Closed form Expression of the Saddle Point in Cognitive Radio and Jammer Power Allocation Game

Feten Slimeni*, Bart Scheers[2], Vincent Le Nir[2], Zied Chtourou[1], and Rabah Attia[3]

[1] VRIT Lab - Military Academy of Tunisia, Nabeul 8000, Tunisia,
[2] CISS Departement - Royal Military Academy (RMA), Brussels 1000, Belgium,
[3] SERCOM Lab - EPT University of Carthage, Marsa 2078, Tunisia
{feten.slimeni,ziedchtourou}@gmail.com,{bart.scheers,vincent.lenir}@rma.ac.be,rabah_attia@yahoo.fr

Abstract. In this paper, we study the power allocation problem for a cognitive radio in the presence of a smart jammer over parallel Gaussian channels. The objective of the jammer is to minimize the total capacity achievable by the cognitive radio. We model the interaction between the two players as a zero-sum game, for which we derive the saddle point closed form expression. First, we start by solving each player's unilateral game to find its optimal power allocation. These games will be played iteratively until reaching the Nash equilibrium. It turns out that it is possible to develop analytical expressions for the optimal strategies characterizing the saddle point of this minimax problem, under certain condition. The analytic expressions will be compared to the simulation results of the Nash equilibrium.

Key words: Cognitive radio, jammer, power allocation, saddle point, Nash equilibrium

1 Introduction

Cognitive radio (CR) technology is presented in [1] as a promising solution to the spectrum scarcity problem due to its dynamic spectrum access capability. However, a challenging problem for a CR is the presence of malicious users such as smart jammers. A jammer equipped with a cognitive technology may always prevent CR users from efficiently exploiting the free frequency bands through a real time adaptation of its transmission parameters such as the jamming power. Since game theory is a process of modeling the strategic interaction between players, it is a suitable tool to understand and analyze this adversarial system.

The optimal power allocation problem in presence of smart jammer has been investigated from game theoretical point of view in both wireless [2–6] and cognitive [7–9] networks with diverse utility functions such as the SINR, transmission capacity and number of successful channel access. Most of related papers proved the existence and uniqueness of the pure strategy Nash equilibrium (NE), but only some papers have dealt with analytic computation of the optimal strategies. In [2] and [3], Altman proved the existence and uniqueness of NE considering the transmission capacity as the utility function. To develop the closed form analytic expressions of the optimal power allocations, in the first paper [2] he proposed an algorithm based on the bisection method. In the second paper [3], he converted the problem to a minimax problem since the NE strategy of a zero-sum game is equal to the optimal minimax strategy [10], and he considered the particular case of proportional channel fading coefficients faced by both the jammer and the transmitter. Considering finite strategy sets for both the transmitter and the jammer, the authors in [6] prove the existence of NE in pure (deterministic) strategies and characterize the optimal power allocations in asymptotic regimes over independent parallel Gaussian wiretap channels.

In the context of cognitive radio networks, the interaction between a jammer and a CR is presented in [7] as Colonel Blotto game where the two opponents distribute limited resources over a number of battlefields with the payoff equal to SINR, and the equilibrium is derived in terms of mixed (probabilistic) strategy via power randomization. Likewise, the authors in [9] adopt a Bayesian approach in studying the power allocation game between the CR and the jammer, and provide the Cumulative Distribution Functions (CDFs) of the transmission powers that should be adopted by the CR and the jammer at NE to optimize the utility function equal to the number of successful transmissions.

In this paper, we model the interaction between a CR and a smart jammer as a two-person zero-sum game, considering the transmission capacity as the utility function, and the power allocation over multiple channels as both players strategy sets. We consider that the game is continuous since the players choose from an uncountably infinite strategy sets (the allocated power to each channel can take any decimal value). The proof of existence of a saddle point is given by Nikaido in [11] who generalizes Von Neumann minimax theorem for infinite strategy sets. The saddle point of this game corresponds to the optimal power allocations for both the jammer and the CR. Computing its closed form through exhaustive search over all the possible power allocations of the two players turns out to be hard to do in terms of resource and time consumption. We start by solving the unilateral games; in each one, only one player has to make the decision about how to distribute his power between the available channels considering that the other player has a fixed power allocation. The unilateral games will be played iteratively until reaching the NE. Then, we analytically determine the closed form expressions of the optimal power allocations characterizing the saddle point, under the assumption that all channels are used by both the CR and the jammer. The explicit solution to this game allows the CR to study the jamming strategy and to proactively use the corresponding optimal anti-jamming power allocation. Finally, we compare the analytical saddle point to the NE simulation result.

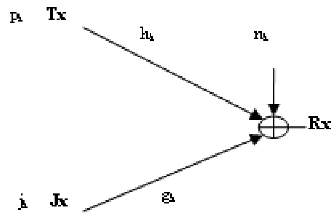

Fig. 1. Scenario of CR jamming attack

2 System model

We consider that the CR has the capacity of accessing multiple frequency bands at the same time with a limited power budget P. The jammer is also assumed to be able to inject interference to all channels with a limited power budget J, which is known as barrage jamming. Whether the attackers can successfully jam communication in a particular channel will depend on how much power the CR and the jammer allocate on that channel. The system is described in Fig. 1. The CR adopt the 'listen-before-talk' rule, that is, sensing for spectrum opportunities at the beginning of each timeslot. On finding M available channels, the CR allocates power $p_k \geq 0$ to the channel $k \in [1, M]$ such that:

$$\sum_{k=1}^{M} p_k = P \tag{1}$$

An action of the CR is designed by the vector $\mathbf{p} = (p_1, \cdots, p_k, \cdots, p_M)$ and the goal is to maximize its transmission capacity subject to (1). At the same time, the jammer injects power $j_k \geq 0$ to the channel k such that:

$$\sum_{k=1}^{M} j_k = J \tag{2}$$

An action of the jammer is designed by the vector $\mathbf{j} = (j_1, \cdots, j_k, \cdots, j_M)$ and the goal is to minimize the transmission capacity of the CR, subject to (2).

In this paper, the CR is trying to maximize its total transmission capacity over the available channels and the jammer is trying to minimize this capacity, so this interaction can be seen as a two person zero-sum game. In each iteration of this game, each player updates its power allocation over the available channels to maximize its payoff. Because each element of the vectors **p** and **j** can take as value any element in $[0, P]$ and $[0, J]$ respectively, we have a continuous set of actions for both the CR and the jammer. The CR's capacity is proportional to

$$f(\mathbf{p}, \mathbf{j}) = \sum_{k=1}^{M} log_2(1 + \frac{|h_k|^2 p_k}{|g_k|^2 j_k + n_k}) \tag{3}$$

n_k is the noise variance of channel k, h_k and g_k are the gains of channel k for the CR and the jammer respectively. In this paper, we assume that all channel gains are common knowledge to both players, and we consider that the M channels are parallel Gaussian channels. We consider $f(\mathbf{p}, \mathbf{j})$ and $-f(\mathbf{p}, \mathbf{j})$ the utility functions of the CR and the jammer, respectively.

This game can be seen as a succession of the two following unilateral games, in which one player is trying to update its power allocation after observing the other player's power allocation.

3 Unilateral games

We start by considering the extreme cases, where only one player has to decide for a one-time how to allocate his total power, the other player has fixed strategy.

3.1 CR Unilateral Game

In each iteration of the game, the CR can consider that the jammer's power allocation is momentarily fixed, and the game degenerates to a classical power allocation problem, where the CR assigns its power into the current noise plus jamming space to maximize the capacity. Mathematically, it can be formulated as the following nonlinear optimization problem:

$$\begin{aligned} \underset{\mathbf{p}}{\text{maximize}} \quad & \sum_{k=1}^{M} log_2(1 + \frac{|h_k|^2 p_k}{|g_k|^2 j_k + n_k}) \\ \text{subject to} \quad & \sum_{k=1}^{M} p_k \leq P \end{aligned} \tag{4}$$

Karush-Kuhn-Tucker (KKT) equations give first order necessary conditions for a solution in nonlinear programming to be optimal, provided that some regularity conditions are satisfied. Allowing inequality constraints, the KKT approach to nonlinear programming generalizes the method of Lagrange multipliers, which allows only equality constraints. The Lagrangian is then

$$L(\mathbf{p}, \mathbf{j}, \lambda) = \sum_{k=1}^{M} log_2(1 + \frac{|h_k|^2 p_k}{|g_k|^2 j_k + n_k}) - \lambda(\sum_{k=1}^{M} p_k - P) \tag{5}$$

Since L is separable in p_k, we can separately optimize each term.

$$\frac{\partial L}{\partial p_k} = \frac{|h_k|^2}{|h_k|^2 p_k + |g_k|^2 j_k + n_k} - \lambda \tag{6}$$

The optimal solution of this optimization problem yields a waterfilling strategy

$$p_k = (\frac{1}{\lambda} - N_k)^+ \tag{7}$$

where $\frac{1}{\lambda}$ is called the waterlevel and the KKT multiplier $\lambda > 0$, that can be found by bisection, is chosen to satisfy (1), N_k is the effective noise power on each channel,

$$N_k = \frac{|g_k|^2 j_k + n_k}{|h_k|^2} \tag{8}$$

and $(x)^+ = \max(0, x)$

3.2 Jammer Unilateral Game

On the other hand, in each iteration the jammer can consider that the CR has a fixed power allocation for the moment of making its decision, and the game degenerates to a jamming unilateral optimization. In such a circumstance, the jammer will allocate its jamming power to minimize the total capacity. Mathematically, this is expressed as the following minimizing problem

$$\begin{aligned} &\underset{\mathbf{j}}{\text{minimize}} \quad && f(\mathbf{p}, \mathbf{j}) \\ &\text{subject to} \quad && \sum_{k=1}^{M} j_k \leq J \end{aligned} \tag{9}$$

We can write the Lagrangian

$$L(\mathbf{j}, \mu) = -f(\mathbf{p}, \mathbf{j}) - \mu(\sum_{k=1}^{M} j_k - J) \tag{10}$$

Since L is separable in j_k, we can separately minimize each term.

$$\frac{\partial L}{\partial j_k} = \frac{|g_k|^2 |h_k|^2 p_k}{(|h_k|^2 p_k + |g_k|^2 j_k + n_k)(|g_k|^2 j_k + n_k)} - \mu \tag{11}$$

After solving the resulting second order equation in j_k, we get

$$j_k = \left(\frac{1}{2} \sqrt{(\frac{|h_k|^2 p_k}{|g_k|^2})^2 + 4 \frac{|h_k|^2 p_k}{|g_k|^2 \mu}} - \frac{|h_k|^2 p_k}{2|g_k|^2} - \frac{n_k}{|g_k|^2} \right)^+ \tag{12}$$

where the KKT multiplier μ should satisfy (2) and can be found by bisection. Note that unlike the CR who uses the waterfilling strategy, the jammer applies a different strategy as given by (12) to dynamically allocate its power.

After solving the optimization problems independently for the CR and the jammer, we can consider Nash game scenario in which both the CR and the jammer make decisions but sequentially. In game theory, a sequential game is a game where one player chooses his action before the others choose theirs. The later players must have some information of the first's choice. The implementation of this game consists in implementing the two unilateral games between the CR and the jammer in an iterative way until convergence to almost fixed power allocation per channel. We consider that the duration of the iterative process until convergence is inferior to the channel coherence time. For the CR, we will use the expression (7) and we proceed by bisection until reaching the value of λ corresponding to the allocation of the total CR power. For the jammer we exploit another strategy with respect the expression (12) and we proceed by bisection until reaching the value of μ corresponding to the allocation of the total jamming power.

In section 5, the NE resulting from the sequential game will be compared to the closed form expression of the saddle point.

4 The closed form expression of the saddle point

The saddle point is so called because if we represent the payoff values as a matrix, the equilibrium value is the minimum in its row and the maximum in its column, this value is the value of the game, and the players' actions are the row and column that intersect at that point. This description of the saddle-point refers to a saddle sits on a horse's back at the lowest point on its head-to-tail axis and highest point on its flank-to flank axis [12]. As example, we determine in Fig. 2 of subsection 5.1 the saddle point of this game over two flat fading channels.

The proof of existence of a saddle point is given by Nikaido in [11] who generalizes Von Neumann minimax theorem for infinite strategy sets. The saddle point of this game corresponds to the optimal power allocations for both the jammer and the CR. Since it is hard to derive it through exhaustive search over the continuous strategy sets, we develop its closed form analytic expression.

4.1 General case

Based on the equations (6) and (11) given to solve each player's decision problem, a vector of powers $(\mathbf{p}, \mathbf{j})$ constitutes the saddle point if and only if there are KKT multipliers λ and μ such that [13]:

$$\frac{\partial f}{\partial p_k} = \frac{|h_k|^2}{|h_k|^2 p_k + |g_k|^2 j_k + n_k} = \lambda \tag{13}$$

and

$$\frac{\partial(-f)}{\partial j_k} = \frac{|g_k|^2 |h_k|^2 p_k}{(|h_k|^2 p_k + |g_k|^2 j_k + n_k)(|g_k|^2 j_k + n_k)} = \mu \tag{14}$$

Equation (13) gives the expression of p_k as

$$p_k = \frac{1}{\lambda} - \frac{n_k + |g_k|^2 j_k}{|h_k|^2} \tag{15}$$

Now we replace p_k in (14) by the expression (15) to find j_k

$$j_k = \frac{|h_k|^2}{\lambda|g_k|^2 + \mu|h_k|^2} - \frac{n_k}{|g_k|^2} \tag{16}$$

If $j_k \geq 0$, we can replace j_k in (15) to get the expression of p_k

$$p_k = \frac{\mu}{\lambda}\frac{|h_k|^2}{\lambda|g_k|^2 + \mu|h_k|^2} \tag{17}$$

So, we can give the equilibrium strategies closed forms for $k \in [1, M]$

$$p_k = \begin{cases} \frac{\mu}{\lambda}\frac{|h_k|^2}{\lambda|g_k|^2+\mu|h_k|^2} & \text{if } \frac{n_k}{|h_k|^2} < \frac{|g_k|^2}{\lambda|g_k|^2+\mu|h_k|^2} \\ \frac{1}{\lambda} - \frac{n_k}{|h_k|^2} & \text{if } \frac{|g_k|^2}{\lambda|g_k|^2+\mu|h_k|^2} \leq \frac{n_k}{|h_k|^2} < \frac{1}{\lambda} \\ 0 & \text{if } \frac{n_k}{|h_k|^2} > \frac{1}{\lambda} \end{cases} \tag{18}$$

and

$$j_k = \begin{cases} \frac{|h_k|^2}{\lambda|g_k|^2+\mu|h_k|^2} - \frac{n_k}{|g_k|^2} & \text{if } \frac{n_k}{|h|^2} < \frac{|g_k|^2}{\lambda|g_k|^2+\mu|h_k|^2} \\ 0 & \text{if } \frac{n_k}{|h_k|^2} \geq \frac{|g_k|^2}{\lambda|g_k|^2+\mu|h_k|^2} \end{cases} \tag{19}$$

To simplify and explain these power allocation expressions, we define a new parameter $\tau_k = \lambda + \mu\frac{|h_k|^2}{|g_k|^2}$. We get $\forall k \in [1, M]$

$$p_k = \begin{cases} \frac{\mu}{\lambda}\frac{|h_k|^2}{\lambda|g_k|^2+\mu|h_k|^2} & \text{if } \frac{n_k}{|h_k|^2} < \frac{1}{\tau_k} \\ \frac{1}{\lambda} - \frac{n_k}{|h_k|^2} & \text{if } \frac{1}{\tau_k} \leq \frac{n_k}{|h_k|^2} < \frac{1}{\lambda} \\ 0 & \text{if } \frac{n_k}{|h_k|^2} > \frac{1}{\lambda} \end{cases} \tag{20}$$

and

$$j_k = \begin{cases} \frac{|h_k|^2}{|g_k|^2}\left(\frac{1}{\tau_k} - \frac{n_k}{|h_k|^2}\right) & \text{if } \frac{n_k}{|h|^2} < \frac{1}{\tau_k} \\ 0 & \text{if } \frac{n_k}{|h_k|^2} \geq \frac{1}{\tau_k} \end{cases} \tag{21}$$

We can draw the following three cases controlled by the three power levels $\frac{1}{\lambda}$ related to the CR, $\frac{1}{\tau_k}$ related to the jammer and $\frac{n_k}{|h_k|^2}$ related to the noise:

- (a) Since $\frac{1}{\tau_k} < \frac{1}{\lambda}$, $\forall k \in [1, M]$, a bad channel for the CR ($\frac{n_k}{|h_k|^2} > \frac{1}{\lambda}$) is also a bad channel for the jammer ($\frac{n_k}{|h_k|^2} > \frac{1}{\tau_k}$). The jammer does not attack a channel which is not occupied by the CR, i.e. if $p_k = 0$ then $j_k = 0$
- (b) In channels verifying $\frac{1}{\tau_k} \leq \frac{n_k}{|h_k|^2} < \frac{1}{\lambda}$, the CR occupies these channels without being jammed; i.e. $p_k > 0$ and $j_k = 0$, these channels are considered unfavorable for the jammer. It avoids these channels may be because of low g_k values which may force it to send with very high power to achieve the CR attack. A solution for the jammer to minimize the number of channels verifying this condition (since it can be considered as favorable opportunity for the CR), is to be close to the receiver node in order to get high g_k values and so $\frac{1}{\tau_k} \approx \frac{1}{\lambda}$.
- (c) If $\frac{n_k}{|h_k|^2} < \frac{1}{\tau_k}$, the channel is considered good for the two players and so occupied by both the CR and the jammer.

We provide in subsection 5.2 an example covering these three situations, see Fig. 3 .

4.2 Case all channels are used by both the CR and the jammer

Under the assumption that the jammer and the CR use all the channels ($p_k, j_k > 0, \forall\, k \in [1, M]$), which means $\frac{|g_k|^2}{\lambda|g_k|^2+\mu|h_k|^2} \geq \frac{n_k}{|h_k|^2}$, then we can give the power allocation closed forms at the NE for $k \in [1, M]$

$$\begin{cases} p_k & = \frac{\mu}{\lambda} \frac{|h_k|^2}{\lambda|g_k|^2+\mu|h_k|^2} \\ j_k & = \frac{|h_k|^2}{\lambda|g_k|^2+\mu|h_k|^2} - \frac{n_k}{|g_k|^2} \end{cases} \tag{22}$$

The power allocations should respect the conditions (1) and (2) which give

$$\begin{cases} \frac{\mu}{\lambda} \sum_{k=1}^{M} \frac{|h_k|^2}{\lambda|g_k|^2+\mu|h_k|^2} & = P \\ \sum_{k=1}^{M} \frac{|h_k|^2}{\lambda|g_k|^2+\mu|h_k|^2} - \sum_{k=1}^{M} \frac{n_k}{|g_k|^2} & = J \end{cases} \tag{23}$$

it gives the following relation between λ and μ

$$\frac{\lambda}{\mu} = \frac{J + \sum_{k=1}^{M} \frac{n_k}{|g_k|^2}}{P} \tag{24}$$

so, we can replace μ in p_k, and λ in j_k to get

$$\begin{cases} p_k & = 1/\lambda \left(1 + \frac{|g_k|^2}{|h_k|^2} \frac{J+\sum \frac{n_k}{|g_k|^2}}{P} \right) \\ j_k & = 1/\mu \left(1 + \frac{|g_k|^2}{|h_k|^2} \frac{\left(J+\sum \frac{n_k}{|g_k|^2}\right)}{P} \right) - \frac{n_k}{|g_k|^2} \end{cases} \tag{25}$$

Using the conditions (1) and (2), we get the closed form expressions of λ and μ

$$\begin{cases} \lambda & = \sum_{k=1}^{M} 1/\left(P + \frac{|g_k|^2}{|h_k|^2} \left(J + \sum \frac{n_k}{|g_k|^2} \right) \right) \\ \mu & = \frac{1}{J+\sum \frac{n_k}{|g_k|^2}} \sum_{k=1}^{M} 1/\left(1 + \frac{|g_k|^2}{|h_k|^2} \frac{\left(J+\sum \frac{n_k}{|g_k|^2}\right)}{P} \right) \end{cases} \tag{26}$$

Finally, replacing λ and μ in (25) gives the closed form expressions of the power allocations at the NE, and the following relation

$$j_k = \frac{J + \sum \frac{n_k}{|g_k|^2}}{P} p_k - \frac{n_k}{|g_k|^2} \tag{27}$$

This analytical result will be compared in subsection 5.3 with the NE found by simulation through playing iteratively the unilateral games.

4.3 Case of proportional fading channels

Now, let's consider the particular case studied by Altman in [3] of proportional fading coefficients,

$$g_k = \gamma h_k, \forall k \in [1, M] \tag{28}$$

and we define

$$\tau = \lambda\gamma + \mu \tag{29}$$

So, the expression of λ in (26) becomes

$$\lambda = \frac{M}{P + \gamma(J + \sum_{k=1}^{M} \frac{n_k}{|g_k|^2})} \tag{30}$$

Replacing λ in (25) results in

$$\begin{cases} p_k & = \frac{P}{M} \\ j_k & = \frac{J + \sum_{k=1}^{M} \frac{n_k}{|g_k|^2}}{M} - \frac{n_k}{|g_k|^2} \end{cases} \tag{31}$$

which brings us to the same conclusion as [3] about uniform power allocation; i.e. if the jammer tries to jam all the channels, then the optimal anti-jamming strategy for the CR is to allocate its power equally over the channels, under the assumption of proportional fading coefficients.

5 Simulation and comparison with the closed form expressions

5.1 Saddle point example

Just to illustrate the concept of saddle-point, we have considered $M = 2$ flat fading channels with gain coefficients $h_k = g_k = 1$, $\forall k \in [1, M]$. we used P=30 and J=20 as the total power for the CR and the jammer respectively. We considered only finite sets of power allocations with steps of 1, so $p_k \in \{0, 1, 2, \cdots, P\}$ and $j_k \in \{0, 1, 2, \cdots, J\}$. We have implemented this scenario in Matlab using the exhaustive search over the finite set of possible power allocations. We calculated a matrix of capacity values, its rows are the possible jammer's power allocations and its columns are the CR's power allocations. We found the optimal maxmin CR's power allocation: $\mathbf{p}^* = (15, 15)$ corresponding to the column number 16, and optimal minimax jammer's power allocation $\mathbf{j}^* = (10, 10)$ corresponding to the row 11. Fig 2 illustrates this saddle point given by the indexes of $\mathbf{p}^*$ and $\mathbf{j}^*$.

5.2 Nash equilibrium in the general case

In this simulation we consider the Nash game between the CR and the jammer, which consists in playing iteratively the two unilateral games presented in section 3. In each iteration of this game, the CR applies waterfilling technique according to equation (7), and the jammer uses bisection and equation (12) to update its power allocation. To cover the general case detailed in 4.1, we consider the system model described in Fig. 1 with $M = 4$ parallel Gaussian channels,

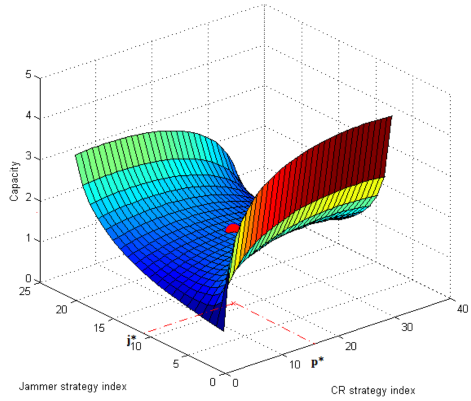

Fig. 2. The saddle point for two channels

$P = 10$ and $J = 10$ as the total CR's and jammer's powers in watts, the background noise over the four channels $\mathbf{n} = (2, 0.75, 0.9, 1.1)$ and the channel gain coefficients $\mathbf{h} = (0.1, 1.1, 1.2, 1.3)$, $\mathbf{g} = (0.7, 0.8, 0.1, 1.2)$.

After convergence of the iterative game to almost fixed power allocations with tolerance $\epsilon = 1e{-}10$, we get $\mathbf{j} = (0, 5.704, 0, 4.296)$ and $\mathbf{p} = (0, 2.5543, 5.5661, 1.8797)$. Which results in a payoff value of $C = 4.5978$ with $\frac{1}{\lambda} = 6.1911$ and $\mu = 0.06$. Fig 3 gives the received power due to the noise, jammer and CR's powers in each channel at the NE.

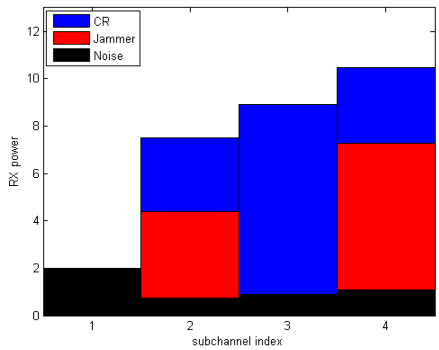

Fig. 3. The strategies at the NE in general case

We can see that in channel 1, $p_k = j_k = 0$ since $\frac{n_1}{|h_1|^2} > \frac{1}{\lambda}$ which corresponds to the case (a) in paragraph 4.1. Channel 3 receives $p_k > 0$ but $j_k = 0$, since

$\frac{1}{3} < \frac{n_3}{|h_3|^2} < \frac{1}{\lambda}$ which corresponds to case (b). Channels 2 and 4 corresponds to case (c) since $\frac{n_k}{|h_k|^2} < \frac{1}{\tau_k}$ which results in $p_k > 0$ and $j_k > 0$.

5.3 Comparison of NE and closed form of the saddle point

In this subsection, we consider the case of all channels used by the CR and the jammer, studied in subsection 4.2. To compare the NE of the iterative power allocation game, with the closed form expressions of the power allocations at the saddle point, we consider the system model described in Fig. 1 with the following parameters: $M = 4$ parallel Gaussian channels, $P = 10$ and $J = 10$ as the total CR's and jammer's powers in watts, the background noise over the four channels $\mathbf{n} = (0.25, 0.75, 0.9, 1.1)$ and the channel gain coefficients $\mathbf{h} = (0.9, 1.1, 1.2, 1.3)$ and $\mathbf{g} = (0.7, 0.8, 1, 1.2)$.

Analytical saddle point

Let's start by replacing the parameters $(|\mathbf{h}|^2, |\mathbf{g}|^2, \mathbf{n}, P, J, M)$ in the closed form expressions given by analytical calculation in subsection 4.2. According to the optimal power allocations given by the expressions (25) and (26), we get $\mathbf{j} = (2.9625, 2.5073, 2.3574, 2.1729)$ and $\mathbf{p} = (2.602, 2.7568, 2.4407, 2.2005)$. Which results in a payoff value of $C = 4.4017$. Let's compare this analytical result with the simulation result found at the convergence of the game considering complete information.

Simulation NE

Under the same conditions, after convergence of the iterative game to almost fixed power allocations with tolerance $\epsilon = 1e - 10$, we get the same power allocation vectors and the same payoff value as given by closed form expressions, which validate our analytical calculation of the NE. Fig 4 gives the received power due to the noise, jammer and CR's powers in each channel at the NE.

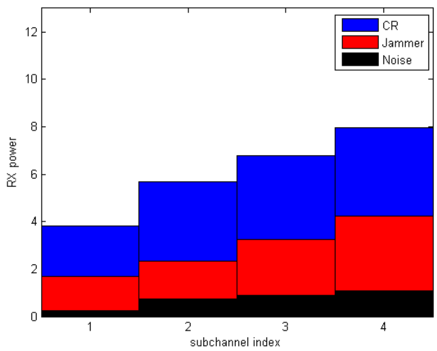

Fig. 4. The strategies at the NE

6 Conclusion

In this paper, we considered a continuous power allocation zero-sum game between a jammer and a CR over parallel Gaussian channels. We provided the optimal strategy for each player depending on the other player's power allocation and on one parameter that usually found using the bisection method. Then, we provided analytical expressions for the optimal power allocations characterizing the saddle point of this game, under the assumption that both the CR and the jammer are using all the channels (i.e. p_k, $j_k > 0$, $\forall\ k \in [1, M]$). Finally, by means of numerical example we found that the analytical expressions are equal to the NE simulation result found by playing iteratively the unilateral games using bisection in each iteration.

References

1. J. Mitola, "Cognitive radio for flexible mobile multimedia communications," in *Mobile Multimedia Communications, 1999. (MoMuC '99) 1999 IEEE International Workshop on*, 1999, pp. 3–10.
2. E. Altman, K. Avrachenkov, and A. Garnaev, "A jamming game in wireless networks with transmission cost," ser. Lecture Notes in Computer Science, vol. 4465. Springer, 2007, pp. 1–12.
3. ——, "Fair resource allocation in wireless networks in the presence of a jammer," in *3rd International ICST Conference on Performance Evaluation Methodologies and Tools, VALUETOOLS 2008, Athens, Greece, October 20-24, 2008*, 2008, p. 33.
4. D. Yang, G. Xue, J. Zhang, A. W. Richa, and X. Fang, "Coping with a smart jammer in wireless networks: A stackelberg game approach." *IEEE Transactions on Wireless Communications*, vol. 12, no. 8, pp. 4038–4047, 2013.
5. R. H. Gohary, Y. Huang, Z.-Q. Luo, and J.-S. Pang, "A generalized iterative water-filling algorithm for distributed power control in the presence of a jammer," in *ICASSP'09*, 2009, pp. 2373–2376.
6. M. Ara, H. Reboredo, S. Ghanem, and M. Rodrigues, "A zero-sum power allocation game in the parallel gaussian wiretap channel with an unfriendly jammer," in *Communication Systems (ICCS), 2012 IEEE International Conference on*, Nov 2012, pp. 60–64.
7. Y. Wu, B. Wang, K. J. R. Liu, and T. C. Clancy, "Anti-jamming games in multi-channel cognitive radio networks," *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 1, pp. 4–15, 2012.
8. K. Dabcevic, A. Betancourt, L. Marcenaro, and C. Regazzoni, "Intelligent cognitive radio jamming-a game-theoretical approach," *EURASIP Journal on Advances in Signal Processing*, vol. 2014, pp. 1–18, 2014.
9. R. El-Bardan, S. Brahma, and P. Varshney, "Power control with jammer location uncertainty: A game theoretic perspective," in *Information Sciences and Systems (CISS), 2014 48th Annual Conference on*, March 2014, pp. 1–6.
10. Z. Yin, D. Korzhyk, C. Kiekintveld, V. Conitzer, and M. Tambe, "Stackelberg vs. nash in security games: interchangeability, equivalence, and uniqueness." in *AAMAS*. IFAAMAS, 2010, pp. 1139–1146.
11. H. Nikaid, "On von neumann's minimax theorem." *Pacific J. Math.*, vol. 4, no. 1, pp. 65–72, 1954.

12. A. Colman, *Game theory and its applications in the social and biological sciences*, 2nd ed., ser. International series in social psychology. Oxford: Butterworth-Heinemann & London: Routledge, 1995.
13. J. Ca, "Optimisation : thorie et algorithmes," Paris, 1971.