Optimal power allocation over parallel Gaussian channels in cognitive radio and jammer games

Feten Slimeni[1,*], Vincent Le Nir[2], Bart Scheers[2], Zied Chtourou[1], Rabah Attia[3]

[1]VRIT Lab, Military Academy of Tunisia, Nabeul 8000, Tunisia
[2]CISS Departement, Royal Military Academy (RMA), Brussels 1000, Belgium
[3]SERCOM Lab, EPT University of Carthage, Marsa 2078, Tunisia
[*]feten.slimeni@gmail.com

Abstract: Cognitive jammers are able to deploy advanced strategies that degrade the performance of cognitive radio user communications. In this paper, we study the problem of power allocation in cognitive radio user and jammer games, over parallel Gaussian channels. We model the interaction between a communicator (a transmitter-receiver pair) and a jammer using zero-sum games with continuous action sets; we describe unilateral, Nash and stackelberg games. We compare the Nash equilibrium, the Stackelberg equilibrium and the minmax/maxmin optimal power allocations through the simulation of the diverse game scenarios. Finally, we give the theoretical proof of existence and uniqueness of the Nash equilibrium.

1. Introduction

Cognitive radio (CR) technology is a promising solution to the shortage of the spectrum due to its capacities of dynamic spectrum access and reconfigurability. A CR device can adjust its operating parameters (such as the frequency and the power level) intelligently in real time to account for the wireless environment changes.

Until recently, the problem of optimal power allocation in cognitive radio networks (CRNs) has been studied to solve either the primary and secondary users (PUs/SUs) coexistence [1, 2, 3, 4], or the spectrum sharing between the CR users [5, 6]. In [1], the goal is to maximize the weighted sum effective capacities of the SUs. The authors determined the optimal power allocation through a convex optimization method using Lagrangian functions with respect to Karush Kuhn-Tucker (KKT) conditions. The authors in [2] studied the impact of channel correlation on the optimal power allocation strategy. Multiple input single output antenna techniques and antenna selection (AS) techniques are studied in [3] to combat the interference constraint and improve the capacity of the SU. The problem in [4], is modeled as a partially observable Markov decision process (POMDP) and the optimal policy is derived for relay selection, channel access, and power allocation through a dynamic programming approach. In order to maximize its transmission rate, each SU in [5] applies a waterfilling scheme and uses the greedy asynchronous distributed interference avoidance algorithm (GADIA) to solve the mutual interference problem. The approach is based on the dynamic adjustment of the number of used frequencies by each user. The problem of power and chunk-based resource allocation is investigated in [6] to maximize the energy efficiency of a multi-carrier CRN. Using Dinkelbach method from non-linear fractional programming and dual optimization method, the authors developed an iterative algorithm to optimize both of the power allocation and chunk-based resource allocation.

The works above deal with the power allocation problem between CR users having the same goal which consists in maximizing an objective function (capacity, SINR, spectrum exploitation, etc) and avoiding interference. However, the CR technology can be exploited by malicious users to prevent the efficient management of the available frequency bands. A jammer may be able to adjust its power allocation across tones in order to cause maximal intentional interference and harm the communication in the most efficient way. Game theory is usefull in this scenario since it helps improve decision making in situations where the best course of action depends upon the decisions made by others. Reference [7], first published in 1997 with substantive revision in 2014, provides and explains almost all possible game scenarios. Power allocation in non-cooperative games against a jammer have been studied in some recent works for MIMO radar system [8], wireless communication networks [9, 10, 11] and cognitive networks [12, 13]. In [8], the interaction between a smart target and a smart MIMO radar is modeled as a two-person zero-sum game (TPZS). The unilateral, hierarchical, and symmetric power allocation games are studied based on the information set available for each player, and the equilibrium solutions are derived. Altman modeled in [9] the jamming game in wireless network as a non zero-sum game with transmission cost, for which he provided analytical expression of the unique Nash equilibrium (NE). Reference [10] can be considered as a generalization of Altman's work to a game scenario between K users and a jammer. The authors develop a generalized version of the iterative waterfilling algorithm (GIWF) whereby all of the users and also the jammer update their power allocations in a greedy manner in order to maximize their respective utilities. Considering finite strategy sets for both the transmitter and the jammer, the authors in [11] prove the existence of NE in pure (deterministic) strategies and characterize the optimal power allocations in asymptotic regimes over independent parallel Gaussian wiretap channels where a legitimate transmitter and a legitimate receiver communicate in the presence of an eavesdropper and a jammer. In the context of CRNs, the interaction between a jammer and a CR user is presented in [12] as Colonel Blotto game where the two opponents distribute limited resources over a number of battlefields with the payoff equal to SINR, and the equilibrium is derived in terms of mixed (probabilistic) strategy via power randomization. Likewise, the authors in [13] adopt a Bayesian approach in studying the power allocation game between the CR user and the jammer, and provide the Cumulative Distribution Functions (CDFs) of the transmission powers that should be adopted by the CR user and the jammer at NE to optimize the utility function equal to the number of successful transmissions.

In this paper, we model the interaction between a CR communicator (a transmitter-receiver pair) and a jammer using zero-sum game scenarios, with the transmission capacity as the objective function. We consider that the actions of both players can be selected from continuous sets, since the allocated power to each channel can take any decimal value with possible infinite sequence of digits to the right of the decimal point. In section 3, we solve the two unilateral games independently. In each game, we consider one player as the unique decision maker and the other player having a fixed power allocation. In section 4, we consider the Nash game in which some player moves first, the other player observes the choice made and then adapts his power allocation. We determine for that scenario the pure strategy Nash equilibrium through simulation of an iterative algorithm of the unilateral games. Then, we study Stackelberg game in section 5 where the first player (the leader) has knowledge of the follower's reaction function and makes the optimal decision reaching the Stackelberg equilibrium. Furthermore, we determine the maxmin and minmax optimal power allocations for the CR user and the jammer under complete knowledge in finite action subsets. The simulation results give equality of the Nash equilibrium, the Stackelberg equilibrium and the minmax/maxmin optimal power allocations. Finally, we theoretically prove the existence and uniqueness of this equilibrium.

2. System model

We consider that the CR user has the capacity of accessing multiple frequency bands at the same time with a limited power budget, this scenario is possible for example by using the OFDM modulation. Each jammer is also assumed to be able to inject interference to all channels which is known as barrage jamming. The scenario is given in Figure 1. The CR user adopts the 'listen-before-talk'

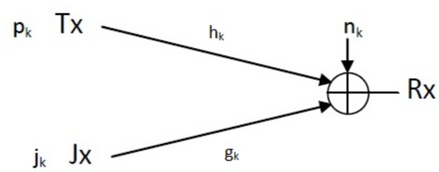


Fig. 1: Scenario of CR jamming attack

rule, that is, sensing for spectrum opportunities at the beginning of each timeslot. On finding M available channels, it allocates power $p_k \geq 0$ to a channel $k \in [1, M]$ such that:

$$\sum_{k=1}^{M} p_k \leq P \tag{1}$$

An action of the CR user is designed by the vector $\mathbf{p} = (p_1, \cdots, p_k, \cdots, p_M)$ in order to maximize its transmission capacity subject to (1) with P as the total power. At the same time, the jammer injects power $j_k \geq 0$ to the channel k such that:

$$\sum_{k=1}^{M} j_k \leq J \tag{2}$$

An action of the jammer is designed by the vector $\mathbf{j} = (j_1, \cdots, j_k, \cdots, j_M)$ in order to minimize the transmission capacity of the CR user, subject to (2) with J as the total power.
n_k is the noise variance of channel k, h_k and g_k are the gains of channel k for the CR user and the jammer respectively. In this paper, we assume that all channel gains are common knowledge to both players, and we consider that the M channels are parallel Gaussian channels.

The Shannon capacity is proportional to

$$f(\mathbf{p}, \mathbf{j}) = \sum_{k=1}^{M} log_2(1 + \frac{|h_k|^2 p_k}{|g_k|^2 j_k + n_k}). \tag{3}$$

We consider $f(\mathbf{p}, \mathbf{j})$ $(-f(\mathbf{p}, \mathbf{j}))$ the utility function of the CR user (the jammer). The CR user is trying to maximize its total transmission capacity over the available channels and the jammer is trying to minimize this capacity, so their interaction can be seen as a two person zero-sum game. In game theory, a zero-sum game is a situation in which one player's gain is equivalent to another's loss. Provided that each element of the vectors **p** and **j** can take any value in $[0, P]$ and $[0, J]$, we have continuous set of actions for the two players.

In the remainder of this paper, we will study diverse scenarios of the game between the two players to find the optimal power allocations. The simulation results will be given in section 7.

3. Unilateral games

We start by considering the extreme cases where only a player has to decide how to allocate his total power against an opponent having a fixed strategy.

3.1. CR user Unilateral Game

If the jammer's strategy is fixed, the game degenerates to a classical power allocation problem where the CR user chooses its power according to the noise plus jamming level in order to maximize the capacity. Mathematically, it can be formulated as the following nonlinear optimization problem:

$$\begin{aligned} \underset{\mathbf{p}}{\text{maximize}} \quad & \sum_{k=1}^{M} log_2(1 + \frac{|h_k|^2 p_k}{|g_k|^2 j_k + n_k}) \\ \text{subject to} \quad & \sum_{k=1}^{M} p_k \leq P \end{aligned} \tag{4}$$

Allowing inequality constraints, the Karush-Kuhn-Tucker (KKT) approach generalizes the method of Lagrange multipliers to nonlinear programming. The Lagrangian is then,

$$L(\mathbf{p}, \mathbf{j}, \lambda) = \sum_{k=1}^{M} log_2(1 + \frac{|h_k|^2 p_k}{|g_k|^2 j_k + n_k}) - \lambda(\sum_{k=1}^{M} p_k - P) \tag{5}$$

Since L is separable in p_k, we can separately optimize each term.

$$\frac{\partial L}{\partial p_k} = \frac{|h_k|^2}{|h_k|^2 p_k + |g_k|^2 j_k + n_k} - \lambda \tag{6}$$

The optimal solution of this optimization problem yields the following strategy

$$p_k^* = (\frac{1}{\lambda} - N_k)^+ \tag{7}$$

known as waterfilling strategy, where $\frac{1}{\lambda}$ is the waterlevel. The KKT multiplier $\lambda > 0$ can be found by bisection and should satisfy

$$\sum_k (\frac{1}{\lambda} - N_k)^+ = P, \tag{8}$$

where $(x)^+ = \max(0, x)$ and N_k is the fictive noise power on each channel given as

$$N_k = \frac{|g_k|^2 j_k^* + n_k}{|h_k|^2} \tag{9}$$

3.2. Jammer Unilateral Game

On the other hand, suppose that the CR user has a fixed power allocation strategy. The game degenerates to a jamming unilateral optimization, as the CR user is not aware of this. In such a circumstance, the jammer will allocate its jamming power to minimize the total capacity. Mathematically, this is expressed as the following minimizing problem

$$\begin{aligned} \underset{\mathbf{j}}{\text{minimize}} \quad & f(\mathbf{p}, \mathbf{j}) \\ \text{subject to} \quad & \sum_{k=1}^{M} j_k \leq J \end{aligned} \tag{10}$$

We can write the Lagrangian as

$$L(\mathbf{j}, \mu) = -f(\mathbf{p}, \mathbf{j}) - \mu(\sum_{k=1}^{M} j_k - J) \tag{11}$$

Since L is separable in j_k, we can separately minimize each term as shown below

$$\frac{\partial L}{\partial j_k} = \frac{|g_k|^2 |h_k|^2 p_k}{(|h_k|^2 p_k + |g_k|^2 j_k + n_k)(|g_k|^2 j_k + n_k)} - \mu \tag{12}$$

After solving the resulting second order equation in j_k, we get

$$j_k = \left(\frac{1}{2} \sqrt{(\frac{|h_k|^2 p_k}{|g_k|^2})^2 + 4 \frac{|h_k|^2 p_k}{|g_k|^2 \mu}} - \frac{|h_k|^2 p_k}{2|g_k|^2} - \frac{n_k}{|g_k|^2} \right)^+ \tag{13}$$

where the KKT multiplier μ is the solution of

$$\sum_{k=1}^{M} j_k \leq J \tag{14}$$

and can be found by bisection.

Unlike the CR user who uses the waterfilling strategy, the jammer applies a different strategy to dynamically allocate its power (as given in equation (13)).

4. Nash game

After solving the optimization problems independently for the CR user and the jammer, we consider here a sequential-moves game in which both the CR user and the jammer make decisions but sequentially. In game theory, a game is said to be sequential if the players choose their actions in a consecutive way and the latter player requires information about the former. The main issue is the convergence of this continuous game to a Nash equilibrium at which no player has interest in changing the power allocation. The theoretical proof of existence and uniqueness of the NE is shown in appendix, according to references [14] and [15].

Since $\mathbf{p}^*$ maximizes $f(\mathbf{p}, \mathbf{j}^*)$ and $\mathbf{j}^*$ minimizes $f(\mathbf{p}^*, \mathbf{j})$, we alternatively determine the CR user's power for a given jamming action, then compute the minimizing jamming power for the CR user's action. That is starting with an initial value $\mathbf{j}_0$, we perform a bisection to determine $\mathbf{p}$. Then for this $\mathbf{p}$, compute the vector $\mathbf{j}$ which minimizes $f(\mathbf{p}, \mathbf{j})$ and these two steps will be repeated.

This game implements the two unilateral games presented in section 3 in an iterative way until convergence to a fixed power allocation per channel within a specific tolerance ϵ. The CR user applies the waterfilling strategy, we proceed by bisection until reaching the value of λ corresponding to the allocation of the total CR user's power (equation (7)). For the jammer, we exploit another strategy and we proceed by bisection until reaching the value of μ corresponding to the allocation of the total jamming power (equation (13)).

5. Stackelberg game

In the previous section, we have considered a sequential-moves game played over time which is usually applied either when the rules of the game are unknown or when directly solving is difficult [16]. In this section we will consider a sequential one-shot game known as a Stackelberg game, in which the leader should anticipate the follower's reaction function in order to alleviate the worst case. Subsequently, the follower observes the action taken by the leader and plays an unilateral game. The solution for this scenario is known as Stackelberg equilibrium and can be found by backward induction. We start by determining the action of the follower, then we derive that of the leader. We will start by a scenario in which the jammer is the leader, then we will solve the game in which the CR user is the leader.

5.1. The jammer as the leader

In this scenario, the jammer knows the reaction function of the CR user which is given in equation (7), and he should use it to substitute p_k in his minimizing problem (equation (10)) to find his optimal power allocation.

Replacing p_k with the expression (7), we get:

$$\begin{aligned} log_2(1 + \frac{|h_k|^2 p_k}{|g_k|^2 j_k + n_k}) &= log_2(1 + \frac{\frac{|h_k|^2}{\lambda} - |g_k|^2 j_k - n_k}{|g_k|^2 j_k + n_k}) \\ &= log_2(\frac{|h_k|^2}{\lambda(|g_k|^2 j_k + n_k)}) \end{aligned} \tag{15}$$

We should get the expression of λ as a function of j_k. For that, the jammer have to consider the following constraint:

$$\sum_{k=1}^{M} p_k = P, \tag{16}$$

which results in,

$$\sum_{k=1}^{M} (\frac{1}{\lambda} - \frac{|g_k|^2 j_k + n_k}{|h_k|^2}) = P. \tag{17}$$

So, we obtain

$$\lambda = \frac{M}{P + \sum_k \frac{g_k^2 j_k + n_k}{h_k^2}}, \tag{18}$$

and the minimizing problem of the jammer becomes

$$\begin{aligned} \underset{\mathbf{j}}{\text{minimize}} \quad & \sum_{k=1}^{M} log_2\Big(\frac{|h_k|^2(P+\sum_k \frac{g_k^2 j_k+n_k}{h_k^2})}{M(|g_k|^2 j_k+n_k)}\Big) \\ \text{subject to} \quad & \sum_{k=1}^{M} j_k \leq J \end{aligned} \tag{19}$$

Since the utility function is no longer separable in j_k, we can no longer derive the Lagrangian expression independently for each j_k. To solve this optimization problem, the jammer can apply a one dimensional exhaustive search over his possible power allocations to find the optimal vector **j** minimizing this new capacity expression, since it is no longer function of $(\mathbf{p}, \mathbf{j})$, it is only a function of **j**. Then, the CR user (as follower) has to exploit the available information about the jammer's power allocation in order to maximize his total capacity. Hence, he plays the unilateral game described in subsection 3.1.

5.2. The CR as the leader

In this scenario, the CR user knows the reaction function of the jammer which is given in equation (13), and he should use it to replace j_k in his maximizing problem (4) to find the optimal power allocation.

To simplify, we replace j_k with $U(p_k)$ which is equal to the expression (13):

$$U(p_k) = \frac{1}{2}\sqrt{(\frac{|h_k|^2 p_k}{|g_k|^2})^2 + 4\frac{|h_k|^2 p_k}{|g_k|^2 \mu}} - \frac{|h_k|^2 p_k}{2|g_k|^2} - \frac{n_k}{|g_k|^2}. \tag{20}$$

Subsequently, we get as new utility function:

$$log_2(1+\frac{|h_k|^2 p_k}{|g_k|^2 j_k + n_k}) = log_2(1+\frac{|h_k|^2 p_k}{|g_k|^2 U(p_k) + n_k}). \tag{21}$$

In $U(p_k)$ we have a Lagrangian parameter which is μ that depends in p_k and we should determine its closed form expression. For that, we have to solve the equation

$$\sum_{k=1}^{M} U(p_k) = J \tag{22}$$

Even it is a complicated equation especially because μ is inside the square root, we can remark that μ is function of all the p_k, $\forall\, k \in [1, M]$ so the utility function of the CR user is not separable in p_k and we can't derive it with respect to each p_k independently.

The maximizing problem (4) of the CR user becomes

$$\begin{aligned} \underset{\mathbf{p}}{\text{maximize}} \quad & \sum_{k=1}^{M} log_2(1+\frac{|h_k|^2 p_k}{|g_k|^2 U(p_k) + n_k}) \\ \text{subject to} \quad & \sum_{k=1}^{M} U(p_k) = J \\ \text{subject to} \quad & \sum_{k=1}^{M} p_k \leq P \end{aligned} \tag{23}$$

We solve this maximizing problem through exhaustive search over all possible power allocations **p** that respect the second constraint. We start by determining μ for each possible **p** through bisection with respect to the first constraint, then using this μ value we calculate the corresponding utility function. The optimal power allocation $\mathbf{p}^*$ corresponds to the maximizer of this function. For the jammer (as follower), we implement the expression (13) found in the jammer unilateral game since he can observe the CR user's power allocation.

6. Optimal solution: minmax/maxmin strategies

Here we consider the perfect scenario of complete knowledge and we define finite action sets for the two players. The minmax search is especially known for its usefulness in calculating the best move in two-player games where all the information is available. Each player in this game, knows that its strategy will be intercepted by its opponent. By considering conservativeness and rationality assumptions of the minmax theorem [17], each player may adopt the strategy which can alleviate the worst case. This means that the strategy of the jammer is the minimizer to the maximum payoff of the CR user, it is also the minimizer to his own maximum loss (since the game is zero-sum). likewise, the CR user's strategy is the maximizer to the worst case (i.e. maximize the minimum payoff of the CR user). So, neither the CR user nor the jammer will profit when changing its strategy and moving from the equilibrium.

6.1. The CR user's maxmin strategy

Consider that the jammer is able to sense the CR user's power allocation and that the CR user is aware of it. Then, a conservative CR user may select its strategy based on the following optimization problem:

$$\begin{aligned} &\underset{\mathbf{p}}{\text{maximize}}\ \underset{\mathbf{j}}{\text{minimize}} && f(\mathbf{p},\mathbf{j}) \\ &\text{subject to} && \sum_{k=1}^{M} p_k \le P, \\ &&& \sum_{k=1}^{M} j_k \le J, \\ &&& \mathbf{p} > 0, \mathbf{j} > 0, \end{aligned} \tag{24}$$

to maximize the capacity in the worst case (i.e. in the situation where the jammer plays the strategy which cause the greatest harm to the CR user).

6.2. The jammer's minmax strategy

The CR device possesses sufficient interception capacity that it can immediately sense interference, due to its wideband spectrum sensing capacity. If the jammer behaves in a conservative way, he will distribute his power as to minimize the possible maximum capacity, which corresponds to solving the two-stage optimization [18],

$$\begin{aligned}
&\underset{\mathbf{j}}{\text{minimize}}\ \underset{\mathbf{p}}{\text{maximize}} && f(\mathbf{p},\mathbf{j}) \\
&\text{subject to} && \sum_{k=1}^{M} p_k \le P, \\
& && \sum_{k=1}^{M} j_k \le J, \\
& && \mathbf{p} > 0, \mathbf{j} > 0
\end{aligned} \tag{25}$$

We implemented this scenario using exhaustive search over a finite set of possible power allocations. We calculate a matrix of capacity values, its rows are the possible jammer's power allocations and its columns are the CR user's power allocations. For the CR user's maxmin, we determine a row of minimum capacity values over all the rows, and finally we determine the column corresponding to the maximum value in this row of minimums. And for the jammer's minmax, we determine a column of maximum capacity values over all the columns, and finally we determine the row corresponding to the minimum value in this column of maximums. The simulation results are given in the next section.

7. Simulation results and discussion

In the following simulations, we consider the game scenario described in section 2. We suppose that there is $M = 4$ available channels, the noise level vector equals $\mathbf{n} = (0.25, 0.75, 0.9, 1.1)$, $P = 10$ and $J = 10$ are the total power respectively for the CR user and the jammer, the channel coefficients are given by $\mathbf{h} = (0.9, 1.1, 1.2, 1.3)$ and $\mathbf{g} = (0.7, 0.8, 1, 1.2)$.

7.1. CR user unilateral game

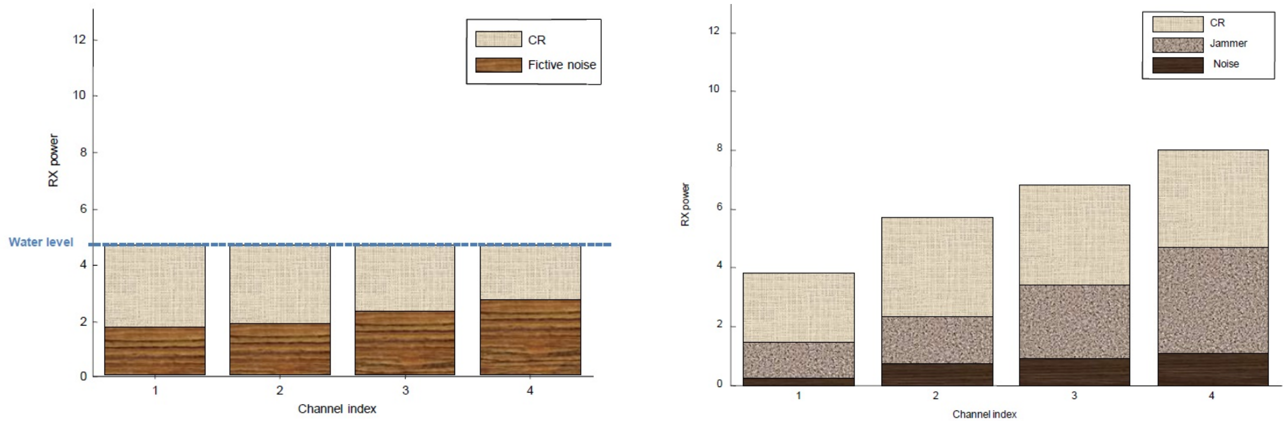
To implement the solution of the CR user unilateral game described in subsection 3.1, we consider the fictive noise level in every channel as given by the expression (9). We proceed by bisection until reaching the maximum water level corresponding to the allocation of the total power of the CR, as illustrated by Figure 2-(a).

As a fixed jamming action, we consider $\mathbf{j} = (2.5, 2.5, 2.5, 2.5)$. The waterfilling strategy of the CR user results in $\mathbf{p}^* = (2.9053, 2.7842, 2.3652, 1.9453)$ and a capacity $C = 4.4254$. Figure 2-(b) gives the total received power per channel ($k \in [1, 4]$) in terms of noise (n_k), jamming signal ($|g_k|^2 j_k$) and CR user's signal ($|h_k|^2 p_k$).

Let us compare the total transmission capacity resulting from the application of the waterfilling strategy to the result of using flat power allocation. If the CR user assigns a power level $p_k = \frac{P}{M}$ to each channel $k \in [1, M]$, the total capacity will be equal to $C = 4.4073$ which results in a payoff loss compared to the optimal waterfilling strategy.

7.2. Jammer unilateral game

To implement the jammer unilateral game described in subsection 3.2, we proceed by bisection and we calculate the sum of the allocated powers to all the channels (using equation (13)) until reaching the value of μ corresponding to the allocation of the total jamming power J.



(a) Waterfilling (b) The received power

Fig. 2: CR user unilateral game

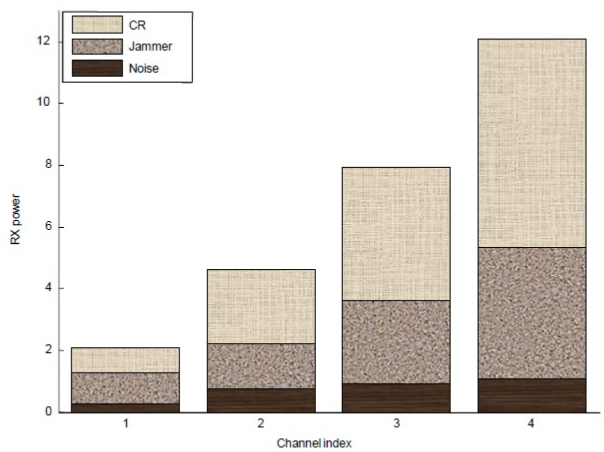


Fig. 3: Jammer unilateral game

Under the same conditions as the above simulation, we consider that the CR user's power allocation is fixed to $\mathbf{p} = (1, 2, 3, 4)$. The reaction of the jammer is given by $\mathbf{j}^* = (2.0723, 2.2849, 2.7064, 2.9364)$ and the resulting capacity is equal to $C = 4.0979$. From this result, note that the jammer pursues the CR user in terms of power allocation. It assigns a higher power to the channels having higher CR user's power. The received power per channel is given in Figure 3.

Under the scenario of imperfect knowledge of the opponent's strategy and the channels gain coefficients, the trivial solution for the jammer would be a flat power allocation. The resulting capacity for the CR user will be $C = 4.1217$ which is higher than the result of applying the described technique based on bisection. Hence, the jammer using flat power allocation loses in terms of payoff since his goal is to minimize the CR user's total transmission capacity.

7.3. Nash game

The Nash game scenario between the CR user and the jammer, described in section 4, consists in playing iteratively the two unilateral games presented in section 3 until convergence to almost fixed power allocation per channel within a specific tolerance $\epsilon = 1e - 15$.

Considering the same conditions as the previous games, we find at the convergence to the NE

$\mathbf{j}^* = (2.9625, 2.5073, 2.3574, 2.1729)$, $\mathbf{p}^* = (2.602, 2.7568, 2.4407, 2.2005)$ and $C = 4.4017$. Figure 4 gives the received power per channel, at the NE. Contrary to the CR user who allocates higher power to the less occupied channels, the jammer allocates higher power to the more occupied channels since he tries to minimize the CR user's payoff.

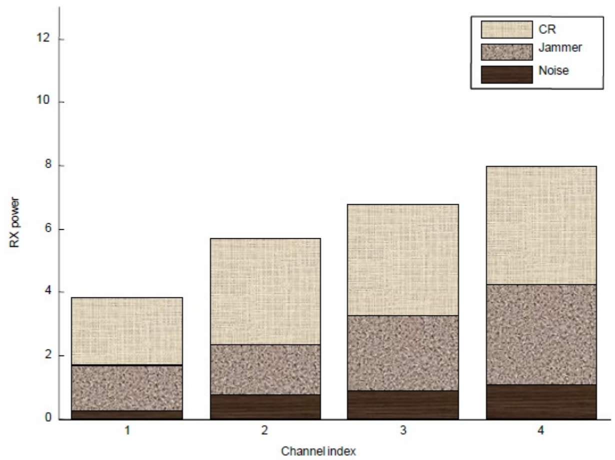


Fig. 4: The strategies at the NE

7.4. *Stackelberg game: Jammer as the leader*

As described in subsection 5.1, we consider that the jammer is the leader and knows the explicit expression of the CR user's reaction function. To implement this game, the jammer does one dimensional exhaustive search over its possible power allocations to find the optimal power allocation $\mathbf{j}^*$ which minimizes the CR user's transmission capacity. The CR user, playing as follower, determines its optimal power allocation $\mathbf{p}^*$ by using equation (7) found in the CR user unilateral game since he can observe the jammer's startegy. We found the same power allocations and the same capacity value as for the NE. Hence, the jammer playing as a leader with knowledge about the reaction function of the opponent finds the same optimal jamming strategy compared to the scenario of playing in iterative way by only observing the instantaneous action of the opponent.

7.5. *Stackelberg game: CR user as the leader*

We consider the Stackelberg game described in subsection 5.2. The CR user is the leader and knows the reaction function of the jammer. Hence, the CR user does exhaustive search over all its possible power allocations $\mathbf{p}$ to find the optimal power allocation $\mathbf{p}^*$ maximizing its transmission capacity. The jammer, as follower, uses the expression (13) found in the jammer unilateral game since it can observe the CR user's power allocation. Also for this scenario, we find the same power allocations and the same capacity value as the result found at the NE.

According to the simulation results of both Stackelberg games (with the jammer or the CR user as the leader), neither the leader wins due to the knowledge of the opponent's reaction nor the follower loses compared to the Nash game.

7.6. Minmax/maxmin optimal solutions

We consider the same parameters considered in the previous simulations to find the NE and the SE. Here, we determine the optimal solutions with the help of the characteristics of the solution found at the NE, otherwise the exhaustive search will be difficult to launch in continuous action sets. We limit the research to the interval $[2, 3]$ where we have found the NE and we consider a step of 0.01. We found the maxmin CR user's power allocation: $\mathbf{p}_{maxmin} = (2.6, 2.76, 2.44, 2.2)$ with $C_{maxmin} = 4.4017$, and the minmax jammer's power allocation $\mathbf{j}_{minmax} = (2.96, 2.51, 2.36, 2.17)$ with the capacity $C_{minmax} = 4.4017$.

Comparing the simulation results, we can note that the optimal values found by exhaustive search under the assumption of finite action subsets and a step of 0.01, give a very near approximation to the power allocations at the NE found in the continuous action sets. Accordingly, the power allocations at the Nash equilibrium and at the Stackelberg equilibrium are equal to the optimal minmax/maxmin power allocations.

8. Conclusion

The jamming attack is a challenging issue in CRNs since it may inhibit the efficient exploitation of the free frequency bands. In this paper, we have exploited the CR technology capacities of simultaneous multi-frequency access and dynamic power allocation as the anti-jamming strategy. We have modeled the interaction between the two players, using different strategies to dynamically update their power allocations, as a zero-sum game with continuous action sets. Then, we have considered different game scenarios, for which we have determined the NE, SE and the optimal minmax/maxmin power allocations. The simulation results have given equality between the solutions of all the considered game scenarios. Finally, we give the theoretical proof of existence and uniqueness of this equilibrium.

9. Acknowledgment

This work has been supported by a grant from the Tunisian defense ministry.

10. References

[1] Xu, D. and Li, Q.: 'Effective capacity region and power allocation for two-way spectrum sharing cognitive radio networks', 2015, **58**, (6), pp. 1-10

[2] Poulakis, M. I., Vassaki, S., Panagopoulos, A. D. and Constantinou, P.: 'Effects of spatial correlation on QoS-driven power allocation over Nakagami-m fading channels in cognitive radio systems', Transactions on Emerging Telecommunications Technologies, 2015, **26**, (5), pp. 851-860

[3] Zhou, F., Li, Z., Si, J., Liu, B. and Huang, H.: 'Optimal power allocation for multiple input single output cognitive radios with antenna selection strategies', IET Communications, 2015, **9**, (10), pp. 1337-1345

[4] Luo, C., Min, G., Yu, F, Zhang, Y., Yang, L. and Leung, V.: 'Joint Relay Scheduling, Channel Access, and Power Allocation for Green Cognitive Radio Communications', Selected Areas in Communications, IEEE Journal, 2015, **33**, (5), pp. 922-932

[5] Baksi, S. and Popescu, D.C: 'Distributed power allocation for rate maximization in cognitive radio networks with horizontal spectrum sharing'. IEEE Wireless Communications and Networking Conference, WCNC, New Orleans, LA, USA, March 9-12, 2015, pp. 932-936

[6] Xu, D. and Li,Q.: 'Energy efficient joint chunk and power allocation for chunk-based multi-carrier cognitive radio networks'. Wireless Communications and Networking Conference (WCNC), 2015, pp. 943-948

[7] , Ross, D.: 'Game Theory', in Edward N. Zalta (Ed.): 'The Stanford Encyclopedia of Philosophy'(Winter 2014)

[8] Song, X., Willett, P., Zhou, S. and Luh, P.: 'The MIMO Radar and Jammer Games', IEEE Transactions on Signal Processing, 2012, **60**, (2), pp. 687-699

[9] Altman, E., Avrachenkov, K. and Garnaev, A.: 'A Jamming Game in Wireless Networks with Transmission Cost', Proc. of NET-COOP (Springer, 2007, pp. 1-12)

[10] Gohary, R.H., Huang, Y., Luo, Z.Q. and Pang, J.S.: 'A generalized iterative water-filling algorithm for distributed power control in the presence of a jammer'. ICASSP'09, pp. 2373-2376

[11] Ara, M., Reboredo, Reboredo, H., Ghanem, S.A.M. and Rodrigues, M.R.D.: 'A zero-sum power allocation game in the parallel Gaussian wiretap channel with an unfriendly jammer'. International Conference on Communication Systems (ICCS), 2012, pp. 60-64

[12] Wu, Y., Wang, B., Liu, K.J.R and Clancy, T.C.: 'Anti-Jamming Games in Multi-Channel Cognitive Radio Networks', IEEE Journal on Selected Areas in Communications, 2012, **30**, (1), pp. 4-15

[13] El-Bardan, R., Brahma, S. and Varshney, P.K.: 'Power control with jammer location uncertainty: A Game Theoretic perspective'. Information Sciences and Systems (CISS), 48th Annual Conference, 2014, pp. 1-6

[14] Rosen, J. B.: 'Existence and Uniqueness of Equilibrium Points for Concave N-Person Games', Econometrica, 1965, **33**, (3), pp. 520-534

[15] Lasaulce, S., Debbah, M. and Altman, E.: 'Methodologies for analyzing equilibria in wireless games', IEEE Signal Processing Magazine, 2009, **26**, (5), pp. 41-52

[16] Sandholm, T.: 'Perspectives on multiagent learning', Artif. Intell., 2007, **171**, (7), pp. 382-391

[17] Kjeldsen, T. H.: 'John von Neumanns Conception of the Minimax Theorem: A Journey Through Different Mathematical Contexts', in Arch. Hist. Exact Sci., 2001, (56), pp. 39-68

[18] Basar, T. and Olsder, G. J.: 'Dynamic Noncooperative Game Theory' (Soc for Industrial & Applied Math, 2nd edn. 1999)

[19] Fudenberg, D. and Tirole, J.: 'Game Theory' (MIT Press, 1991)

[20] Boyd, S. and Vandenberghe, L.: 'Convex Optimization' (Cambridge University Press, 2004)

[21] Aumann, R.J. and Hart, S.: 'Handbook of Game Theory with Economic Applications', Elsevier, 1994, **2**

[22] Ozdaglar, A.: 'Strategic Form Games and Nash Equilibrium' (MIT Press, 2013)

11. Appendix

Existence and uniqueness of Nash equilibrium in pure strategies

In this paper, the jamming scenario is described as a two-player zero-sum game with continuous action sets. The existence of NE can be proved from the properties of the action sets and the utility functions:

- The action sets, $[0, P]^M$ and $[0, J]^M$ are non-empty convex and compact.
- The utility functions are continuous in (p, j).

So, this game is said to be a continuous game for which the Nash equilibrium (NE) is guaranteed [19], but we have to determine if the NE exists in pure strategies or mixed strategies.

11.1. Existence of Nash equilibrium in pure strategies

According to the definition of quasi-convex and quasi-concave utility functions given in [20], the utility function $f(\mathbf{p}, \mathbf{j})$ is quasi-concave in **p** and quasi-convex in **j**.

We can conclude that we have a non-empty compact convex action sets and the utility function is continuous, quasi-concave in **p** and quasi-convex in **j**. Then according to [21] and [19],

$$sup_{\mathbf{p}\in A} inf_{\mathbf{j}\in B} f(\mathbf{p}, \mathbf{j}) = inf_{\mathbf{j}\in B} sup_{\mathbf{p}\in A} f(\mathbf{p}, \mathbf{j}), \tag{11.1}$$

which is equal to the optimal value of the game. So, this game has a Nash equilibrium in pure strategies.

The uniqueness of the NE issue with continuous action sets, can be dealt by verifying Rosen's sufficient condition of diagonally strictly concavity, given in [14].

11.2. Uniqueness of the Nash equilibrium in pure strategies

Let's define the pseudo-gradient vector [22]

$$\mathbf{gr}(\mathbf{p}, \mathbf{j}) = [\nabla_{\mathbf{p}} u_{CR}(\mathbf{p}, \mathbf{j}), \nabla_{\mathbf{j}} u_{Jx}(\mathbf{p}, \mathbf{j})]^T \tag{11.2}$$

Where, u_{CR} and u_{Jx} are respectively the utility functions of the CR user and the jammer verifying: $u_{CR} = -u_{Jx} = f(\mathbf{p}, \mathbf{j})$ and their gradient vectors are

$$\nabla_{\mathbf{p}} u_{CR}(\mathbf{p}, \mathbf{j}) = \nabla_{\mathbf{p}} f(\mathbf{p}, \mathbf{j}) \quad = \left[\frac{\partial f}{\partial p_1}, \cdots, \frac{\partial f}{\partial p_k}, \cdots, \frac{\partial f}{\partial p_M}\right]^T \tag{11.3}$$

and

$$\nabla_{\mathbf{j}} u_{Jx}(\mathbf{p}, \mathbf{j}) = -\nabla_{\mathbf{j}} f(\mathbf{p}, \mathbf{j}) \quad = -\left[\frac{\partial f}{\partial j_1}, \cdots, \frac{\partial f}{\partial j_k}, \cdots, \frac{\partial f}{\partial j_M}\right]^T \tag{11.4}$$

Let $\mathbf{G}(\mathbf{p},\mathbf{j})$ denote the Jacobof the pseudo-gradient $\mathbf{gr}(\mathbf{p},\mathbf{j})$. To justify the diagonally strictly concavity (DSC) condition, we have to prove that the symmetric matrix $(\mathbf{G}(\mathbf{p},\mathbf{j})+\mathbf{G}^T(\mathbf{p},\mathbf{j}))$ is negative definite for all possible $(\mathbf{p},\mathbf{j})$, which is a sufficient condition [14].

$\mathbf{G}(\mathbf{p},\mathbf{j})$ is $2M*2M$ matrix, in which the first M columns are the partial derivatives of $\mathbf{gr}(\mathbf{p},\mathbf{j})$ with respect to the M elements of the vector $\mathbf{p}$ and the second M columns are its partial derivatives with respect to the vector $\mathbf{j}$, so we can represent the matrix $\mathbf{G}=(g_{lc})_{1\leq l,c\leq 2M}$ using four $M*M$ sub-matrices

$$\mathbf{G}(\mathbf{p},\mathbf{j})=\begin{bmatrix} [\mathbf{A}][\mathbf{B}] \\ [\mathbf{C}][\mathbf{D}] \end{bmatrix} \tag{11.5}$$

let's give the expressions of these sub-matrices, using l to denote the row index and c for the column index

- $\forall 1\leq l,c\leq M$, (the submatrix A)

$$\begin{aligned} g_{lc}=a_{lc}&=\frac{\partial^2 f(\mathbf{p},\mathbf{j})}{\partial p_l \partial p_c} \\ &=\begin{cases} -\left(\frac{h_l^2}{h_l^2 p_l+g_l^2 j_l+n_l}\right)^2 & if \quad c=l \\ 0 & else \end{cases} \end{aligned} \tag{11.6}$$

- $\forall M+1\leq l,c\leq 2M$, let's $x=l-M$ and $y=c-M$, (the submatrix D)

$$\begin{aligned} g_{lc}=d_{xy}&=-\frac{\partial^2 f(\mathbf{p},\mathbf{j})}{\partial j_x \partial j_y} \\ &=\begin{cases} -\frac{h_x^2 g_x^2 p_x(2g_x^2(g_x^2 j_x+n_x)+h_x^2 g_x^2 p_x)}{(h_x^2 p_x+g_x^2 j_x+n_x)^2(g_x^2 j_x+n_x)^2} & if \quad x=y \\ 0 & else \end{cases} \end{aligned} \tag{11.7}$$

- $\forall M+1\leq l\leq 2M$ and $1\leq c\leq M$, let's $x=l-M$, (the submatrix C)

$$\begin{aligned} g_{lc}=c_{xc}&=-\frac{\partial^2 f(\mathbf{p},\mathbf{j})}{\partial j_x \partial p_c} \\ &=\begin{cases} \frac{h_x^2 g_x^2}{(h_x^2 p_x+g_x^2 j_x+n_x)^2} & if \quad c=x \\ 0 & else \end{cases} \end{aligned} \tag{11.8}$$

- $\forall 1\leq l\leq M$ and $M+1\leq c\leq 2M$, let's $y=c-M$, (the submatrix B)

$$\begin{aligned} g_{lc}=b_{ly}&=\frac{\partial^2 f(\mathbf{p},\mathbf{j})}{\partial p_l \partial j_y} \\ &=\begin{cases} -\frac{h_l^2 g_l^2}{(h_l^2 p_l+g_l^2 j_l+n_l)^2} & if \quad y=l \\ 0 & else \end{cases} \end{aligned} \tag{11.9}$$

As we can see from these expressions, all the four sub-matrices are diagonal matrices, we also have $\mathbf{B}=-\mathbf{C}<0$, $\mathbf{A}<0$ and $\mathbf{D}<0$. Now we can calculate the symmetric matrix $(\mathbf{G}(\mathbf{p},\mathbf{j})+\mathbf{G}^T(\mathbf{p},\mathbf{j}))$ and determine if it is a negative definite matrix.

$$\mathbf{G}(\mathbf{p},\mathbf{j})+\mathbf{G}^T(\mathbf{p},\mathbf{j})=\begin{bmatrix} [2\mathbf{A}] & [\mathbf{B}+\mathbf{C}] \\ [\mathbf{B}+\mathbf{C}] & [2\mathbf{D}] \end{bmatrix} = \begin{bmatrix} [2\mathbf{A}] & [\mathbf{0}] \\ [\mathbf{0}] & [2\mathbf{D}] \end{bmatrix} \tag{11.10}$$

Since the diagonal sub-matrices **A** and **D** are negative definite, we can conclude that $(\mathbf{G}(\mathbf{p}, \mathbf{j}) + \mathbf{G}^T(\mathbf{p}, \mathbf{j}))$ is a negative definite matrix, which is sufficient to prove the condition of diagonally strictly concavity. So, this game has a unique NE.

In this appendix, we have proved that the described two-person zero-sum game between the jammer and the CR user with continuous action sets, has a unique NE in terms of pure strategies.