Spectrum sensing method based on Goodness of Fit test using chi-square distribution

D. Teguig, V. Le Nir and B. Scheers

In cognitive radio, spectrum sensing is a challenging task. In this letter, a new spectrum sensing method is proposed based on Goodness of Fit test (GoF) of the energy of the received samples with a chi-square distribution. We derive the test statistic and evaluate the performance of the proposed method by Monte Carlo simulations. It is shown that our proposed spectrum sensing method outperforms the conventional energy detection (ED) without increasing the complexity of the sensing.

Introduction: One of the most important task in cognitive radio (CR) is spectrum sensing. The main function of spectrum sensing is to detect the presence of primary users utilizing the channel, in order to access the channel without causing interference [1].

Spectrum sensing methods are classified into two categories, coherent sensing methods and blind sensing methods. In coherent sensing methods, such as Cyclostationarity , matched filtering and waveform-based sensing [2] [3], the CR node uses a priori knowledge of the waveform of the primary user (PU). In case of blind sensing methods, the CR node does not require any prior knowledge of the transmitted waveform. Some examples are Energy Detection (ED) [4] and Goodness of Fit (GoF) tests such as Kolmogorov-Smirnov (KS) [5], Anderson Darling (AD) [6] and order statistics [7]. Due to its low complexity, the ED is the most common method for spectrum sensing in CR. Nevertheless, the performance of the ED is deeply affected by noise unceratinty at low signal to noise ratio (SNR).

The GoF test is a nonparametric hypothesis test problem which can be used to detect the presence of a PU by determining whether the received samples are (are not) drawn from a distribution with a Cumulative Distribution Function (CDF) F_0, representing the noise distribution. The hypothesis to be tested can be formulated as follows:

$$\begin{array}{c} H_0 : F_n(x) \cong F_0(x) \\ H_1 : F_n(x) \ncong F_0(x), \end{array} \tag{1}$$

where $F_n(x)$ is the empirical CDF of the sample and can be calculated by:

$$F_n(x) = |\{i : x_i \leq x, 1 \leq i \leq n\}/n|, \tag{2}$$

where $| \bullet |$ indicates cardinality, $x_1 \leq x_2 \leq .... \leq x_n$ are the samples under test and n represents the total number of samples.

There have been many goodness of fit test proposed in literature. The most important ones are the Kolmogorov- Smirnov test, the Cramer-von Mises test, and the Anderson-Darling test. In the following, we recall briefly these GoF tests.

A. Kolmogorov- Smirnov test (KS test): In this test the distance between $F_n(x)$ and $F_0(x)$ is given by:

$$D_n = max|F_n(x) - F_0(x)|, \tag{3}$$

where $F_n(x)$ is the empirical distribution which is defined in (2). If the samples under test are coming from $F_0(x)$, then, D_n converges to 0.

B. Cramer-Von Mises (CM test): In this test, the distance between $F_n(x)$ and $F_0(x)$ is defined as:

$$T_n^2 = \int_{-\infty}^{\infty} [F_n(x) - F_0(x)]^2 \, dF_0(x). \tag{4}$$

According to [8], and by breaking the integral in (5) into n parts, T_n^2 can be writen as:

$$T_n^2 = \sum_{i=1}^{n} [z_i - (2i-1)/2n]^2 + (1/12n), \tag{5}$$

with $z_i = F_0(x_i)$

C. Anderson-Darling test (AD test): This test can be considered as a weighted Cramer-Von Mises test where the distance between $F_n(x)$ and $F_0(x)$ is given by:

$$A_n^2 = \int_{-\infty}^{\infty} [F_n(x) - F_0(x)]^2 \, \frac{dF_0(x)}{F_0(x)(1 - F_0(x))}. \tag{6}$$

The expression of A_n^2 can be also simplified according to [8] to:

$$A_n^2 = -n - \frac{\sum_{i=1}^{n} (2i-1)(\ln z_i - \ln z_{(n+1-i)})}{n}, \tag{7}$$

with $z_i = F_0(x_i)$.

Goodness of Fit testing for spectrum sensing: As a strarting point, we recall the model in [5] in which the authors consider an AWGN channel.

$$\begin{array}{rcl} H_0 : r_i & = & w_i \\ H_1 : r_i & = & \sqrt{\rho} m + w_i, \end{array} \tag{8}$$

where H_0 and H_1 represent the hypothesis of absence and presence of a primary signal, respectively. m represents the transmitted signal, ρ is the signal to noise ratio (SNR), w_i is the real Gaussian noise with zero mean and unit variance and r_i are real valued. In [5], the sensing method is based on testing the GoF of the received samples compared to the Gaussian distribution.

The authors in [5] assumed that the transmitted signal m=1, in other words, the data is represented as $r_i = \sqrt{\rho} + w_i$. The model in (8) does not reflect a realistic scenario, as normally the received signal is complex and can vary in time.

In this letter, we propose to start from the more general hypothesis test:

$$\begin{array}{rcl} H_0 : X_i & = & W_i \\ H_1 : X_i & = & S_i + W_i, \end{array} \tag{9}$$

where S_i are the received complex samples of the transmitted signal and W_i is the complex Gaussian noise. We now consider the random variable $Y_i = |X_i|^2$ which corresponds to the received energy. It is proven that the variable Y_i is chi-squared distributed with 2 degree of freedom under H_0 hypothesis.

Proof:
Let $Z_1, Z_2 \cdots Z_n$ be real independent random variable with $Z_i \backsim N(0,1)$. If $Y = \sum_{i=1}^{n} Z_i^2$ then Y follows the chi-square distribution with n degrees of freedom, and denoted as $Y \backsim \chi_n^2$. In our case, we consider Z_i complex normal distributed variable and $Y_i = |Z_i|^2 = \alpha_i^2 + \beta_i^2$, where α_i and β_i are real and imaginary part of Z_i which are normal distributed variable. Therefore Y_i is chi-square distributed variable with 2 degree of freedom under hypothesis H_0.
The spectrum sensing problem can be transformed to testing the GoF of the received energy compared to the chi-square distribution. The CDF of the chi-square distribution is given by:

$$F_0(y) = 1 - e^{-y/2\sigma_n^2} \sum_{k=0}^{m-1} \frac{1}{k!} (\frac{y}{2\sigma_n^2})^k, y > 0, \tag{10}$$

with m is the degree of freedom (in our case m=2).

As a GoF test, we apply the Anderson-Darling test in (6) or (7) on the energy samples Y_i.

$$A_n^2 = -n - \frac{\sum_{i=1}^{n} (2i-1)(\ln z_i - \ln z_{(n+1-i)})}{n}, \tag{11}$$

with $z_i = F_0(y_i)$. z_i can be retrieved using the expression in (10) for m=2.

The proposed sensing method

The spectrum sensing problem is formulated as:

$$H_0 : A_n^2 \leq \lambda$$
$$H_1 : A_n^2 > \lambda, \tag{12}$$

where λ is a threshold. The proposed spectrum sensing method can be summarised in the following steps:

Step1 from the complex received samples X_i, calculate the energy samples $Y_i = |X_i|^2$

Step2 Sort the sequence $\{Y_i\}$ in increasing order such as $Y_1 \leq Y_2 \leq \cdots \leq Y_n$

Step3 Calculate A_n^2 according to (11)

Step4 Find the threshold λ for a given probability of false alarm such that:

$$Pfa = P\{A_n^2 > \lambda | H_0\}. \tag{13}$$

Step5 Accept the null hypothesis H_0 if $A_n^2 \leq \lambda$. Otherwise, reject H_0 in favour of the presence of the primary user signal.

To find λ, it is worth to mention that the distribution of A_n^2 under H_0 is independent of the $F_0(y)$ [5],[9]. The value of λ is determined for a specific value of Pfa. A table listing values of λ corresponding to different false alarm probabilities Pfa is given in [8]. For example, for Pfa=0.05, the value of λ equals 2.492.

Simulation Results: In this section, simulation results are presented to show the sensing performance of the proposed spectrum sensing method compared to ED. Figure 1 shows the ROC (Receiver Operating Characteristic) curves (detection probability versus false alarm probability) of the two spectrum sensing methods: ED and the spectrum sensing method based on GoF using chi-square. Both methods perform sensing on 20 samples of the received signal with a signal to noise ratio (SNR) equal to $-6dB$. The ROC curves were obtained for 10000 Monte-Carlo simulations. The detection probability for the proposed sensing method is given by:

$$Pd = P\{A_n^2 > \lambda | H_1\}. \tag{14}$$

From Fig. 1, it can be seen that the proposed spectrum sensing method outperforms the ED spectrum sensing algorithm. It is also clear that the probability of detection goes to 1 much faster in the case of our proposed method comparing to the ED spectrum sensing method. For example, for $Pfa = 0.2$, the probabilities of detection of the ED sensing can achieve 0.492 only, while for the proposed sensing method Pd equals 0.815.

In Figure 2, the values of the detection probability versus SNR are plotted for the two sensing methods.

The Pfa is set to 0.05 and the SNR varies from $-20dB$ to $10dB$, keeping the number of samples n to 20 samples. It can be seen that the proposed sensing has almost $5dB$ gain over ED sensing with $Pd = 0.8$ and $Pfa = 0.05$, hence the performance of the proposed sensing is indeed better than that of ED sensing.

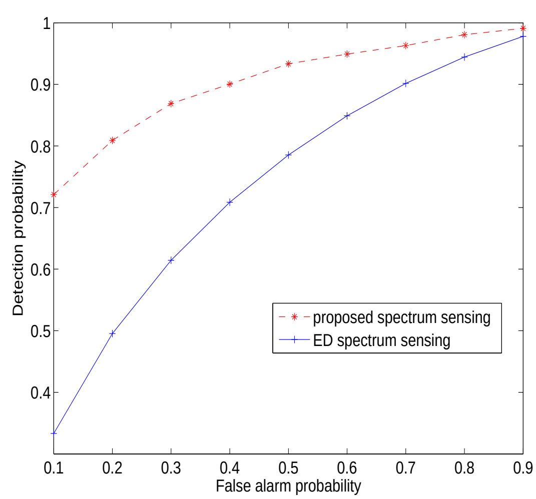

Fig. 1 *Detection probability versus false alarm probability over AWGN channels with SNR=-6 dB and n=20 samples*

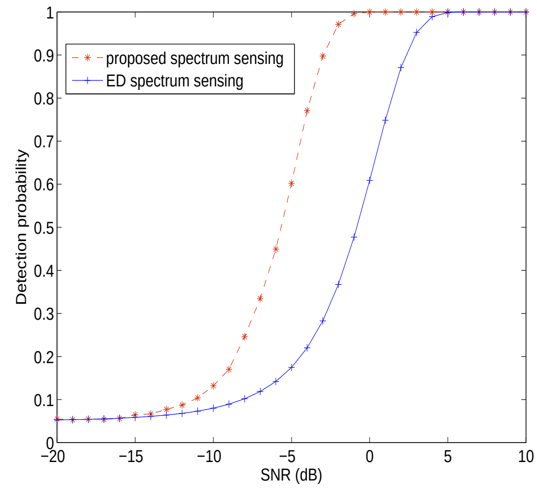

Fig. 2 *Detection probability versus SNR over AWGN channels with Pfa=0.05 and n=20 samples*

Conclusion: In this letter, a spectrum sensing method based on goodness of fit testing using chi-square distribution is proposed. We consider the energy of the received samples and apply the GoF test to compare its CDF with a chi-square CDF with 2 degree of freedom. Simulations have been performed to compare the ED sensing method with the proposed one. It is shown that the proposed sensing method outperforms ED sensing method by 5 dB at Pd = 0.8 and Pfa = 0.05. It is worth noting that the ED sensing and the proposed sensing method present similar complexity. Our future works will focus on the case where the noise is not Gaussian. A future work is to extend the proposed method to cooperative spectrum sensing.

D. Teguig, V. Le Nir and B. Scheers (*Royal Military Academy, Brussels, Belgium*)

E-mail: djamel.teguig@rma.ac.be

References

1 J. Mitola and G. Q. Maguire: 'Cognitive radio: making software more personel', IEEE Personal Commun Mag., vol. 6, no. 4, pp. 13-18, Aug.1999.
2 T. Yucek and H. Arslan: 'A survey of spectrum sensing algorithms for cognitive radio applications', IEEE Commun. Surveys and Tutorials, vol. 11, no. 1, pp. 116-130, Quat. 2009.
3 E. Axell, G. Leus, E. G. Larsson and H. V. Poor: 'Spectrum sensing for cognitive Radio : State-of-the-art and recent advances', IEEE Signal Processing Mag., vol. 29, no. 3, pp. 101-116, May. 2012.
4 H. Urkowitz: 'Energy detection of unknown deterministic signals', *Proc. IEEE*, vol. 55, no. 4, pp. 523-531, Apr. 1967.
5 H. Wang, E. H. Yang, Z. Zhao and W. Zhang: 'Spectrum sensing in cognitive radio using goodness of fit testing', IEEE Trans. Wireless Commun., vol. 8, no. 11, pp. 5427-5430, Nov. 2009.
6 G. Zhang, X. Wang, Y. C. Liang and J. Liu: 'Fast and robust spectrum sensing via kolmogorov-smirnov test', IEEE Trans. Commun. vol. 58, no. 12, pp. 3410-3416, Dec. 2010.
7 S. Rostami, K. Arshad and K. Moessner: 'Order-Statistic Based Spectrum Sensing for Cognitive Radio', IEEE Commun. Lett.,vol. 16, no. 5, pp. 592-595, May. 2012.
8 M. A. Stephens: 'EDF Statistics for Goodness of Fit and Some Comparisons', Journal of the American Statistic Association, vol.69, No.347, pp.730-737, Sep. 1974.
9 R. D.Agostino and M. Stephens: 'Goodness of Fit Techniques' ser. Statistics: Textbooks and Mono-graphs, M. Dekker, 1986.