 low SNR signals: $N = 1024$, $K = 40$, $\lambda = 3.89$ [7].

4.9 Conclusion

In this chapter, we have proposed a blind spectrum sensing method based on GoF test. The novelty in the proposed GoF sensing methods was to consider the energy of the received samples and test them against a chi-square distribution under hypothesis H_0. Firstly, the chapter has provided a comparative study among existing GoF sensing methods as well as some other adapted and modified GoF tests. It has been shown that ED sensing has better detection performances compared to AD, CM and KS GoF sensing. Besides, It has been shown that the modified AD GoF sensing outperforms the conventional AD GoF sensing. Moreover, we have proposed the LLR-GoF sensing method and it has been found that LLR-GoF sensing outperforms AD-GoF sensing and it has comparable performance with ED based sensing. To show the effectiveness of the proposed GoF sensing methods, we have studied some typical impairment for spectrum sensing, i.e. the effect of a non Gaussian noise and noise uncertainty on the performance of GoF based sensing and ED based sensing. As a model for the non Gaussian noise, we used the Gaussian mixture (GM). It has been observed that a non Gaussian noise can affect noticeably the performance of ED, but has only a limited influence on the performance of the GoF based sensing methods. The same conclusion can be drawn for the noise uncertainty. This is mainly due to the fact that the test statistics in GoF testing is based on the difference of the measured CDF and the reference CDF and hence only indirectly influenced by noise parameters. Then, we have proposed a new GoF test statistic which takes into account the physical characteristic of spectrum sensing. It has be found that the resulting spectrum sensing method of the proposed GoF tests statistic achieves a significant improvement compared to other GoF sensing methods and approaches more the ED detection performance. Finally, we have presented a wide-band spectrum sensing based on the distribution of the power coefficients of DFT. It has been shown that an accurate decision per frequency bin can be made after only a few DFTs. This work can be investigated by deriving the theoretical expressions for the probabilities of detection and false alarm of GoF sensing in order to testify the simulation results.

Distributed Consensus Spectrum Sensing For CRN

5

5.1 Introduction

In this chapter, we study a consensus algorithm for distributed spectrum sensing (DSS) in cognitive radio networks (CRN) integrating a Goodness of Fit based spectrum sensing scheme. Existing work in this area often applies energy detector as a local spectrum sensing method for DSS, however in this case one needs to make the assumption that the noise level is the same at every node in the network, otherwise the threshold can not be set properly. Most energy detection schemes are based on constant noise power [98], [41], [99], [59] and [100]. In GoF based spectrum sensing, the distribution of the test statistic is independent of the noise power, hence the threshold for the binary test depends only on the desired false alarm probability and not on the local noise powers. Motivated by this nice feature of GoF based spectrum sensing, we consider the goodness of fit (GoF) test statistic to be exchanged among cognitive radio (CR) users (consensus variable) instead of the energy. Moreover, a weighted consensus based DSS scheme is proposed and compared to the conventional consensus based on DSS. Simulations are conducted to show the effectiveness of the consensus algorithm based on GoF test. In order to test the optimality of that proposed method, we implement some optimal schemes such as an exhaustive search scheme and Genetic algorithm schemes using GoF sensing as local detection and compared their performances with the weighted consensus based DSS scheme.

5.2 Related Works

Cooperative spectrum sensing can be performed in two models: centralized or distributed as illustrated in figure 5.1. The former requires a common receiver (fusion center) to collect sensing results from all CR users in order to make final decision about the presence of a PU signal. However, a distributed scheme permits to CR users to share individual sensing results with their neighbors in order to make their own sensing decisions. This scheme is more suitable for cognitive radio ad-hoc networks (CRAHN), in which no hierarchical structure is involved, therefore, any node failure would not result in the failure of the entire network [101].

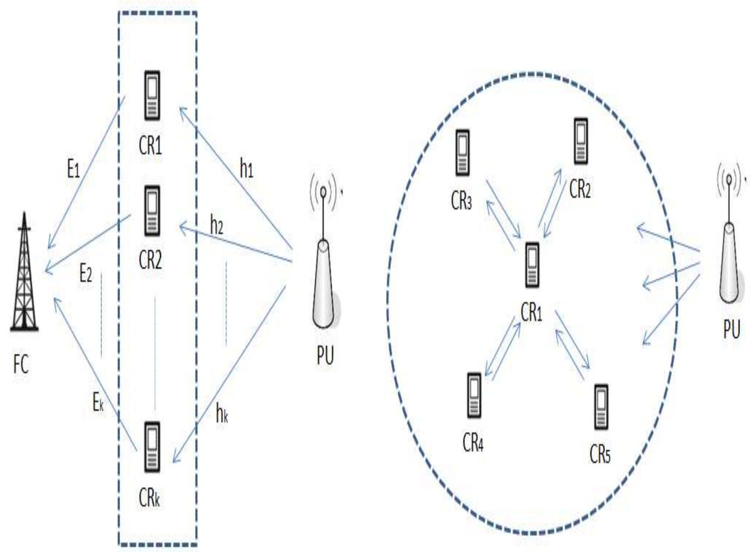

Figure 5.1 Centralized Cooperative Spectrum Sensing (left) and Distributed Cooperative Spectrum Sensing (right)

A large number of studies have adopted a centralized cooperative spectrum sensing such [100] [66] [102], where a central unit (fusion center) collects hard or soft sensing information from cognitive radios, makes a final decision about the presence of PU, and broadcasts this information to other CR users.

Beside, distributed spectrum sensing has been a subject of several studies in recent years. Consensus algorithms have been utilized in order to obtain an agreement value in distributed systems [103].

In [104], a biologically inspired consensus-based spectrum sensing scheme without a fusion centre was proposed. A fully distributed spectrum sensing scheme is presented, where each CR user uses a biologically inspired computation rule to generate an updated state of the consensus variable. Authors in [105], extend the latter work with fixed bidirectional and random graphs. In the proposed scheme, CR users exchange messages based on local interaction without a centralized common receiver, and the consensus of the CR users is used to make the final decision. In [106], consensus-based spectrum sensing similar to that proposed in [105], is used in order to improve the security of CRAHNs using ID-based cryptography with threshold secret sharing. A weighted consensus-based spectrum sensing scheme is proposed in [107]. The CR users measure energy based on energy detection and then exchange the measured energy with its neighbors. The information exchanged is weighted according to its own estimated SNR value. These algorithms perform detection in two time phases, one phase to take the measurement and an another phase to run the consensus algorithm. In [108],a distributed detection scheme based on diffusion strategies which can track changes in the PU state is proposed, i.e. a new measurement is incorporated into the algorithm on the fly. It is worth to mention that in the cited works, they assume that the noise is the same in every CR node.

In this chapter, we aim to perform detection in a distributed way, i.e., without fusion center, relying on a new metric to be exchanged among CR users known as a GoF test statistic. In [95], it was shown that the GoF based spectrum sensing outperforms the conventional energy detection, moreover, the method is less sensitive to noise uncertainty [109] and the test statistic is independent of noise power [81]. Hence, the distributed consensus based on spectrum sensing is presented relying on the communication of GoF test statistic values among CR users. Moreover, a weighted consensus based DSS scheme is proposed and compared to the conventional consensus based DSS.

5.3 Network Model for Distributed Spectrum sensing

We model a cognitive radio network as a graph $G = (V, E)$ with N CR users collaborating to detect the presence or absence of a signal, where V is the vertices of the graph (identified by the index of the CRs $i = \{1, ..., N\}$) and E is the set of edges of the graph represented as the set of links between each pair of CRs. Links (i, j) is denoted e_{ij} and refers to the information flowing from vertex j to vertex i which is equivalent to information flowing from vertex i to vertex j, if no direction is assigned to the edges (undirected graph). The set of all CR neighbors of a vertex i is defined as $N_i = \{j \in V : e_{ij} \in E\}$. The maximum degree of the CRN is defined as the maximum number of neighbors of a CR node.

The adjacency matrix A of G is the matrix with entries a_{ij} is given by

$$a_{ij} = \begin{cases} 1, & \text{if } e_{ij} \in E \\ 0 & otherwise \end{cases} \tag{5.1}$$

We assume a bidirectional communication between any two CR users, i.e.; $a_{ij} = a_{ji}, \forall i, j \in N$. The Laplacian L of the graph G is defined as :

$$l_{ij} = \begin{cases} |N_i|, & \text{if } j = i \\ -1, & \text{if } j \in N_i \\ 0 & otherwise \end{cases} \tag{5.2}$$

The matrix L is positive semi-definite.

5.4 Spectrum sensing Model

The first stage of distributed spectrum sensing based on consensus scheme is a local measurement performed by each CR user. The statistic hypothesis test for local spectrum sensing can be modeled as:

$$y_i(n) = \begin{cases} w_i(n) & H_0 \\ h_i(n)x(n) + w_i(n) & H_1 \end{cases} \tag{5.3}$$

where $x(n)$ is the unknown signal of the primary user, $w_i(n)$ is the zero-mean additive white Gaussian receiver noise of the ith CR user, and $h_i(n)$ is the channel gain from the primary user to the ith CR user.

As mentioned before, on previous works [105] [104] and [107], the distributed consensus based sensing schemes use energy detector as a local sensing method. The energy detection based spectrum sensing [72] consists of passing the received signal through a band-pass filter of bandwidth W and center frequency f_s. The filtered signal then is squared and integrated over the sensing period T (m=2TW, m: sample size). The output from integrator is distributed among neighbor CR users for consensus. Y_i is formulated based ED as:

$$Y_i = \sum_{n=1}^{m} |y_i(n)|^2, i = 1, 2, ..., N(CRusers). \tag{5.4}$$

Y_i in the above equation is the sum of the square of m independent Gaussian distributed random variables. As a result, Y_i follows the central chi-square distribution under hypothesis H_0, otherwise,Y_i follows the non-central chi-square distribution.

Once the local sensing is performed, CR users communicate their local information (in a soft format) with their neighbors until reaching the consensus. The local CR nodes can then take a decision based on this consensus value by comparing it to a threshold. The problem is that to set this threshold, one needs to know the noise power, however the noise could vary due to local interference, differences in AGC setting , hardware impairments etc, so the threshold can not be set properly. The GoF based sensing requires only the knowledge of the noise distribution under H_0 hypothesis and the threshold for the binary test depends only on the desired false alarm probability and not the local noise power seen by the CR nodes.

As it is mentioned, in this chapter, instead of communicating the energy, we propose communicate the GoF test statistic (for example: the Anderson

Darling test) among CR users, which means that the local sensing is a GoF based spectrum sensing method.

GoF tests were proposed in mathematical statistics by measuring a distance between the empirical distribution of the observation made and the assumption distribution. In CRNs, the GoF test is used to solve a binary detection problem and decide whether the received samples are drawn from a distribution with a Cumulative Distribution Function (CDF) F_0, representing the noise distribution, or they are drawn from some distribution different from the noise distribution. The statistical hypothesis test is given by:

$$\begin{aligned} H_0 &: F_m(x) = F_0(x) \\ H_1 &: F_m(x) \neq F_0(x), \end{aligned} \tag{5.5}$$

for a random sample of m independent and identically distributed observations, where $F_m(x)$ is the empirical CDF of the received sample and can be calculated by:

$$F_m(x) = |\{i : x_i \leq x, 1 \leq i \leq m\}/m|, \tag{5.6}$$

where $|\bullet|$ indicates cardinality, $x_1 \leq x_2 \leq .... \leq x_m$ are the samples under test and m represents the total number of samples.

There have been many goodness of fit test proposed in literature. The most important one is the Anderson-Darling test $A^2_{m,i}$. The expression of $A^2_{m,i}$ can be given according to [95] as:

$$A^2_{m,i} = -m - \frac{\sum_{k=1}^{m} (2k-1)(\ln z_k + \ln(1 - z_{(m+1-k)}))}{m}, \tag{5.7}$$

for $i = 1, .., N$ and with $z_k = F_0(x_k)$.

Each CR user exchanges the GoF test value $A^2_{m,i}$ with its neighbors, and then update it based on the received GoF test values from neighbors using consensus algorithms (details in the next section).

5.5 The Consensus Algorithms for Distributed Spectrum Sensing

For the N CR users distributed according to the graph G, we assign them a set of states variable X_i (consensus variable) for $i \in N$. The consensus algorithms aim to distribute the X_i's through an iterated process. By achieving consensus, the consensus variable X_i progressively converges to the common value X^* such as $X_i(k) \to X^*$, $k \to \infty$, where k is the discrete time.

We distinguish different cases:

1- It is said that the average consensus is achieved if all the individual state variable x_i, asymptotically converge to the average value i.e $X^* = mean(X) = \frac{1}{N}\sum_{i=1}^{N} X_i(0)$.

2- It is said that the maximum consensus is achieved if all the individual state variable x_i, asymptotically converge to the maximum value i.e $X^* = max_{i=1}^{N} X_i(0)$.

3- It is said that the minimum consensus is achieved if all the individual state variable X_i, asymptotically converge to the minimum value i.e $X^* = min_{i=1}^{N} X_i(0)$.

It is worth noting that the OR rule and the AND rule for cooperative spectrum sensing can be viewed as a form of max-consensus and min-consensus respectively. Likewise, the voting rule for CSS can be viewed as a form of weighted average consensus which will be described in the next section.

The performance of consensus algorithms is associated with the connectivity of CRN. The consensus based spectrum sensing algorithms can be expressed using a discrete-time state equation:

$$X_i(k+1) = X_i(k) + \varepsilon \sum_{j \in N_i} (X_j(k) - X_i(k)) \tag{5.8}$$

where $X_i(k)$ is the updated state at time k of CR user i, N_i denotes the neighbor set of CR user i and ε is a consensus parameter (step-size) satisfies:

$$0 < \varepsilon < (\max_i |N_i|)^{-1} \doteq 1/\Delta \tag{5.9}$$

where Δ is called the maximum degree of the network.

The consensus algorithm can be written in the vector form as:

$$X(k+1) = PX(k) \tag{5.10}$$

where $X = [X_1, ..., X_N]^T$ and $P = I - \epsilon L$ is called the Perron matrix. P is a stochastic matrix if the condition in (9) is ensured. Since G is an indirect connected graph, therefore, P is a doubly stochastic matrix, which means that P is a non negative matrix and all of its row sums and column sums are equal to one.

In our proposed method, we construct P based on the Metropolis weights [110] where the p_{ij} are defined as:

$$p_{ij} = \begin{cases} \frac{1}{1+max(|N_i|,|N_j|)}, & \text{if } (i,j) \in E \\ 1 - \sum\limits_{j \in N_i} N_j, & \text{if } i = j \\ 0 & otherwise \end{cases} \tag{5.11}$$

In this alternative method, the knowledge of the maximum degree of the network is not needed. Since that any two neighboring nodes exchange their degree.

There are two stages in the consensus spectrum sensing scheme.

In the first stage of spectrum sensing, every CR user performs GoF based spectrum sensing to get a local measurement $A^2_{m,i}$. We set up the initial GoF test vector such as: $X_i(0) = A^2_{m,i}$.

In the second stage, the average consensus algorithm or the maximum-consensus algorithm is conducted iteratively based on the fixed graph model at time $k = 0, 1, 2, ...$ The iterative process is done until all the individual states $X_i(k)$ converge towards a common value X^*. Then, a decision is taken by every CR user by comparing the common value X^* with a pre-defined threshold λ, every CR user obtains the global decision as:

$$
\begin{aligned}
H_0 &: X^* < \lambda \\
H_1 &: X^* \geq \lambda
\end{aligned}
\tag{5.12}
$$

λ is chosen on the function of the predefined Pfa according to table 5.1. The table 5.1 gives a corresponding λ for some critical values of Pfa through Monte-Carlo simulations.

5.6 Weighted Average Consensus for Distributed Spectrum Sensing

In this section, motivated by [107], we present a weighted consensus for DSS. Compared to [107], our scheme use weights based on the local measured value by each CR node and its neighbors values (GoF test statistic). Moreover, the weights are updated at each step. In other terms, the weights are set according to the channel condition. Knowing that the GoF test statistic reflects the channel condition, we can use the GoF test statistic itself as a weight. Setting weights in the consensus algorithm favorites the CR nodes with higher values of GoF statistic measurement. The proposed weighted consensus is formulated as:

$$X_i(k+1) = \varpi_i(k)X_i(k) + \varepsilon \sum_{j \in N_i} \varpi_j(k)(X_j(k) - X_i(k)) \tag{5.13}$$

with ϖ_i and ϖ_j are the weights associted to nodes i and j respectively.

where: $\varpi_i(k) = \frac{X_i(k)}{X_i(k)+\sum\limits_{l \in N_i} X_l(k)}$ and $\varpi_j(k) = \frac{X_j(k)}{X_i(k)+\sum\limits_{l \in N_i} X_l(k)}$

with $\varpi_i(k) + \sum\limits_{j \in (N_i)} \varpi_j(k) = 1$

For convenience, we re-write the equation (5.13) in the following compact form:

$$X(k+1) = P_{\varpi} X(k) \tag{5.14}$$

Consensus based DSS	Pfa	0.1	0.05	0.01
	Threshold	1.180	1.249	1.348
Weighted consensus based DSS	Pfa	0.1	0.05	0.01
	Threshold	2.036	2.217	2.710

Table 5.1 Threshold values for some given Pfa

where $X = [X_1, ..., X_N]$ and P_{ϖ} is the weighted Perron matrix.

It can be shown that by using (5.13) the consensus value X^* will converge to the weighted average of initial GoF test statistic values $\frac{1}{N}\sum_{i=1}^{N} \varpi_i(0)X_i(0)$. This convergence is concluded from the famous Perron Frobenius Theorem [111].

Likewise, each CR user performs sensing based on GoF based detector, and simultaneously collects GoF test statistics from its connecting neighbor CR users. It then updates its sensing value (GoF statistic) iteratively using its own and neighbors sensing information according to the algorithm (5.13). As time elapses, the sensing information will be diffused through the network and finally each CR user obtains a consensus value x^*. Every CR user is able to take the global decision by comparing the common value X^* with a predefined threshold λ such as:

$$\begin{aligned} H_0 &: X^* < \lambda \\ H_1 &: X^* \geq \lambda \end{aligned} \tag{5.15}$$

The value of λ is determined for a specific value of Pfa. A table listing values of λ corresponding to different false alarm probabilities can be computed by Monte Carlo approach. The table 5.1 gives a corresponding λ for some critical values of Pfa. As the test statistic values for the Consensus based DSS and the Weighted consensus based DSS are different, the values of λ will be different for both methods. The value of λ is dependent of Pfa and the test statistic value.

In this section, we conduct simulation to study the performances of the proposed weighted consensus scheme. We show the convergence of the weighted scheme and evaluate the detection performance of the weighted consensus based DSS through Monte Carlo simulation. The simulations are done such that, each CR user has different SNR values varying randomly from $-20dB$ to $0dB$. We consider a network topology with 50 nodes as depicted in figure 5.2.

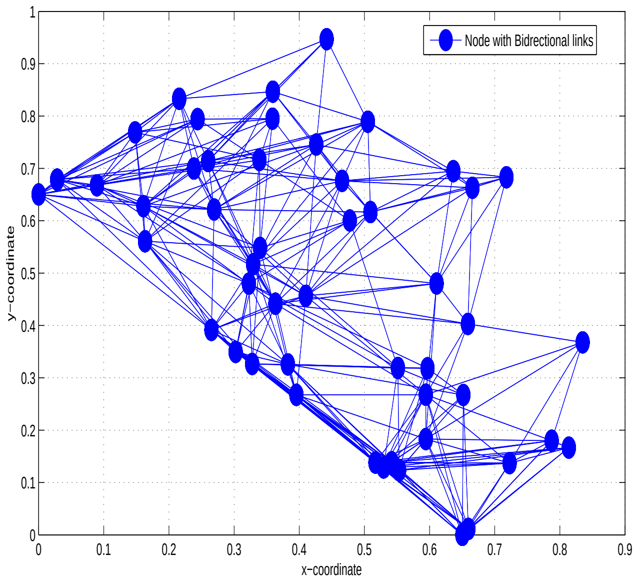

Figure 5.2 The network with 50 CR users and fixed graph

Figure 5.3 shows the convergence of the GoF statistic calculated based on the received signal from a PU in the network. As one can observe, despite the fact that the initially sensed measurement varies greatly due to their different wireless channel conditions for different CR users, a consensus is achieved after several iterations. The same goes for the proposed weighted scheme, when we show that the GoF test values, calculated initially by different CR users, tend to converge towards a consensus value after several iterations. It is observed that the consensus value, reached by the proposed weighted consensus, is higher than the consensus value reached by the conventional consensus.

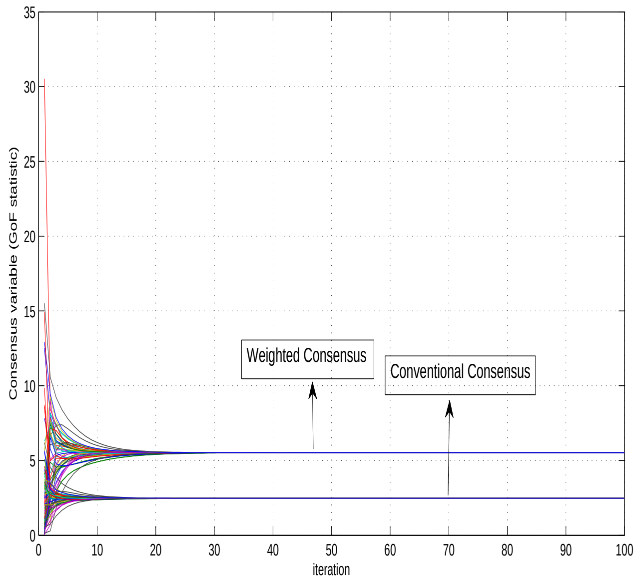

Figure 5.3 Convergence of the network for conventional consensus based GoF test

In figure 5.4, we plot the ROC curves (detection probability versus false alarm probability) for the proposed weighted consensus based DSS and the conventional consensus based DSS under AWGN channel with CRN size 50 nodes. From figure 5.4 , it is shown that the proposed weighted consensus detection based on GoF test have a significant improvement compared the conventional consensus detection based on GOF, in terms of detection performances.

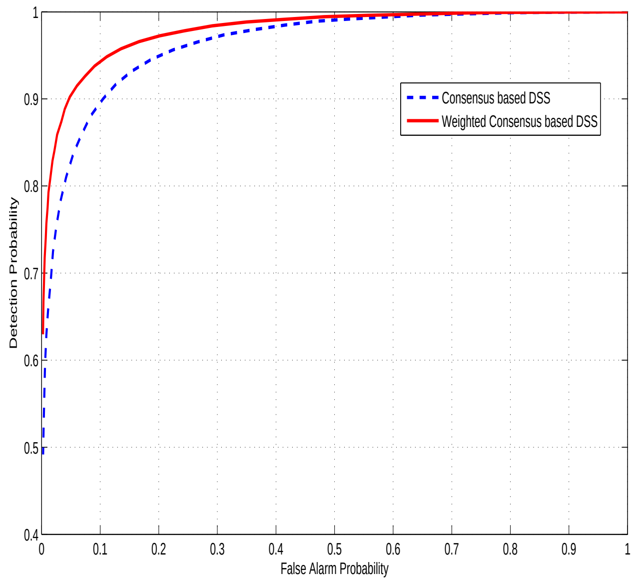

Figure 5.4 Detection probability versus false alarm probability for proposed weighted consensus based DSS using GoF for local sensing

In this section, we have studied the performance of the proposed method and compared to the conventional consensus scheme, based on GoF as local spectrum sensing. It is shown that the proposed method outperforms the conventional one in terms of detection performances. In the next section, we will test how our proposed weights for DSS perform compared to the optimal weights.

5.7 Test the optimality of the proposed weighted consensus DSS scheme

In the previous section, we have proposed a weighted consensus DSS scheme and in order to test its optimality, we will study in this section some optimal cooperative spectrum sensing methods using GoF statistics. In this section, we assume a centralized scheme for CSS. Our objective is to optimize the weighting coefficients from the received observation vector such that the global probability of detection is maximized. Meaning that we have to solve the following problem

$$\omega^* = argmaxQ_d(\omega_i Z_i) \tag{5.16}$$

where ω_i are the weights associated to the CR user i, with $|\omega^2| = 1$ is a condition to be satisfied. Z_i is a local measurement performed by CR user i (Z_i supposed to be the GoF test statistic).

Several methods have been proposed to solve the problem in (5.16) such as Exhaustive Search (ES) based algorithm. In spite of its huge computational complexity, many works [112], [113] refer to this optimal method in order to compare the performance of their proposed methods.

Beside, ES based algorithm, Genetic Algorithm (GA) based algorithm is recently utilized to solve optimization problems in distributed spectrum sensing [114], [115]. Motivated by their performances (ES, GA) discussed in literature, we propose in the next section to test the optimality of our proposed weighted consensus based DSS method with some optimal (sub-optimal) schemes such exhaustive search and generic algorithms.

5.7.1 Exhaustive Search (ES) based algorithm

For determining the optimal set of weights assigned to the CR users based on the criterion given in (5.16),we shall first look at how an exhaustive search can be made to solve the problem in (5.16). For that, we sweep over all possible values of ω and then by exhaustive search we select the optimal $\omega*$ corresponding to the maximum value of detection probability.

The computational complexity associated with this approach is huge, since for every ω, all possible value of $Q_d(\omega_i Z_i)$ need to be computed. The main inconvenient of this algorithm is that all of these computations are wasteful except for those corresponding to $\omega*$. Moreover, this approach is not capable of adapting to the time variations in the underlying channel.

5.7.2 GA based GoF cooperative spectrum sensing

In this section, we present an other method which is considered as a self-adaptive global searching optimization algorithm, known as Genetic Algorithm (GA). The GA is an evolutionary technique used to develop many algorithms in order to search for optimal (or nearly optimal) solution and select those best suited for a particular environment [116]. Therefore, it can be used to solve the problem in (5.16) by performing a quick search in order to assign optimum (or nearly optimal) weights for CR users given the parameters of the radio environment.

The genetic algorithm (GA) mechanism consists on randomly generating a set of chromosomes that constitute a population (pops). The natural processes in GA are modeled by using function such as Selection, Crossover and Mutation. The selection function consists on the choice of the best chromosomes for reproduction through crossover and mutation. Once the selection function is done, the crossover function starts with pairing to produce new offspring. After crossover function, the mutation operation is performed to provide new search space [117].

In the proposed GA based GoF cooperative spectrum sensing, we proceed as follows:

Step 1: we randomly generate a population of pops chromosomes with $N * nbits$ bits long, where N is the number of CR users in the network and $nbits$ is the number of bits represent each chromosomes.

Step 2: we consider the weighting coefficients vector $\overrightarrow{\omega} = [\omega_1, \omega_2, ..., \omega_N]^T$ with $||\overrightarrow{\omega}||$. We decode each chromosome in the random population into its corresponding weighting coefficients vector.

Step 3: each weighting coefficient is normalized to satisfy the constraints related to the optimization problem (5.16).

Step 4: Compute the fitness value of every normalized decoded weighting vector. According to their fitness value, we rank their corresponding chromosomes and identify the best chromosomes.

Step 5: For next time, we reproduce new chromosomes using genetic algorithm operations: selection, crossover and mutation.

Step 6: The newly reproduced chromosomes are concatenated with the best chromosomes found on the previous time.

Step 7: Perform operations in step 2 and step 3 on the new population.

Step 8: Perform process in step 4 and test if is equal to predefined number of iterations to stop. Otherwise go to Step 5.

5.7.3 Simulation results and comparison

In this section, we try to test the proposed weighted consensus based DSS using GoF and compare its performances with some optimal schemes in the literature such as GA and exhaustive search. It is worth to mention that all these schemes use GoF sensing as local detection method. In figure 5.5, we present the ROC curves for some centralized schemes using 4 CR users and each CR user has different SNR values varying randomly from $-10dB$ to $-5dB$. Each CR user performs a local detection based on GoF sensing using 50 samples. It can be seen that the exhaustive search based CSS outperforms slightly the GA based CSS and it is much better than OR hard based CSS and EGC based CSS.

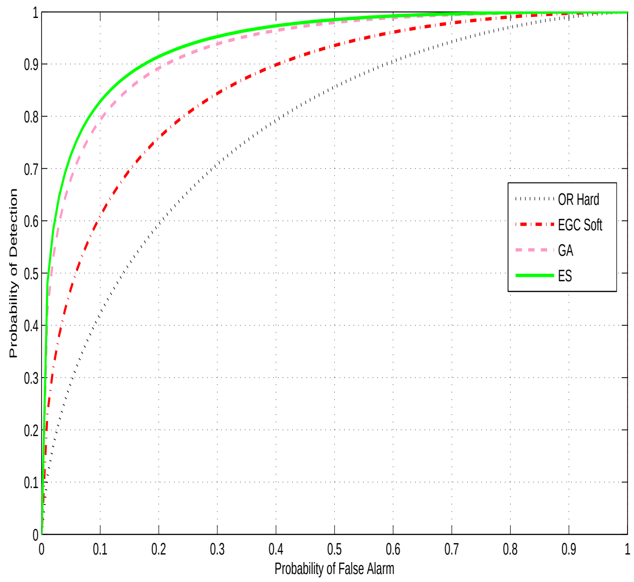

Figure 5.5 Detection probability versus false alarm probability for some optimal schemes using GoF for local sensing

In figure 5.6, we aim to test the optimality of the proposed weighted consensus based DSS using GoF and considering a topology with 10 CR users (nodes) and and each CR user has different SNR values varying randomly from $-10dB$ to $0dB$. We have kept the same sample size of 50 . Knowing that the computational complexity associated with the ES approach is huge, we will limit to GA based CSS as a reference for the optimality (as it was shown that ES based CSS outperforms slightly GA based CSS).

For GA based CSS, we need some default parameters, because finding the optimal parameters is out of scope of my thesis. Most of these parameters have been used in some well trusted sources in the field of genetic algorithms for cooperative spectrum sensing as in [115]. These parameters can be selected through numerical experiments by repetitive simulation considering one GA parameters at a time, by integrating GA and cooperative spectrum sensing optimization model. The parameters of GA based CSS are as follow:

Size of a chromosome population=20,

Number of genes in a trait=10,
Crossover probability=0.95,
Mutation probability=0.1,
Number of generations=100,
Percentage number of chromosomes for reproduction=0.9.

It is shown in figure 5.6 that the performance of our proposed method approaches the performance of the GA based CSS. However, it can be observed that we can improve the proposed method to be converged to the optimal solution. The proposed weights in our scheme are defined based on the value of the GoF test statistic. It was found that the optimal (or nearly optimal) weights are defined based on the value of the SNR, hence, a relationship between the GoF test statistic and the SNR can be resulted in a definition of optimal weights.

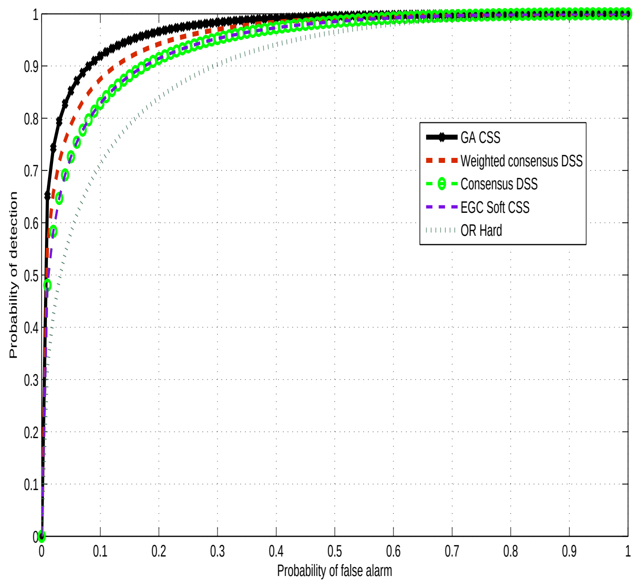

Figure 5.6 Detection probability versus false alarm probability for proposed weighted consensus based DSS using GoF for local sensing

5.8 Conclusion

In this chapter, we have presented a distributed spectrum sensing, based on consensus algorithms. This scheme, compared to centralized spectrum sensing scheme does not require a central unit. The detection problem is modeled as a graph networking topology. We have proposed to use a GoF test as a local spectrum sensing, because this test statistic is independent of the local noise power seen by the CR nodes and then considering a GoF test statistic to be exchanged among CR users to reach consensus. The effectiveness of the distributed consensus-based cooperative sensing has been shown when GoF sensing is used as a local sensing method. In order to optimize the proposed scheme, a weighted consensus based DSS is proposed by assigning different weights to the collaborating CR users. The performance of the proposed method is studied and compared to the conventional consensus scheme, based on GoF as local spectrum sensing. It has been shown that the proposed weighted consensus based DSS presents better performances, in terms of decision and efficient detection, compared to the conventional consensus based spectrum sensing. Beside, the optimality of the proposed method is tested with some optimal schemes such exhaustive search and generic algorithms (sub-optimal scheme). It was found that the proposed method approaches the performance of the optimal schemes, however, the proposed weights can be improved to converge better to the optimal solutions. Therefore, we suggest to study the relationship between SNR and GoF test statistic in order to optimally define the weights.

Conclusions and Future Work

6

In this concluding chapter, first, a summary of the main contributions from different chapters is given, then, several suggestions for future research areas are presented.

6.1 Conclusions

This thesis presented one of the key enabling techniques related to cognitive radio functionalities which is spectrum sensing as well as cooperative spectrum sensing.

Initial chapters have been devoted to the introduction to cognitive radio concept (definition, cognitive cycle and dynamic spectrum access). It has been shown that cognitive radio provides the ability to adapt to real-time spectrum conditions, by offering regulators allowing efficient and comprehensive use of the spectrum.

One of the important elements of cognitive radio is sensing the available spectrum opportunities. spectrum sensing topics have been treated in chapter 2, addressing various methods, their classifications and their advantages and drawbacks. We have discussed and studied some main spectrum sensing techniques for CR.

On the other hand, as cooperative spectrum sensing (CSS) approaches are commonly used for combating fading and improving detection performance, the performance of cooperative spectrum sensing algorithms using different combining rules has been analyzed in Chapter 3. It has been shown that soft combining rule for CSS outperforms hard combining rule in term of detection performance, to the detriment of overhead in terms of reporting channel bandwidth. To realize a a trade-off between overhead and detection performance, we have proposed the quantized combining scheme which benefits from the advantages of the soft and the hard combining rules.

Beside, the performance of CSS has been also developed under two operational modes scenarios, namely, CPUP (Constant Primary User Protection) and CSUSU (Constant Secondary User Spectrum Usability).

Further, the relationship between the throughput and sensing time has been studied for both scenarios and under different combining rules. The simulation results have shown that under CPUP, there is an optimal sensing time for which the CR throughput of the CR network is maximized and the highest value of the throughput can be obtained by the EGC soft combining rule. The two-bit quantized combining rule which has been derived in this chapter, could be an appropriate combining rule to provide a better compromise between performances (in terms of detection and throughput) and overhead (in terms of complexity and reporting channel bandwidth).

One of the key contributions of this thesis is the study of blind detection method based on Goodness of Fit test. GoF based spectrum sensing has been used for cognitive radio applications. In Chapter 4, it has been shown that in the GoF sensing, the threshold for the binary test depends only on the desired false alarm probability and not the local noise power seen by the CR node. This nice feature has motivated the study of the GoF sensing which has been compared to ED based sensing. Although the energy detection outperforms the GoF sensing in terms of detection performance, it has been shown that impact of some sensing impairments (noise uncertainty and non Gaussian noise) on GoF sensing is much less compared to the ED. In order to approaching the performance of ED, several GoF sensing has been proposed (or adapted) taking into account complex input samples model such LLR-GoF sensing, Modified AD-sensing and OS GoF sensing.

Moreover, two new GoF sensing methods have been proposed, resulting in significant improvement in terms of sensing performance. Besides, we have presented how GoF based spectrum sensing can be integrated in a conventional wideband spectrum sensing scheme. The effectiveness of GoF sensing methods is emphasized in a classical wideband spectrum sensing approach where an accurate decision per frequency bin can be made.

Motivated by its nice features of local sensing, in Chapter 5, a distributed consensus spectrum sensing for CR, has been presented, integrating a Goodness of Fit based spectrum sensing scheme. Distributed consensus spectrum sensing provides an improvement for primary user detection. Where existing work in this area often applies energy detector as a local spectrum sensing method for DSS, it makes the assumption that the noise level is the same at every node in the network, otherwise the threshold can not be set properly. In GoF based spectrum sensing, the distribution of the test statistic is independent of the noise power, hence the threshold for the binary test depends only on the desired false alarm probability and not on the local noise powers. Therefore, we have proposed to exchange the Goodness of Fit (GoF) test statistic (consensus variable) among cognitive radio (CR) users rather than the energy. It has been shown through simulations the effectiveness of the consensus algorithm based on GoF test in terms of detection performance keeping the same complexity compared to the existing works with high resistance against noise variation. Furthermore, a weighted consensus based DSS scheme has been proposed which assigns weights to each CR nose, based on the its local measured value (GoF test statistic) and its neighbors measured values. Compared to the conventional consensus based on DSS, it has been found that our proposed scheme outperforms the conventional one it term of detection performance. At the end, the optimality of the proposed scheme has been tested with some optimal schemes such exhaustive search and generic algorithms. Simulations results have shown that the proposed scheme approaches the optimal scheme.

6.2 Future Work

In this thesis, there are a number of research issues which have not been addressed. We list below some of them that merit much more research.

In Chapter 3, the detection performance of CSS, with a Gaussian distribution assumption, is expressed in two different scenarios. For a comprehensive study of the cooperative spectrum sensing algorithms, one needs to consider a chi-square distribution and accordingly derive the different expressions for the detection probability and false alarm probability considering different combining rules and under the two different operational modes (CPUP and CSUSU).

In chapter 4, the performance of Goodness of Fit statistical testing technique in spectrum sensing applications, has be studied using theoretical analysis and Monte Carlo simulation. However, for the new proposed GoF test statistic which takes into account the physical characteristic of spectrum sensing, one needs to derive the theoretical expressions for the probabilities of detection and false alarm. It will be very helpful to follow and adapt the steps proposed in [81]. This work could be interesting to testify if the proposed expression closely fits the simulation results.

In chapter 5, a weighted consensus based DSS scheme has been proposed and its optimality is tested. However, the proposed weights need to be improved to converge better to the optimal scheme. For that, we propose to deeply study a relationship between the signal to noise ratio (SNR) and GoF statistical test. Based on that relationship, we will be able to propose weights resulting in the optimal scheme for DSS.

List of Publications

Journal Papers

- D. Teguig, V. Le Nir and B. Scheers, "Spectrum sensing method based on goodness of fit test using chi-square distribution," published in *IET Journal (Electronics Letters)*, vol. 50(9), pages 713 - 715, 24 April 2014.
- D. Teguig, V. Le Nir and B. Scheers, "Spectrum Sensing Method Based on the Likelihood Ratio Goodness of Fit Test" published in *IET Journal (Electronics Letters)*, vol. 51(3), pages 253 - 255, 02 February 2015.
- B. Scheers, D. Teguig and V. Le Nir, "Modified Anderson-Darling detector for Spectrum Sensing," accepted in *IET Journal (Electronics Letters)*
- D. Teguig, V. Le Nir, B. Scheers and F. Horlin, "Spectrum sensing based on GoF testing: Analysis and Comparative Study," *ELSEVIER Journal on Computer Networks* , Revision.

Conference Papers

- D. Teguig, B. Scheers and V. Le Nir, "Data Fusion Schemes for Cooperative Spectrum Sensing in Cognitive Radio Networks," submitted in *Military Communications and Information Systems Conference*, Gdansk (Poland), Sep. 2012.
- D. Teguig, B. Scheers and V. Le Nir, "Throughput Optimization for Cooperative Spectrum Sensing in Cognitive Radio Networks" in *Next*

Generation Mobile Apps, Services and Technologies (NGMAST), Prague (Czech Republic), Oct. 2013.

- D. Teguig, B. Scheers and V. Le Nir, "Evolutionary Theoretical Game for Cooperative Spectrum Sensing in Cognitive Radio Networks: A Survey, " in *Information Systems Technology Panel Symposium on Cognitive Radio and Future Networks*, The Hague (Netherlands), May. 2014.

- D. Teguig, B. Scheers, V. Le Nir and F. Horlin, "Consensus Algorithms for Distributed Spectrum Sensing Based on Goodness of Fit Test in Cognitive Radio Networks," in *International Conference on Military Communications and Information Systems ICMCIS*, Cracow (Poland),May. 2015.

- B. Scheers, D. Teguig and V. Le Nir, "Wideband Spectrum Sensing technique based on Goodness-of-Fit testing," in *International Conference on Military Communications and Information Systems ICMCIS*, Cracow (Poland), May 2015.

Bibliography

[1] I. Akyildiz, W. Lee, M. Vuran, and S. Mohanty, "Next generation dynamic spectrum access cognitive radio wireless networks: a survey," *Computer Networks.*, vol. 50, no. 13, pp. 2127–2159, 2006.

[2] J. Mitola, "Cognitive radio an integrated agent architecture for software defined," *PhD Thesis Royal Institut of Technology (KTH)*, 2000.

[3] DySPAN, "Proceedings of the first ieee symposium on new frontiers in dynamic spectrum access networks," *DySPAN*, Nov 2005.

[4] S. Filin, H. Harada, H. Murakami, and K. Ishizu, "International standardization of cognitive radio systems," *Communications Magazine, IEEE*, vol. 49, no. 3, pp. 82–89, March 2011.

[5] Z. Tian and G. Giannakis, "A wavelet approach to wideband spectrum sensing for cognitive radios," *1st International Conference on Cognitive Radio Oriented Wireless Networks and Communications*, pp. 1–5, June 2006.

[6] I. F. Akyildiz, B. F. Lo, and R. Balakrishnan, "Cooperative spectrum sensing in cognitive radio networks: A survey," *in: Proc. of Asilomar Conf. on Signals, Systems, and Computers*, vol. 4, no. 1, pp. 40–62, March 2011.

[7] B. Scheers, D. Teguig, and V. L. Nir, "Wideband spectrum sensing technique based on goodness-of-fit testing," *ICMCIS 2015.*, 2015.

[8] I. Johnstone, "On the distribution of the largest eigenvalue in principle components analysis," *The Annals of Statistics*, vol. 29, no. 2, pp. 295–327, 2001.

[9] R.Kamath, “Spectrum sensing in cognitive radio networks,” *International Conference on Control, Communication and Copmuting*, 2010.

[10] Ofcom, “Technology research programme: Research and development,” *http://www.ofcom.org.uk/research/technology/overview/techrandd200405/*, p. 37, Oct. 2004/05.

[11] FCC, “Federal communications commission: Spectrumpolicy task force report,” *http://fjallfoss.fcc.gov/*, Nov. 2002.

[12] S. Atapattu, C. Tellambura, and H. Jiang, *Energy Detection for Spectrum Sensing in Cognitive Radio*. New York Heidelberg Dordrecht London: SpringerBriefs in Computer Science., 2013.

[13] J. Mitola and G. Q. Maguire, “Cognitive radio: making software radios more personal,” *IEEE Pers. Commun.*, vol. 6, pp. 13–18, Aug. 1999.

[14] S. Haykin, “Cognitive radio: brain-empowered wireless communications,,” *IEEE J. Sel. Areas Commun.*, vol. 23, no. 2, pp. 201–220, Feb. 2005.

[15] F. K. Jondral, “Software defined radio: basics and evolution to cognitive radio,” *Journal on Wireless Communications and Networking*, vol. 5, no. 3, pp. 275–283, 2005.

[16] J. Mitola, “Cognitive radio for flexible multimedia communications,” *IEEE International Workshop on.*, pp. 3–10, 1999.

[17] FCC and Docket, “Cognitive radio definition,” no. 3, Mar 2005.

[18] D. Document, “Standard definitions and concepts for spectrum management and advanced radio system technologies,” *Draft Document*, Jun 2006.

[19] [Online]. Available: http://www.ieeeusa.org/policy/positions/cognitiveradio.asp

[20] W. Xu, J. Zhang, P. Zhang, and C. Tellambura, “Outage probability of decode and forward cognitive relay in presence of primary user interference,” *IEEE Communications Letters*, vol. 16, no. 8, pp. 1252–1255, 2012.

[21] D. Hatfield and P. Weiser, "Property rights in spectrum: taking the next step," *in Proc. of the first IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks*, Nov 2005.

[22] L. Xu, R. Tonjes, T. Paila, W. Hansmann, M. Frank, and M. Albrecht, "Drive-ing to the internet: Dynamic radio for ip services in vehicular environments," *in Proc. of 25th Annual IEEE Conference on Local Computer Networks*, Nov 2000.

[23] C. Raman, R. Yates, and N. Mandayam, "Scheduling variable rate links via a spectrum server," *in Proceedings of the first IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks*, pp. 110–118, 2005.

[24] Ieee 1900 standards committee, ieee scc 41. [Online]. Available: http://www.scc41.org

[25] I. S. 802.22, "Cognitive wireless ran medium access control (mac) and physical layer (phy) specifications: Policies and procedures for operation in the tv bands," *IEEE Std 802.22-2011*, pp. 1–680, 2011.

[26] A. Flores, R. Guerra, E. Knightly, P. Ecclesine, and S. Pandey, "Ieee 802.11af: A standard for tv white space spectrum sharing," *IEEE Communications Magazine*, vol. 51, no. 10, pp. 92â–100, 2013.

[27] C. Cordeiro, L. Challapali, D. Birru, and S. Shankar, "Ieee 802.22: The first worldwide wireless standard based on cognitive radios," *IEEE DySPAN2005*, Nov 2005.

[28] K. Sithamparanathan and A. Giorgetti, *Cognitive Radio Techniques, Spectrum Sensing, Interference Mitigation, and Localization*. Boston-London: Mobile communications series., 2012.

[29] [Online]. Available: http://gnuradio.org/redmine/projects/gnuradio/wiki

[30] [Online]. Available: Ettus,ETTUSwebsite,http://www.ettus.com/

[31] [Online]. Available: http://warp.rice.edu/

[32] [Online]. Available: http://bee2.eecs.berkeley.edu/

[33] T. J. Lim, R. Zhang, Y. C. Liang, and Y. Zeng, "Glrt-based spectrum sensing for cognitive radio," *Proc. of IEEE GLOBECOM, New Orleans, LA*, 2008.

[34] H.Urkowitz, "Energy detection of unknown deterministic signals," *IEEE Proceeding*, vol. 55, no. 4, pp. 523–531, 1967.

[35] F. Digham, M. Alouini, and M. Simon, "on the energy detection of unknown signals over fading channels," *in: Proc. IEEE ICC*, vol. 5, pp. 3575–3579, 2003.

[36] R. Tandra and A. Sahai, "Fundamental limits on detection in low snr under noise uncertainty," *International Conference on Wireless Networks, Communications and Mobile Computing*, vol. 1, pp. 464–469, June 2005.

[37] ——, "Snr walls for signal detection," *IEEE J. Sel. Topics Signal Process.*, vol. 2, pp. 4–17, Feb 2008.

[38] H. L. van Trees, *Detection, estimation and modulation theory: Part I: Detection, Estimation and Linear Modulation Theory*. New York, U.S.A: John Wiley, 2001.

[39] J. Proakis, *Digital Communications*. United States: McGraw Hill, 2001.

[40] A. Sahai, R. Tandra, S. M. Mishra, and N. Hoven, "Fundamental design tradeoffs in cognitive radio systems," *Proc. First intl. workshop on Technology and policy for accessing spectrum*, pp. 2–5, 2006.

[41] D. Cabric, A. Tkachenko, and R. Brodersen, "Spectrum sensing measurements of pilot, energy, and collaborative detection," in *In Proc. of IEEE Military Communications Conference*, Oct 2006.

[42] A. V. Dandawate and G. B. Giannakis, "Statistical tests for presence of cyclostationarity," *IEEE Transactions on Signal Processing*, vol. 42, no. 3, pp. 2355â–2369, 1994.

[43] W. Gardner, "Exploitation of spectral redundancy in cyclostationary signals," *IEEE Signal Processing Magazine*, vol. 8, no. 2, pp. 14â–36, 1991.

[44] W. A. Gardner, A. Napolitano, and L. Paura, "Cyclostationary: Half of century of research," *Signal Processing*, vol. 86, pp. 639â–697, 2006.

[45] P. D. Sutton, K. E. Nolan, and L. E. Doyle, "Cyclostationary signatures in pratical cognitive radio applications," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 1, pp. 13â–24, 2008.

[46] K. Kyouwoog, I. A. Akbar, K. K. Bae, J.-S. Urn, and C. M. Spooner, "Cyclostationary approaches to signal detection and classification in cognitive radio," *IEEE International Symposiumon on New Frontiers in Dynamic Spectrum Access Networks (DySPAN)*, pp. 212â–215, 2007.

[47] F. Penna, R. Garello, D. Figlioli, and M. Spirito, "Exact non-asymptotic threshold for eigenvaluebased spectrum sensing," *in Proc. Int. Conf. on Cognitive Radio Oriented Wireless Network and Commun.*, 2009.

[48] Y. Zeng and Y. C. Liang, "Eigenvalue based spectrum sensing algorithms for cognitive radio," *IEEE Trans. and Commun.*, vol. 57, no. 6, pp. 1784–1793, 2009.

[49] M. Chiani and M. Z. Win, "Estimating the number of signal observed by multiple sensor," *IEEE Int. Workshop on Cognitive Inf. Process. (CIP 2010)*, June 2010.

[50] Y. Zeng and Y. C. Liang, "Spectrum sensing algorithms for cognitive radio based on statistical covariances," *IEEE Transactions On Vehecular Technology*, vol. 58, no. 4, pp. 1804–1815, 2009.

[51] B. Farhang-Boroujeny, "Filter bank spectrum sensing for cognitive radios," *IEEE Transactions on Signal Processing*, vol. 56, pp. 1801–â1811, May 2008.

[52] D. J. Thomson, "Spectrum estimation and harmonic analysis," *Proc. IEEE*, vol. 70, no. 9, pp. 1055–1096, Sep 1982.

[53] K. S. RIEDEL and A. SIDORENKO, "Minimum bias multiple taper spectral estimation," *IEEE Transactions on Signal Processing*, vol. 43, pp. 188–195, Jan 1995.

[54] S. haykin, "cognitive radio: brain-empowered wireless communications," *IEEE J. Select.Areas commun*, vol. 23, no. 2, pp. 201–220, Feb 2005.

[55] T. Bogale and L. Vandendorpe, "Moment based spectrum sensing algorithm for cognitive radio networks with noise variance uncertainty,"

47th Annual Conference on Information Sciences and Systems (CISS), pp. 1–5, Mar 2013.

[56] K. Cao and H. Shao, "Compressive wideband spectrum sensing using high-order statistics for cognitive radio," *IEEE Global High Tech Congress on Electronics (GHTCE)*, pp. 187–190, Nov 2013.

[57] S. Mishra, A. Sahai, and R. Brodersen, "Cooperative sensing among cognitive radios," *in Proc. IEEE Int. Conf. on Commun., (ICC 2006), Istanbul, Turkey*, vol. 1, pp. 1658â–1663, Jun 2006.

[58] T. Yucek and H. Arslan, "A survey of spectrum sensing algorithms for cognitive radio application," *IEEE Communications Surveys and Tutorials*, vol. 11, no. 1, 2009.

[59] D. Cabric, S. Mishra, and R. Brodersen, "Implementation issues in spectrum sensing for cognitive radios," *in: Proc. of Asilomar Conf. on Signals, Systems, and Computers*, vol. 1, pp. 772â–776, 2004.

[60] A. Ghasemi and E. Sousa, "Collaborative spectrum sensing for opportunistic access in fading environments," *DySPANn*, pp. 131â–136, Nov 2005.

[61] E. Peh and Y.-C. Liang, "Optimization for cooperative sensing in cognitive radio networks," *WCNC*, pp. 27–32, March 2007.

[62] J. Unnikrishnan and V. V. Veeravalli, "Cooperative spectrum sensing and detection for cognitive radio," *IEEE GLOBCOM*, pp. 2972–2976, Nov. 2007.

[63] T. Jiang and D. Qu, "On minimum sensing error with spectrum sensing using counting rule in cognitive radio networks," *in Proc. 4th Annual Int. Conf.Wireless Internet (WICONâ08)*, pp. 1–9, Aug. 2008.

[64] W. Zhang, R. Mallik, and K. Letaief, "Cooperative spectrum sensing optimization in cognitive radio networks," *in Proc. IEEE Int. Conf.Commun.*, pp. 3411â–3415, 2008.

[65] Z. Quan, S. Cui, and A. H. Sayed, "Optimal linear cooperation for spectrum sensing in cognitive radio networks," *IEEE Journal of Selected Topics in Signal*, 2009.

[66] E. Visotsky, S. Kuffner, and R. Peterson, "On collaborative detection of tv transmissions in support of dynamic spectrum sharing," *in Proc. 1st IEEE Int. Symp. New Frontiers in Dynamic Spectrum Access Netw. (DySPAN)*, pp. 338â–345, 2005.

[67] J. Ma and Y. Li, "Soft combination and detection for cooperative spectrum sensing in cognitive radio networks," *in Proc. IEEE Global Telecomm. Conf.*, pp. 3139â–3143, 2007.

[68] C. T. S.P. Herath, N. Rajatheva, "On the energy detection of unknown deterministic signal over nakagami channel with selection combining," *Proc. IEEE Symp. CCECCâ09 Canadian conference on.*, pp. 745–749, 2009.

[69] S. Herath and N. Rajatheva, "Analysis of equal gain combining in energy detection for cognitive radio over nakagami channels," *IEEE. GLOBECOM*, pp. 1–5, 2008.

[70] Y. Liu and D. Yuan, "Analysis of square-law combining for cognitive radios over nakagami channels," *WiComâ09 5th International Conference on*, pp. 1–4, 2009.

[71] V. I. Kostylev, "Energy detection of a signal with random amplitude," *In Proc. IEEE ICC*, pp. 1606–â1610, Apr 2002.

[72] F. F. Digham, M.-S. Alouni, and M. K. Simon, "On the energy detection of unknown signals over fading channels," *IEEE. Trans. Commun.*, vol. 55, pp. 21–â24, Jun 2007.

[73] Z. Quan, S. Cui, H. V. Poor, , and A. H. Sayed, "Collaborative wideband sensing for cognitive radios," *IEEE Signal Processing Magazine*, vol. 25, no. 6, pp. 63–â70, 2008.

[74] Z. Li, P. Shi, W. Chen, and Y. Yan, "Square law combining double threshold energy detection in nakagami channel," *Internation Journal of Digital Content Technology and its Application*, vol. 5, no. 12, Dec 2011.

[75] Ieee 802.22 working group, spectrum sensing requirement summary, doc num.22-06-0089-04-0000. [Online]. Available: http://grouper.ieee.org/groups/802/22/Meetingdocument/2006July/22-06-0089-04-0000

[76] E. C. Y. Peh, Y.-C. Liang, and Y. L. Guan, "Optimization of cooperative sensing in cognitive radio networks: A sensing-throughtput tradeoff view," *IEEE Transactions on Vehicul Technology*, vol. 58, no. 9, pp. 5294–5299, 2009.

[77] R. Fan and H. Jiang, "Optimal multi-channel cooperative sensing in cognitive radio networks," *IEEE Transactions on Wireless Communications*, vol. 9, no. 3, pp. 1128â–1138, 2010.

[78] S. J. Y, J. T, and L. S. Y, "Maximum channel throughput via cooperative spectrum sensing in cognitive radio networks," *IEEE Transactions on Wireless Communications*, vol. 8, no. 10, pp. 5166–5175, 2009.

[79] Y.-C. Liang, Y. Zeng, E. Peh, and A. Hoang, "Sensing-throughput tradeoff for cognitive radio networks," *IEEE Trans Wirel Commun.*, vol. 7, no. 4, pp. 1326â–1337, 2008.

[80] F. Massey, "The kolmogorov-smirnov test for goodness of fit," *Journal of the American Statistical Association*, vol. 69, pp. 730–737, 1974.

[81] T. W. Anderson and D. A. Darling, "Asymptotic theory of certain goodness of fit criteria based on stochastic process," *Annals of Mathematical Statistic*, vol. 23, pp. 193–212, 1952.

[82] H. Wang, E. H. Yang, Z. Zhao, and W. Zhang, "Spectrum sensing in cognitive radio using goodness of fit testing," *IEEE Trans. Wireless Commun*, vol. 8, pp. 5427–5430, 2009.

[83] M. A. Stephens, "Edf statistics for goodness of fit and some comparisons," *Journal of the American Statistic Association*, vol. 69, pp. 730–737, 1974.

[84] C. Sinclaira, B. Spurra, and M. Ahmada, "Modified anderson darling test," *Commun. Statist. Theory and Math*, vol. 19, pp. 3677–3686, 2007.

[85] A. G. Glen, L. M. Leemis, and D. R. Barr, "Order statistic in goodness-of-fit testing," *IEEE Transaction on Reliability*, vol. 50, pp. 209–213, 2001.

[86] S. Rostami, K. Arshad, and K. Moessner, "Order-statistic based spectrum sensing for cognitive radio," *IEEE Trans. Commun.*, vol. 16, pp. 592–595, 2012.

[87] J. Zhang, "Powerful goodness-of-fit tests based on the likelihood ratio," *J. R. Statist. Soc.B 64, Part 2*, pp. 281–294, 2002.

[88] G. Zhang, X. Wang, Y. C. Liang, and J. Liu, "Fast and robust spectrum sensing via kolmogorov-smirnov test," *IEEE Trans. Commun.*, vol. 58, pp. 3410–3416, 2010.

[89] T. Kieu-Xuan and I. Koo, "Cramer-von mises test spectrum sensing for cognitive radio systems," *Wireless Telecommunication Symposium (WTS)*, 2011.

[90] L. Shen, H. Wang, W.Zhang, and Z. Zhao, "Blind spectrum sensing for cognitive radio channels with noise uncertainty," *IEEE Commun. Lett.*, vol. 10, pp. 1721–1724, 2011.

[91] S. Bengio, "Statistical machine learning from data: Gaussian mixture models," *http://bengio.abracadoudou.com/lectures/gmm.pdf*.

[92] A. Saha and G. Anand, "Design of detectors based on stochastic resonance," *Signal Processing*, vol. 83, pp. 1193–1212, 2003.

[93] ——, "A robust detector of known signal in non-gaussian noise using threshold systems," *Signal Processing*, vol. 92, pp. 2676–2688, 2012.

[94] W. Jouini, "Energy detection limits under log-normal approximated noise uncertainty," *IEEE Signal Process. Lett.*, vol. 18(7), pp. 423–426, 2011.

[95] D. Teguig, V. L. Nir, and B. Scheers, "Spectrum sensing method based on goodness of fit test using chi-square distribution," *Electronics Letters IET*, vol. 50, no. 9, pp. 713â–715, April 2014.

[96] H. Sun and A. Nallanathan, "Wideband spectrum sensing for cognitive radio networks: A survey," *IEEE Wireless Communications.*, vol. 20, pp. 74–81, 2013.

[97] E. Axell, G. Leus, E. G. Larsson, and H. V. Poor, "Spectrum sensing for cognitive radio : State-of-the-art and recent advances," *IEEE Signal Processing Mag.*, vol. 29, pp. 101–116, 2012.

[98] F. A. Ian, Y. L. Won, and C. V. Mehmet, "A survey on spectrum management in cognitive radio networks," *IEEE Communications Magazine*, vol. 46, no. 4, pp. 40–48, April 2008.

[99] S. M. Mishra, A. Sahai, and R. W. Broderson, "Cooperative sensing among cognitive radios," *Proc. ICC 2005, Istanbul, Turkey*, 2006.

[100] E. Sousa and AGhasemi, "Collaborative spectrum sensing for opportunistic access in fading environments," *DySPANn*, pp. 131â–136, Nov 2005.

[101] I. F. Akyildis, W.-Y. Lee, and K. R. Chowdhurye, "Crahn: cognitive radio ad hoc networks," *AD HOC NETWORKS*, vol. 7, no. 5, pp. 810–836, Jul. 2009.

[102] G. Ganesan and T. Lim, "Cooperative spectrum sensing in cognitive radio networks," *IEEE, Dynamic Spectrum Access Networks (DySPAN)*, Nov. 2005.

[103] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proc. of the IEEE*, vol. 95, no. 1, pp. 215–233, Jan. 2007.

[104] F. Yu, M. Huang, and H. Tang, "Biological inspired consensus-based spectrum sensing in mobile ad hoc networks with cognitive radios," *IEEE Netw.*, vol. 24, no. 3, pp. 26–30, 2010.

[105] Z. Li, F. Yu, and M. Huang, "A distributed consensus-based cooperative spectrum sensing scheme in cognitive radios," *IEEE Trans. Veh. Technol.*, vol. 59, no. 1, pp. 383â–393, 2010.

[106] H. Tang, F. Yu, and M. Huang, "Distributed consensus-based security mechanisms in cognitive radio mobile ad hoc networks," *IET Commun.*, vol. 6, no. 8, pp. 974â–983, 2012.

[107] W. Zhang, Z. Wang, Y. Guo, H. Liu, Y. Chen, and J. M. III, "Distributed cooperative spectrum sensing based on weighted average consensus," *in: Proc. of IEEE Global telecommunication conference (GLOBECOM)*, pp. 1â–6, Dec 2011.

[108] F. S. Cattivelli and A. H. Sayed, "Distributed detection over adaptive networks using diffusion adaptation," *IEEE Transactions On Signal Processing*, vol. 59, no. 5, pp. 974â–983, May 2011.

[109] D. Teguig, V. L. Nir, B. Scheers, and F. Horlin, "Spectrum sensing method based on the likelihood ratio goodness of fit test under noise uncertainty," *IJERT*, vol. 3, no. 9, Sep 2014.

[110] L. Xiao, S. Boyd, and S. Lall, "A scheme for robust distributed sensor fusion based on average consensus," *in Proc. Fourth International Symposium on Information Processing in Sensor Networks*, pp. 63–70, 2005.

[111] M. Carl, *Matrix Analysis and Applied Linear Algebra*. SIAM, 2000.

[112] L. T. Tan and L. B. Le, "Joint cooperative spectrum sensing and mac protocol design for multi-channel cognitive radio networks," *EURASIP Journal on Wireless Communications and Networking*, vol. abs/1406.4125, Jun 2014.

[113] Z. ZHANG, Q. WU, and J. WANG, "Optimal energy-efficient cooperative spectrum sensing in cognitive radio networks," *RADIOENGINEERING*, vol. 22, no. 4, Dec 2013.

[114] R. Deka, S. Chakraborty, and J. S. Roy, "Optimization of spectrum sensing in cognitive radio using genetic algorithms," *Ser: Elec. Energ.*, vol. 25, no. 3, pp. 235–243, Dec 2012.

[115] M. K. Hossain and A. A. El-Saleh, "Cognitive radio engine model utilizing soft fusion based genetic algorithm for cooperative spectrum optimization," *International Journal of Computer Networks Communications (IJCNC)*, vol. abs/1304.3273, May 2013.

[116] E. S. Gopi, "Algortithm collections for digital signal processing applications using matlab," *Springer*, 2007.

[117] K. Arshad, M. A. Imran, and K. Moessner, "Collaborative spectrum sensing optimisation algorithms for cognitive radio networks," *International Journal of Digital Multimedia Broadcasting*, vol. 2010, 2010.