Spectrum Sensing Method Based On The Likelihood Ratio Goodness of Fit test

D. Teguig, V. Le Nir and B. Scheers

In this letter, a blind spectrum sensing method based on goodness-of-fit (GoF) test using likelihood ratio (LLR) is studied. In the proposed method, a chi-square distribution is used for GoF testing. The performance of the method is evaluated through Monte Carlo simulations. It is shown that the proposed spectrum sensing method outperforms the GoF test using Anderson Darling (AD) and the conventional energy detection (ED) in case of a low signal to noise ratio (SNR).

Introduction: The main function of spectrum sensing is to detect the presence of other users within a frequency band, in order to access the channel without causing interference [1]. Spectrum sensing methods are classified into two categories, coherent spectrum sensing methods and blind sensing methods. In coherent spectrum sensing methods, such as Cyclostationarity , matched filtering and waveform-based spectrum sensing [2] [3], the CR node uses a priori knowledge of the waveform of the considered signal. In case of blind spectrum sensing methods, the CR node does not require any prior knowledge of the transmitted waveform. Some examples are Energy Detection (ED) [4] and Goodness of Fit (GoF) tests [5]. Due to its low complexity, the ED is the most common method for spectrum sensing in CR. Nevertheless, the performance of the ED is deeply affected by noise uncertainty at low signal to noise ratio (SNR)[7].
The GoF test is a blind nonparametric hypothesis test problem which can be used to detect the presence of signals in noise by determining whether the received samples are (are not) drawn from a distribution with a Cumulative Distribution Function (CDF) F_0, representing the noise CDF. The hypothesis to be tested can be formulated as follows:

$$\begin{array}{c} H_0 : F_n(x) = F_0(x) \\ H_1 : F_n(x) \neq F_0(x), \end{array} \tag{1}$$

where $F_n(x)$ is the empirical CDF of the received sample and can be calculated by:

$$F_n(x) = |\{i : x_i \leq x, 1 \leq i \leq n\}/n|, \tag{2}$$

where $|\bullet|$ indicates cardinality, $x_1 \leq x_2 \leq .... \leq x_n$ are the samples under test and n represents the total number of samples.
There are many goodness of fit test based spectrum sensing proposed in literature. The most important ones are the Kolmogorov- Smirnov test [6], the Cramer-Von Mises test [8] and the Anderson-Darling test [5]. All these tests are based on the hypothesis test as formulated in (1), but differ in the way the distance between the empirical cumulative distribution of the observations made locally at the CR user and the noise CDF $F_0(x)$ is calculated. The calculated distance is compared with a threshold to decide whether the signal is present or not, given a certain probability of false alarm.
The GoF test based spectrum sensing was first presented in [5]. It is based on the Anderson-Darling GoF test to decide whether the received samples are drawn from the noise CDF F_0 (Gaussian CDF) or an alternative CDF. Authors in [5], show by simulations that AD-sensing outperforms the ED-sensing at low SNR. All above mentioned methods take as noise CDF a normal distribution F_0 for the GoF test. Meaning that they all assume that the samples of the received signal are real valued. As cognitive radio is based on the SDR technology, the received baseband samples in the digital domain are complex in nature. In this case, the most practical approach to apply the GoF test for spectrum sensing is to considering the squared magnitude of the complex samples (i.e energy of the samples) and test their empirical distribution against the hypothetical noise energy distribution [10].
In this letter, we will evaluate the performance of a more recent GoF test, i.e. the likelihood ratio (LLR) test, in the application of GoF based spectrum sensing for CR. The simulation results illustrate that the proposed LLR-GoF sensing method is performing better than the one based on AD-GoF [10] and ED spectrum sensing methods.

Likelihood based Goodness of fit test: In [9], the authors propose a new, more general approach of parametrization to construct a general GoF test. With this approach, they could generate the traditional GoF tests including KS, CM and AD. Moreover, they provided also a new, more powerful GoF test, based on likelihood ratio. The authors in [9] formulated the hypothesis test as follows:

$$\begin{array}{r} H_0 :\ H_0(t) : F_n(t) = F_0(t) \quad for\ all\ t \in (-\infty, \infty) \\ H_1 :\ H_1(t) : F_n(t) \neq F_0(t) \quad for\ some\ t \in (-\infty, \infty) \end{array} \tag{3}$$

meaning that testing H_0 versus H_1 is equivalent to testing $H_0(t)$ versus $H_1(t)$ for every $t \in (-\infty, \infty)$.
Two types of statistic for testing H_0 versus H_1 were proposed :

$$Z = \int_{-\infty}^{\infty} Z_t \, dw(t), \;\; and \tag{4}$$

$$Z_{max} = \sup_{t \in (-\infty, \infty)} \{Z_t \, w(t)\} \tag{5}$$

with Z_t a statistic for testing $H_o(t)$ versus $H_1(t)$ and $w(t)$ some weight function. Large values of Z or Z_{max} will reject a null hypothesis H_0. In [9], authors present two natural candidates for Z_t, the Pearson χ^2 test statistic and the likelihood ratio (LLR) test statistic. The LLR test statistic is given by:

$$G_t^2 = 2n[F_n(t) \log\{\frac{F_n(t)}{F_0(t)}\} + (1 - F_n(t)) \log\{\frac{1 - F_n(t)}{1 - F_0(t)}\}]. \tag{6}$$

where $F_n(t)$ is the empirical distribution function of the received samples. Taking in (4) Z_t as G_t^2 and choosing an appropriate weight function $w(t)$, produces a powerful goodness of fit tests statistic Z_A, comparing to the traditional tests.

$$Z_A = -\sum_{i=1}^{n} [\frac{log\{F_0(X_{(i)})\}}{n - i + \frac{1}{2}} + \frac{log\{1 - F_0(X_{(i)})\}}{i - \frac{1}{2}}]. \tag{7}$$

For the proposed spectrum sensing method in this paper, we will use the test statistic Z_A as LLR-GoF test. Once the test Z_A is computed, it will be compared to a predefined threshold λ with:

$$\begin{array}{c} H_0 : Z_A \leq \lambda \\ H_1 : Z_A > \lambda, \end{array} \tag{8}$$

Goodness of Fit testing for spectrum sensing: We have proposed in [10] to start from the more general model:

$$\begin{array}{rcl} H_0 : X_i & = & W_i \\ H_1 : X_i & = & S_i + W_i, \end{array} \tag{9}$$

where S_i are the received complex samples of the transmitted signal and W_i is the complex Gaussian noise. We now consider the random variable $Y_i = |X_i|^2$ which corresponds to the received energy. It is known that, if the real and the imaginary part of X_i are normally distributed, which is the case under H_0 hypothesis, the variable $Y_i = |X_i|^2$ is chi-squared distributed with 2 degree of freedom.
The spectrum sensing problem can now be reformulated as an hypothesis represented in (1) where we will test whether the received energy $Y_i = |X_i|^2$ are drawn from a chi-square distribution with 2 degree of freedom or not [10]. F_0, the CDF of the chi-square distribution is given by:

$$F_0(y) = 1 - e^{-y/2\sigma_n^2} \sum_{k=0}^{m-1} \frac{1}{k!} (\frac{y}{2\sigma_n^2})^k, y > 0, \tag{10}$$

with m is the degree of freedom (in our case m=2) and σ_n^2 is the noise power.

One of the nice features of GoF based spectrum sensing is that it needs fewer samples than ED to achieve the same sensing performance as presented in figure 1. It can be seen that the AD based sensing outperforms ED sensing under a limited number of samples and that the ED based sensing yields the same performance as GoF based sensing in terms of detection probability if the sample size is approximately 2.5 times the number of samples used for GoF based sensing.

The proposed spectrum sensing (LLR-GoF): The proposed spectrum sensing method can be summarised in the following steps:

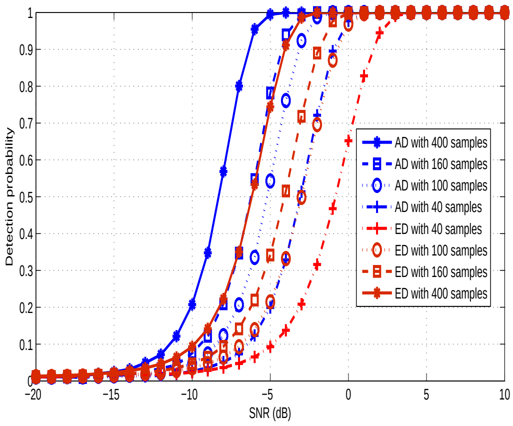

Fig. 1 *Detection probability versus SNR for AD detector and ED for different number of samples (40 100 160 and 400 samples) with* $Pfa = 0.01$

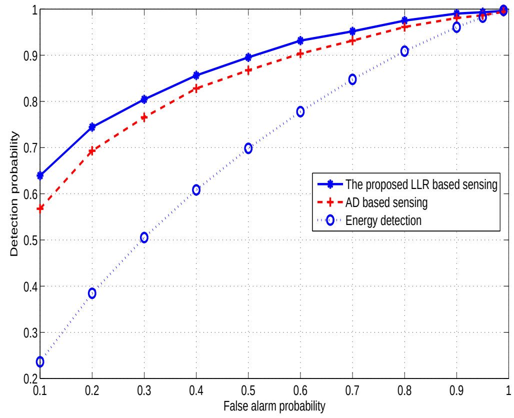

Fig. 2 *Detection probability versus false alarm probability over AWGN channels with* $SNR = -6$ *dB and* $n = 20$ *samples*

Step1 from the complex received samples X_i, calculate the energy samples $Y_i = |X_i|^2$
Step2 Sort the sequence $\{Y_i\}$ in increasing order such as $Y_1 \le Y_2 \le \dots \le Y_n$
Step3 Calculate the test Z_A according to (7), with F_0 given in (10).
Step4 Find the threshold λ for a given probability of false alarm such that:

$$Pfa = P\{Z_A > \lambda | H_0\}. \tag{11}$$

Step5 Accept the null hypothesis H_0 if $Z_A \le \lambda$. Otherwise, reject H_0 in favour of the presence of the primary user signal.

To find λ, it is worth to mention that the distribution of Z_A under H_0 is independent of the $F_0(y)$ [5],[11]. The value of λ is determined for a specific value of P_{fa}. A table listing values of λ corresponding to different false alarm probabilities P_{fa} is given in [9]. Otherwise, these values can be computed in advance by Monte Carlo approach.

Simulation Results: Figure 2 presents the detection probability as a function of the false alarm probability (ROC curves) of the proposed LLR-GoF based spectrum sensing method compared to the AD-GoF based sensing and the energy detection (ED). The results are obtained by 10000 Monte-Carlo simulations. For the AD-GoF method, the same 5 steps as for the LLR-GoF are followed, except for step 3 in which we took as a statistic test A_n^2 as given in [10]. The simulations are performed using only 20 samples of the received signal with a signal to noise ratio (SNR) equal to $-6dB$. It can be seen in figure 1 that the proposed LLR-GoF based sensing outperforms both AD-GoF based sensing and ED. For example, for $Pfa = 0.2$, the probability of detection P_d for the ED sensing equals 0.392, for AD based sensing P_d equals 0.695. However, for the proposed LLR-GoF sensing, P_d equals 0.745 .
In figure 3, the values of the detection probability versus SNR are plotted for the three sensing methods. The Pfa is set to 0.05 and the SNR varies from $-20dB$ to $10dB$, keeping the number of samples n to 20 samples. It can be seen that the proposed LLR-GoF based sensing has almost $1dB$ gain over AD based sensing and almost $5dB$ over ED sensing with $Pd = 0.8$ and $Pfa = 0.05$, hence the performance of the proposed LLR based sensing is indeed better than that of AD based sensing and ED sensing.

Conclusion: In this letter, we have proposed a blind spectrum sensing method based on GoF test. The novelty in the proposed spectrum sensing method was to consider the energy of the received samples and test them against a chi-square distribution under hypothesis H_0 using the likelihood ratio test statistic. It was shown by Monte-Carlo simulations that the proposed LLR-GoF sensing method outperforms both AD-GoF based sensing and ED based sensing, particularly for low SNR values.

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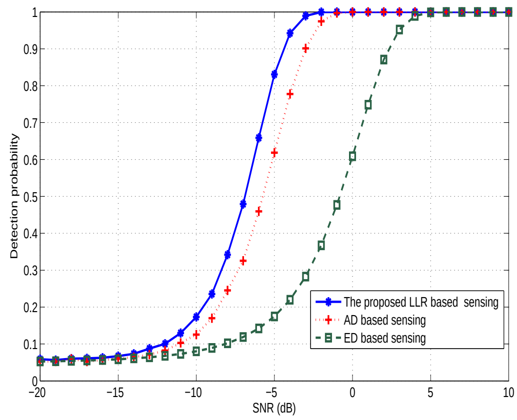

Fig. 3 *Detection probability versus* SNR *over AWGN channels with* $Pfa = 0.05$ *and n=20 samples*

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