Pseudo-Random Binary Sequence Selection for Delay and Add Direct Sequence Spread Spectrum Modulation Scheme

Bart Scheers, Vincent Le Nir

***Abstract*—Recently, correlation delay shift keying (CDSK) has been proposed as a modulation scheme for noncoherent detection which inherits the advantages of conventional spread-spectrum communications using a chaotic reference signal. The CDSK modulation scheme transmits the sum of a chaotic reference signal with its delayed modulated version. In this paper, we consider the utilisation of pseudo-random binary sequences (PRBS) instead of chaotic reference signals. This new modulation scheme is defined as the delay and add direct sequence (DADS) modulation scheme. DADS offers some interesting properties compared to CDSK, mainly the possibility to select some PRBS which improves the BER performance. To this end, a new criterion for PRBS selection is proposed. Theoretical analysis and simulation results show that the BER performance of CDSK is comparable to DADS with arbitrary PRBS, however the DADS performance can be improved by 3 dB with PRBS selection.**

***Index Terms*—Spread spectrum modulation scheme, transmit reference, noncoherent detection,**

I. INTRODUCTION

Correlation delay shift keying (CDSK) has been introduced by Sushchik in 2000 [1]. CDSK is a modulation scheme primarily designed for noncoherent detection in which a chaotic signal is used on one hand as an embedded reference signal, and on the other hand for modulating the data information. This modulation scheme provides a processing gain and therefore inherits the advantage of conventional spread-spectrum communications such as low probability of detection, multipath mitigation, anti-jamming and multiple access capabilities. Moreover, contrary to coherent detection, noncoherent detection does not require the phase synchronisation of the spreading code at the receiver, resulting in a very simple receiver structure.

In this paper, we consider the utilisation of pseudo-random binary sequences (PRBS) instead of chaotic reference signals. This new modulation scheme is defined as the delay and add direct sequence (DADS) modulation scheme. DADS offers some interesting properties compared to the CDSK, mainly the possibility to select some PRBS which improves the BER performance without any additional complexity at the receive side. Therefore, we develop a new criterion for PRBS selection. Theoretical analysis and simulation results show that the BER performance of CDSK is comparable to DADS with arbitrary PRBS, however the DADS performance can be improved by 3 dB with PRBS selection.

The remainder of this paper is organized as follows. In Section II, the DADS modulation scheme is presented and the differences with the CDSK modulation scheme are discussed. In section III, the new criterion for PRBS selection is proposed. In section IV, simulation results are given in AWGN and flat Rayleigh channels.

II. DELAY AND ADD DIRECT SEQUENCE

The CDSK and DADS transmission chains are shown on Figure 1. The main difference between the two modulation schemes is the reference signal used to bear the data information, i.e. a chaotic reference signal for the CDSK modulation scheme and a bipolar PRBS for the DADS modulation scheme. The transmitted signal is the sum of two signals, namely a reference signal and its delayed version multiplied by the information signal. Then it passes through a propagation channel which affects the quality of the transmission. The structures of the CDSK and DADS demodulators are identical, i.e. the received signal is correlated with its delayed conjugate version. Let us derive the mathematics behind Figure 1. First, we consider a flat Rayleigh channel with additive white Gaussian noise (AWGN). The received signal $\{r_i\}$ can be modeled as

$$r_i = h_i(d_k x_{i-L} + x_i) + n_i \tag{1}$$

with h_i the complex-valued channel attenuation at time i, d_k the information bits taking values in {-1,1} with data rate $1/M$, x_i the generated chaotic reference signal (for the CDSK modulation scheme) or the generated bipolar PRBS (for the DADS modulation scheme) of length M and n_i the AWGN with variance $N_0/2$ per dimension. The correlator output is given by

$$S_k = \sum_{i=(k-1)M+1}^{kM} r_i r^*_{i-L}, \tag{2}$$

with

$$\begin{aligned} r_i r^*_{i-L} &= \underbrace{d_k h_i h^*_{i-L} x^2_{i-L}}_{\text{useful part } a_i} \\ &\underbrace{\begin{aligned} &+d_k h_i h^*_{i-L} x_i x_{i-2L} \\ &+h_i h^*_{i-L} x_{i-L} x_{i-2L} + h_i h^*_{i-L} x_i x_{i-L} \end{aligned}}_{\text{interference part } b_i} \\ &\underbrace{\begin{aligned} &+d_k h_i x_{i-L} n^*_{i-L} \\ &+d_k h^*_{i-L} n_i x_{i-2L} \\ &+h_i x_i n^*_{i-L} + h^*_{i-L} n_i x_{i-L} + n_i n^*_{i-L} \end{aligned}}_{\text{noise part } c_i} \end{aligned} . \tag{3}$$

The first line of the equation corresponds to the useful part a_i. The second and third lines corresponds to the interference part b_i, which has three terms due to the cross-correlation between two delayed time intervals of the reference signal. If there is no correlation between x_i and its delayed versions, then $E[b_i] = 0$. The remaining lines correspond to the noise part c_i, which has five terms due to the cross-correlation between the reference signal and the noise (the last component being the cross-correlation of the noise with its delayed version). Through the paper, we assume that the channel is a block Rayleigh channel, which can be represented mathematically as

$$h_i = h_k \quad \forall i \in \{(k-1)M + 1 \dots kM\} \ . \tag{4}$$

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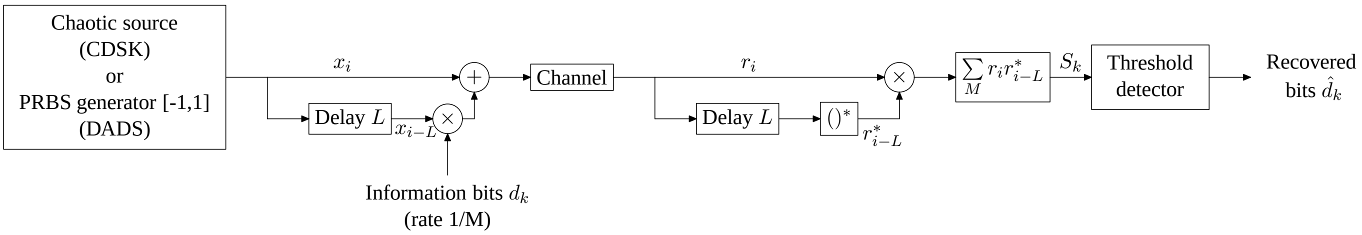

Fig. 1. Transmission chains of the CDSK and DADS modulation schemes

Let us define the output corresponding to the useful part A_k, interference part B_k and noise part C_k

$$\{A_k, B_k, C_k\} = \sum_{i=(k-1)M+1}^{kM} \{a_i, b_i, c_i\}. \tag{5}$$

Considering a PRBS for the DADS modulation scheme and assuming that the delay L $\ll$ M, the useful part becomes $A_k = d_k|h_k|^2 M P_s$ with P_s the energy per chip. Because x_i is statistically independent from x_j for any $i \neq j$, x_i is statistically independent from n_j for any (i, j), and n_i is statistically independent from n_j for any $i \neq j$, the interference part B_k and the noise part C_k can be approximated as zero-mean Gaussian variables as M increases with variances

$$\begin{aligned} var[B_k|d_k] &= 3|h_k|^4 M P_s^2 \\ var[C_k|d_k] &= 4|h_k|^2 M P_s \frac{N_0}{2} + M\frac{N_0^2}{2} \end{aligned} \;. \tag{6}$$

A semi-analytical approach to evaluate the bit error rate (BER) performance is to average the conditional BER over the channel realisations h_k [2]. The BER formula [3] is then given by

$$\begin{aligned} BER &= E[P((B_k + C_k) > A_k|d_k)] \\ &= \frac{1}{2}E\left[\text{erfc}\left(\frac{|h_k|^2 M P_s}{\sqrt{2(var[B_k|d_k] + var[C_k|d_k])}}\right)\right] \end{aligned} \tag{7}$$

with $E[.]$ the expected value over the channel realisations and erfc(.) the complementary error function. Knowing that a transmitted data bit is the sum of two sequences of length M, the energy per bit E_b can be written as $E_b = 2MP_s$, and the BER formula becomes

$$BER = \frac{1}{2}E\left[\text{erfc}\left(\sqrt{\frac{|h_k|^2 E_b}{8N_0(1 + \frac{3|h_k|^2 E_b}{4MN_0} + \frac{MN_0}{2|h_k|^2 E_b})}}\right)\right] \tag{8}$$

Note that in an AWGN channel ($h_k = 1, \forall k$) the performance can be improved by multiplying the received signal by its non-conjugate delayed version [1]

$$S_k = \sum_{i=(k-1)M+1}^{kM} r_i r_{i-L}. \tag{9}$$

The useful part A_k and the variance of the interference part $var[B_k|d_k]$ are unchanged. However, the variance of the noise part becomes $var[C_k|d_k] = 4MP_s\frac{N_0}{2} + M\frac{N_0^2}{4}$, leading to

$$BER = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_b}{8N_0(1 + \frac{3E_b}{4MN_0} + \frac{MN_0}{4E_b})}}\right). \tag{10}$$

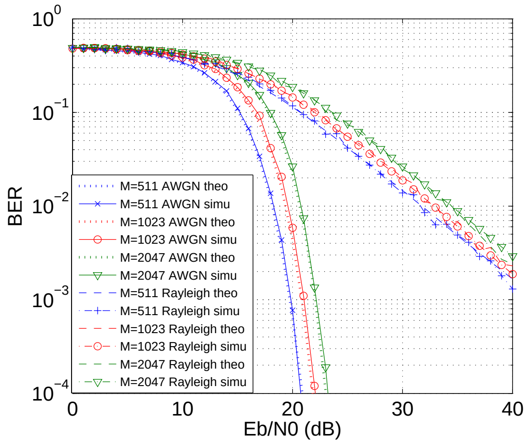

Fig. 2. Simulation results of the DADS modulation scheme in AWGN and flat Rayleigh channels

In order to evaluate the theoretical formulas given in (7) and (8), we have simulated the baseband transmission chain given in Figure 1. Figure 2 shows the theoretical and simulated BER performance of the DADS modulation scheme using arbitrary PRBS in AWGN and flat Rayleigh channels for various lengths of the spreading code (reference signal) M=511,1023 and 2047 with delay L=2. For CDSK, different BER formulas can be obtained depending on the chaotic map [1], [4]. However, simulations reveal that for BER values higher than 10^{-5}, the BER curves for CDSK and DADS modulation coincide for both AWGN and Rayleigh channels. The figure shows that there is a performance degradation for both propagation channels as the length of the spreading code M increases. An explanation for this behavior is the variance of the interference and noise parts in (6) which are function of M. If we look at the performance results, a loss of about 1.0 dB appears when the length of the reference signal M doubles. This means that a processing gain of about 2.0 dB can still be achieved by doubling the length of M.

III. PSEUDO RANDOM BINARY SEQUENCE SELECTION FOR DADS

PRBS selection plays a prominent role in the performance of a DADS modulation scheme. In this Section, a new criterion is proposed for PRBS selection. This criterion may be used to select a finite number of codes from a given codeset. It is found that with the proposed criterion there is about 3 dB improvement in the BER performance compared to arbitrary PRBS. The idea is to select the sequence $\{x_i\}$ such that the first term of the interference part b_i in (3) moves to the useful part a_i while keeping the same properties of the other components in the interference part. This can be done under the following conditions

$$\text{select } \{x_i\} = \left\{ \begin{array}{ll} x_i = x_{i-2L} & \forall i \\ E[x_i x_{i-L}] = 0 & \end{array} \right. . \tag{11}$$

Using this criterion and considering a block Rayleigh channel, (3) can be re-written as

$$\begin{aligned} r_i r_{i-L}^* &= \underbrace{d_k |h_k|^2 (x_i^2 + x_{i-L}^2)}_{\text{useful part } a_i} \\ &\underbrace{+ |h_k|^2 x_{i-L} x_{i-2L} + |h_k|^2 x_i x_{i-L}}_{\text{interference part } b_i} \\ &+ d_k h_k x_{i-L} n_{i-L}^* \\ &+ d_k h_k^* n_i x_{i-2L} \\ &\underbrace{+ h_k x_i n_{i-L}^* + h_k^* n_i x_{i-L} + n_i n_{i-L}^*}_{\text{noise part } c_i} \end{aligned} . \tag{12}$$

Therefore, by selecting a spreading code according to (11), a 3 dB gain on the useful part is achieved while reducing the interference part by one component. Following the same derivations as in Section II, the BER formula in a block Rayleigh channel is

$$BER = \frac{1}{2} E\left[\text{erfc}\left(\sqrt{\frac{|h_k|^2 E_b}{2N_0 \left(1 + \frac{|h_k|^2 E_b}{2MN_0} + \frac{MN_0}{2|h_k|^2 E_b}\right)}} \right) \right] . \tag{13}$$

By multiplying the received signal by its non-conjugate delayed version in an AWGN channel, the BER formula is

$$BER = \frac{1}{2} \text{erfc}\left(\sqrt{\frac{E_b}{2N_0 \left(1 + \frac{E_b}{2MN_0} + \frac{MN_0}{4E_b}\right)}} \right) . \tag{14}$$

A method to generate an optimal PRBS of length M is to repeat a pseudorandom code of length $N = 2L$ ($N \ll M$), leading to 2^N possible codes satisfying already the first condition of (11). From these 2^N codes, a code satisfying the second condition of (11) has to be selected. An exhaustive search on the set of all the possible codes of length $N = 2L$ justifies the choice of an even delay L. The ratio between the number of codes satisfying the second condition of (11) and the total number of codes 2^N for delays $L = 2, 4, 6, 8$ are 0.5, 0.375, 0.3125, and 0.2734 respectively. However, as N increases, the central limit theorem states that the second condition of (11) approaches the normal distribution with a mean 0 and variance $2/N$. Therefore, for large delays L, one can choose a pseudorandom code arbitrarily. For a multiple access scheme, each transmitter needs a different delay L, as suggested in [4]. Hence criterion (11) will not limit the use of DADS for multiple access, as for each L at least one code satisfying (11) can be found.

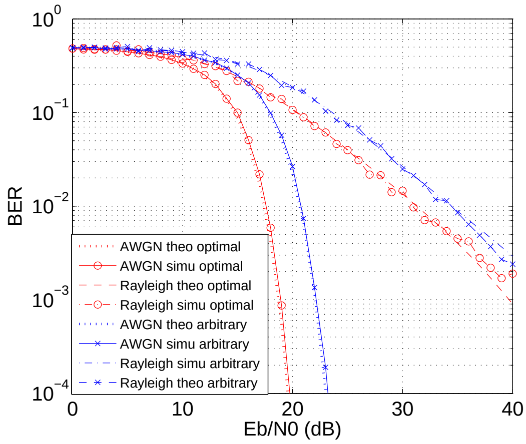

Fig. 3. Simulation results of the DADS modulation scheme with optimal and arbitrary PRBS in AWGN and flat Rayleigh channels for M=2047

IV. SIMULATION RESULTS

Figure 3 shows the theoretical and simulated BER performance comparison of the DADS modulation scheme between arbitrary PRBS of length M (blue) and an PRBS satisfying the criterion (11) (red) in AWGN and flat Rayleigh channels for M=2047 with delay L=2. To generate the PRBS satisfying (11), we choose a pseudo code [-1 -1 1 -1] of length $N = 4$ which repeats itself to generate the spreading code for length M (the last value is removed to get an odd value for M). The simulations show that the theoretical and simulated BER curves satisfying the criterion (11) coincide for both AWGN and Rayleigh channels. A gain in performance of about 3.5 dB appears in an AWGN channel and 3 dB in a flat Rayleigh channel compared to arbitrary PRBS. Similar results are obtained for different values of L as long as (11) is satisfied.

V. CONCLUSION

In this paper, we have introduced a new modulation scheme defined as the delay and add direct sequence (DADS) modulation scheme. This modulation scheme considers the utilisation of pseudo-random bit sequences (PRBS) instead of chaotic signals. A new criterion has been proposed for PRBS selection. Simulation results have shown that the proposed criterion leads to about 3 dB improvement in the BER performance compared to CDSK and DADS with arbitrary PRBS. Future studies will evaluate the performance of the DADS modulation scheme in a multiple access scheme considering the length of the pseudo code N, the delay L and the length of the reference signal M.

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