Multiple Jammer Localization and Transmission Power Estimation for Radio Environment Map

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***Abstract*—The problem of jammer localization is an important problem in a tactical context. This paper describes a method for multiple jammer localization and transmission power estimation using only received signal strength (RSS) from spectrum sensing devices for radio environment map (REM). This method is able to localize multiple jammers and to estimate their transmission powers in the presence of known transmitters. Simulations show the efficiency of the method compared to existing methods in the literature such as inverse distance weighting (IDW), kriging, or LiveREM.**

I. Introduction

The problem of jammer localization is an important problem in a tactical context. Indeed, a jammer can be used to disrupt wireless communications operated by transmitting in the same frequency band.

This paper describes a method for multiple jammer localization and transmission power estimation using only received signal strength (RSS) from spectrum sensing devices for radio environment map (REM). REM is an intelligent network entity that can process multi-domain information (e.g. known transmitters, policies, terrain data) to derive a map of the radio frequency (RF) environment [1].

Multiple surveys of methods for single and multiple jammer localization can be found in the literature [2], [3] and references herein. For single jammer localization, the full search maximum likelihood (ML) method leads to a non convex problem and can be solved via different approaches [4], [5]. The full search least square (LS) method leads to a suboptimal solution [6] and several variants of the LS method have been proposed for unknown power via a constrained optimization problem [1], [9]. Contrary to full search methods, grid search methods do not require the knowledge of the path loss model and consider every point as a potential jammer position. The grid search ML and LS methods are presented respectively in [7] and [8]. For multiple jammer localization, ML methods have been proposed in the literature via particle swarm optimization (PSO) or the iterative expectation-maximization (EM) with unknown number of jammers [2], [10], [11], [12], [13].

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More specifically, [9] proposed a constrained optimization method to solve the problem of the localization and transmission power estimation of a single jammer and showed that the constrained optimization method has better performance than the standard least square method with disturbance in the measurements. [14] also proposed the same constrained optimization method for the localization and transmission power estimation of a single jammer without disturbance in the measurements which is another way to solve the standard least square method. The method presented in this paper generalizes the work of [1], [9] for the localization and transmission power estimation of multiple jammers in the presence of known transmitters.

Simulations show the efficiency of the method compared to existing techniques in the literature such as inverse distance weighting (IDW), kriging, or LiveREM [16], [17], [18], [1].

The paper is organized as follows. In section II, we describe the method for the localization and transmission power estimation of multiple jammers in the presence of known transmitters. In Section III, simulations are conducted to show the efficiency of the method. Finally, section V concludes the paper.

II. Multiple Jammer Localization and Transmission Power Estimation for Radio Environment Map

In this paragraph, we generalize the work of [1], [9] for the localization and transmission power estimation of multiple jammers in the presence of known transmitters.

Assuming multiple jammers and known transmitters whose signals are uncorrelated, we can consider the received signal strength (RSS) from spectrum sensing device $i \in \{1 \ldots N\}$ as the sum of the independent contributions from each jammer and known transmitter

$$RSS_i = \sum_{j=1}^{M} \left(P_{T_j} - L(d_{i,T_j})\right) + \sum_{k=1}^{K} \left(P_{J_k} - L(d_{i,J_k})\right) \tag{1}$$

with P_{T_j} the transmit power of known transmitter $j \in \{1 \ldots M\}$, $L(d_{i,T_j})$ the path loss corresponding to the distance d_{i,T_j} between known transmitter j and spectrum sensing device i in meters, P_{J_k} the transmit power of jammer $k \in$

$\{1 \dots K\}$, and $L(d_{i,J_k})$ the path loss corresponding to the distance d_{i,J_k} between jammer k and spectrum sensing device i in meters.

Assume the knowledge of the log-distance path loss model. If there is no disturbance in the measurements, the log-distance path loss model is defined as

$$L(d) = L_0 + 10\alpha \log(d) \tag{2}$$

with L_0 the path loss in one meter, d the distance in meters, and α the path loss exponent. The first step after the collection of the RSS from all the spectrum sensing devices is to remove the contribution of the known transmitters to the RSS

$$v_i = RSS_i - \sum_{j=1}^{M} \left(P_{T_j} - L(d_{i,T_j})\right) \tag{3}$$

with v_i the contribution of the K jammers to the RSS of spectrum sensing device i

$$v_i = \sum_{k=1}^{K} \left(P_{J_k} - L_0 - 10\alpha \log(d_{i,J_k})\right) \tag{4}$$

We assume that a single jammer has the most contribution to the power of each spectrum sensing device i, therefore v_i in equation (4) can be approximated by $\tilde{v}_i$ defined as

$$\tilde{v}_i = \max_k \left(P_{J_k} - L_0 - 10\alpha \log(d_{i,J_k})\right) \tag{5}$$

This assumption requires the knowledge of the assignment of a dominant jammer to each spectrum sensing device

$$q_i = \operatorname*{argmax}_k \left(P_{J_k} - L_0 - 10\alpha \log(d_{i,J_k})\right) \tag{6}$$

The distance d_{i,J_k} between the jammer k and spectrum sensing device i is represented as the following equation

$$d_{i,J_k} = \sqrt{(x_{J_k} - x_i)^2 + (y_{J_k} - y_i)^2} \tag{7}$$

Replacing (7) in (5) gives the following equation

$$\sqrt{(x_{J_{q_i}} - x_i)^2 + (y_{J_{q_i}} - y_i)^2} = 10^{\frac{P_{J_{q_i}} - L_0 - \tilde{v}_i}{10\alpha}} \tag{8}$$

Raising to the power of two and rearranging equation (8) leads to the following equation

$$x_i^2 + y_i^2 = 2x_{J_{q_i}} x_i + 2y_{J_{q_i}} y_i + 10^{\frac{P_{J_{q_i}} - L_0 - \tilde{v}_i}{5\alpha}} - R_{q_i}^2 \tag{9}$$

with $R_{q_i}^2 = x_{J_{q_i}}^2 + y_{J_{q_i}}^2$. We can express (9) in a matrix form

$$\mathbf{A}\theta = \mathbf{b} \tag{10}$$

with

$$\mathbf{A} = \begin{pmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} & \dots & \mathbf{a}_{1K} \\ \mathbf{a}_{21} & \mathbf{a}_{22} & \dots & \mathbf{a}_{2K} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{a}_{N1} & \mathbf{a}_{N2} & \dots & \mathbf{a}_{NK} \end{pmatrix} \text{ of size } N\text{x}4K,$$

$$\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_K \end{pmatrix}, \text{ and } \mathbf{b} = \begin{pmatrix} x_1^2 + y_1^2 \\ x_2^2 + y_2^2 \\ \vdots \\ x_N^2 + y_N^2 \end{pmatrix},$$

with

$$\mathbf{a}_{ik} = \begin{cases} \begin{pmatrix} 0 & 0 & 0 & 0 \end{pmatrix}, & \text{if } k \neq q_i \\ \begin{pmatrix} 2x_i & 2y_i & 10^{\frac{-\tilde{v}_i - L_0}{5\alpha}} & -1 \end{pmatrix}, & \text{if } k = q_i \end{cases}$$

and $\theta_k = \begin{pmatrix} x_{J_k} & y_{J_k} & 10^{\frac{P_{J_k}}{5\alpha}} & R_k^2 \end{pmatrix}^T$

Similarly to [1], [9] for the localization and transmission power estimation of a single jammer, the localization and transmission power estimation of multiple jammers in the presence of known transmitters can be formulated in a matrix form and can be solved using least square methods. The number of sensors N should be at least four times the number of jammers K to solve the $4K$ unknowns for θ, providing that the matrix $\mathbf{A}$ is nonsingular. The constraints corresponding to the relationships of the intermediate variables R_k in (9) and (10) can be introduced in the least square problem to build a constrained least square problem. The constraints ensure that the solutions are valid according to the Pythagoras' theorem. This constrained least square problem can be written as

$$\begin{aligned} \hat{\theta} &= \text{argmin}||\mathbf{A}\theta - \mathbf{b}||^2 \\ &\text{s.t. } \mathbf{r}_l^T \theta + \theta^T \mathbf{P}_l \theta = 0 \quad \forall l \in \{1 \dots K\} \end{aligned} \tag{11}$$

with

$$\mathbf{P}_l = \begin{pmatrix} \mathbf{P}_{l1} & \mathbf{0}_{4\text{x}4} & \dots & \mathbf{0}_{4\text{x}4} \\ \mathbf{0}_{4\text{x}4} & \mathbf{P}_{l2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0}_{4\text{x}4} \\ \mathbf{0}_{4\text{x}4} & \dots & \mathbf{0}_{4\text{x}4} & \mathbf{P}_{lK} \end{pmatrix} \text{ of size } 4K\text{x}4K,$$

$$\mathbf{r}_l = \begin{pmatrix} \mathbf{r}_{l1} \\ \mathbf{r}_{l2} \\ \vdots \\ \mathbf{r}_{lK} \end{pmatrix}$$

with

$$\mathbf{P}_{lk} = \begin{cases} \mathbf{0}_{4\text{x}4}, & \text{if } k \neq l \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, & \text{if } k = l \end{cases}$$

and

$$\mathbf{r}_{lk} = \begin{cases} \begin{pmatrix} 0 & 0 & 0 & 0 \end{pmatrix}^T, & \text{if } k \neq l \\ \begin{pmatrix} 0 & 0 & 0 & -1 \end{pmatrix}^T, & \text{if } k = l \end{cases}$$

The constrained optimization problem can be solved by its Lagrangian

$$\mathcal{L}(\theta, \lambda) = (\mathbf{A}\theta - \mathbf{b})^T (\mathbf{A}\theta - b) + \sum_{l=1}^{K} \lambda_l (\mathbf{r}_l^T \theta + \theta^T \mathbf{P}_l \theta) \tag{12}$$

with Lagrange multipliers $\lambda_1, \dots, \lambda_K$ and optimality Karush-Kuhn-Tucker (KKT) conditions

$$\frac{\partial \mathcal{L}(\theta,\lambda)}{\partial \theta} = 0$$
$$\frac{\partial \mathcal{L}(\theta,\lambda)}{\partial \lambda_l} = 0 \quad \forall l \in \{1 \dots K\} \tag{13}$$

Derivatives are given by

$$\frac{\partial \mathcal{L}}{\partial \theta} = 2\theta^T(\mathbf{A}^T\mathbf{A} + \sum_{l=1}^{K} \lambda_l \mathbf{P}_l) - 2\mathbf{b}^T\mathbf{A} + \sum_{l=1}^{K} \lambda_l \mathbf{r}_l^T \tag{14}$$

and

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \lambda_l} &= \mathbf{r}_l^T\theta + \theta^T\mathbf{P}_l\theta \quad \forall l \in \{1 \dots K\} \\ &= -R_l^2 + x_l^2 + y_l^2 \quad \forall l \in \{1 \dots K\} \\ &= 0 \quad \forall l \in \{1 \dots K\} \end{aligned} \tag{15}$$

Therefore the set of equations satisfy the second set of conditions. Setting the first equation (14) to zero leads to the following equation

$$\theta = (\mathbf{A}^T\mathbf{A} + \sum_{l=1}^{K} \lambda_l \mathbf{P}_l)^{-1} \left(\mathbf{A}^T\mathbf{b} - \sum_{l=1}^{K} \frac{\lambda_l}{2} \mathbf{r}_l \right) \tag{16}$$

The convergence to the global optimal solution is guaranteed for a single jammer localization [15]. For multiple jammer localization and transmission power estimation, the convergence of the algorithm to the global optimal solution requires that matrix $\mathbf{A}^T\mathbf{A} + \sum_{l=1}^{K} \lambda_l \mathbf{P}_l$ is non singular and that the functions of Pythagoras' theorem constraints are monotone in λ_l. One can observe that $\mathbf{A}^T\mathbf{A} + \sum_{l=1}^{K} \lambda_l \mathbf{P}_l$ is a block diagonal matrix, therefore the inverse of this block diagonal matrix is another block diagonal matrix composed of the inverse of each block $\sum_{i=1}^{N} \mathbf{a}_{il}^T\mathbf{a}_{il} + \lambda_l \mathbf{P}_{ll}$. Similarly to the single jammer localization [15], each block matrix is non singular in its trust region or interval and each function of Pythagoras' theorem constraints $\mathbf{r}_l^T\theta(\lambda_l) + \theta(\lambda_l)^T\mathbf{P}_l\theta(\lambda_l)$ is stricly decreasing on its interval. Therefore, the proposed solution will converge to a unique optimal solution under the assumption of a dominant jammer assigned to each spectrum sensing device.

A bisection method can be used to find the Lagrange multipliers that satisfies the Pythagoras' theorem constraints. Algorithm 1 describes the different steps of the method for multiple jammer localization and transmission power estimation. The first step after the collection of the RSS from all the spectrum sensing devices is to remove the contribution of the known transmitters to the RSS. Then, each Lagrange multiplier is initialized as the center point of its trust region [15]. An initial solution vector is calculated based on all the Lagrange multipliers. The Pythagoras' theorem constraints are checked for this initial solution vector and the Lagrange multipliers are iteratively updated by bisection until convergence of each the Pythagoras' theorem constraints, leading to the final solution vector.

Algorithm 1 Multiple Jammer Localization and Transmission Power Estimation Algorithm

Loop : for all spectrum sensing devices $i \in \{1 \dots N\}$
    Compute equation (3)
End loop
Generate : $\mathbf{A}$, $\mathbf{b}$, $\mathbf{P}_l$ $\forall l$, $\mathbf{r}_l$ $\forall l$
Initialisation : $\lambda_{l,a} = -1/\lambda_{l,max}$ $\forall l$, $\lambda_{l,b} = 10^{10}$ $\forall l$
Generate: $\theta_a = (\mathbf{A}^T\mathbf{A} + \sum_{l=1}^{K} \lambda_{l,a}\mathbf{P}_l)^{-1} \left(\mathbf{A}^T\mathbf{b} - \sum_{l=1}^{K} \frac{\lambda_{l,a}}{2}\mathbf{r}_l \right)$
Generate: $\theta_b = (\mathbf{A}^T\mathbf{A} + \sum_{l=1}^{K} \lambda_{l,b}\mathbf{P}_l)^{-1} \left(\mathbf{A}^T\mathbf{b} - \sum_{l=1}^{K} \frac{\lambda_{l,b}}{2}\mathbf{r}_l \right)$
Generate : $F_{l,a} = \mathbf{r}_l^T\theta_a + \theta_a^T\mathbf{P}_l\theta_a$ $\forall l$
Generate : $F_{l,b} = \mathbf{r}_l^T\theta_b + \theta_b^T\mathbf{P}_l\theta_b$ $\forall l$
If $F_{l,a}F_{l,b} < 0$ $\forall l$
    Generate : $\lambda_l = (\lambda_{l,a} + \lambda_{l,b})/2$ $\forall l$
    Generate: $\theta = (\mathbf{A}^T\mathbf{A} + \sum_{l=1}^{K} \lambda_l\mathbf{P}_l)^{-1} \left(\mathbf{A}^T\mathbf{b} - \sum_{l=1}^{K} \frac{\lambda_l}{2}\mathbf{r}_l \right)$
    Generate : $F_l = \mathbf{r}_l^T\theta + \theta^T\mathbf{P}_l\theta$ $\forall l$
    Loop : while $abs(F_l) > 10^{-5}$ $\forall l$
        Loop : For all $l \in \{1 \dots K\}$
            If $F_{l,a}F_l < 0$
                $\lambda_{l,b} = \lambda_l$
            Else
                $\lambda_{l,a} = \lambda_l$
            End if
        End loop for
        Generate : $\lambda_l = (\lambda_{l,a} + \lambda_{l,b})/2$ $\forall l$
        Generate : $\theta = (\mathbf{A}^T\mathbf{A} + \sum_{l=1}^{K} \lambda_l\mathbf{P}_l)^{-1} \left(\mathbf{A}^T\mathbf{b} - \sum_{l=1}^{K} \frac{\lambda_l}{2}\mathbf{r}_l \right)$
        Generate : $F_l = \mathbf{r}_l^T\theta + \theta^T\mathbf{P}_l\theta$ $\forall l$
    End loop while
End if

III. Simulation Results

The first simulation is evaluated in the following scenario. We consider a square of 1000mx1000m with a known transmitter at the position (670,470) with transmit power 16 dBW and a jammer at the position (270,770) with transmit power 17 dBW. There are 16 spectrum sensing stations equally spaced on a square grid and placed at the center positions of a subregion with an edge size of 250m. The spectrum sensing devices positions can easily be seen on Figure 1 (b) in which the inverse distance weighting (IDW) shows deep fades and peaks at these positions as artifacts of the algorithm. The path loss in one meter is defined as

$$L_0 = 20\log(f) - 27.55 \tag{17}$$

with f the frequency in megahertz. The path loss exponent is set to 3.5 and the frequency is set to 400 MHz.

Figure 1 shows the results of the different algorithms. Figure 1 (a) shows the true REM (true positions of the known transmitter and the jammer). Figure 1 (b) shows the REM using the inverse distance weighting (IDW) algorithm [16]

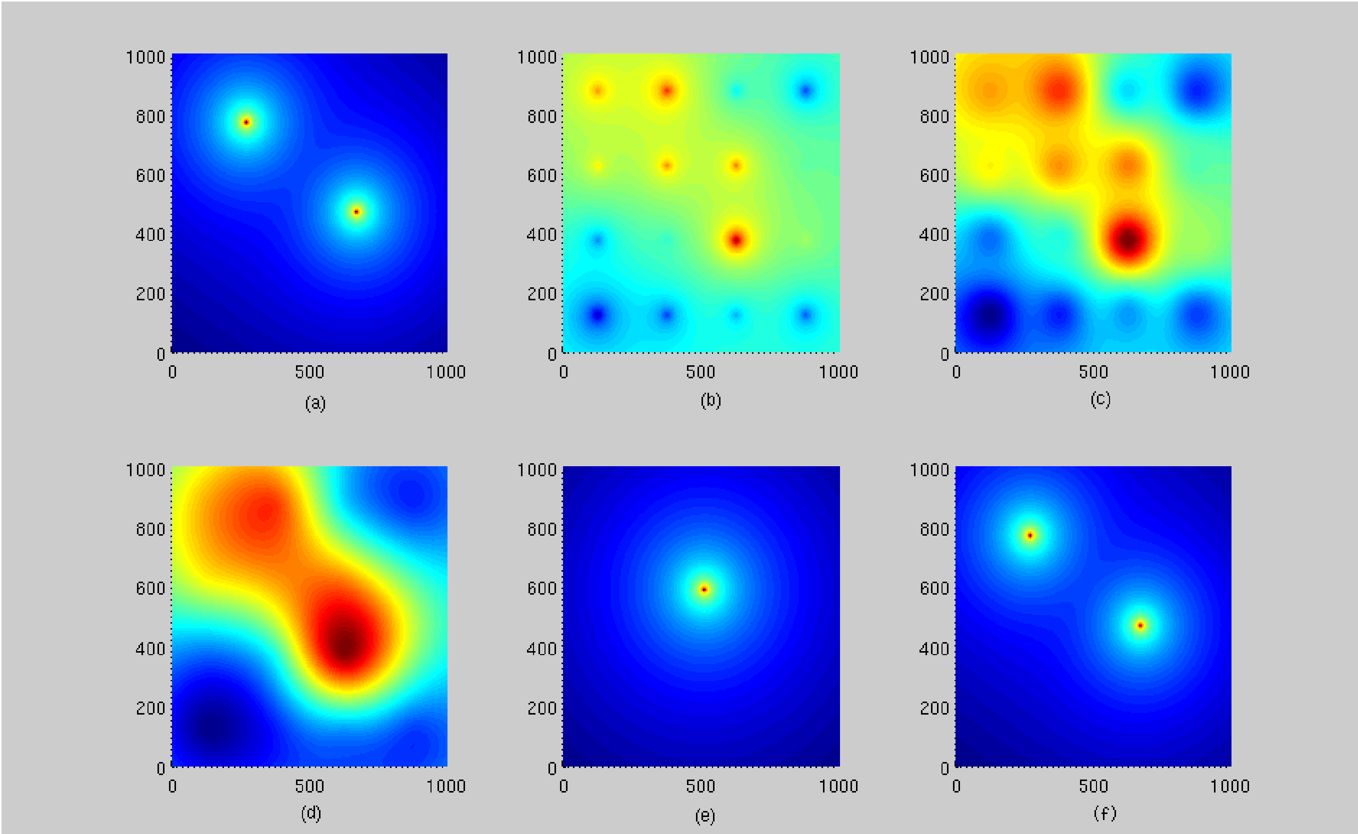

Fig. 1. Results of the different algorithms for one known transmitter and one jammer

with power parameter $\beta = 1$. Figure 1 (c) shows the REM using the IDW algorithm with power parameter $\beta = 2$. Figure 1 (d) shows the REM using the kriging algorithm [17], [18]. Figure 1 (e) shows the REM using the LiveREM algorithm by determining the position of a single jammer [14]. Figure 1 (f) shows the REM using the proposed Algorithm 1. One can see that the proposed algorithm leads to best performance in comparison with the true REM.

Monte carlo trials have been performed for the first scenario, in which the known transmitter and the jammer have been placed at random positions with random transmit powers in the range $[1\ 20]$ dBW. Over 10^5 Monte Carlo trials, only 3 trials did not converge (ill-conditioned matrix inversion). From the remaining trials the distance error mean and standard deviation are $4.47\ 10^{-2}$ and 1.82 meters respectively. The power error mean and standard deviation are $-6.22\ 10^{-4}$ and $3.15\ 10^{-2}$ dBW respectively. Reducing the transmit powers in the range $[15\ 20]$, only 1 trial did not converge. From the remaining trials the distance error mean and standard deviation are $1.3\ 10^{-8}$ and $1.75\ 10^{-6}$ respectively. The power error mean and standard deviation are $-1.86\ 10^{-10}$ and $5.18\ 10^{-8}$ dBW respectively.

The second simulation is evaluated without known transmitters and with two jammers at the positions (670,470) with transmit power 16 dBW and (270,770) with transmit power 17 dBW. Figure 1 (a) shows the true REM (true positions of the two jammers). Figure 2 shows the results for algorithm 1 and scenario 2. One can see that the proposed algorithm leads to a good performance in comparison with the true REM.

Monte carlo trials have been performed for the second scenario, in which the two jammers have been placed at random positions with random transmit powers in the range $[1\ 20]$ dBW. Over 10^5 Monte Carlo trials, 46.65% of the trials did not converge (ill-conditioned matrix inversion). From the remaining trials the distance error mean and standard deviation are 51.64 and 37.41 meters respectively. The power error mean and standard deviation are -1.62 and 1.28 dBW respectively. Reducing the transmit powers in the range $[15\ 20]$, 24.94% of the trials did not converge. From the remaining trials the distance error mean and standard deviation are 50.60 and 29.50 meters respectively. The power error mean and standard deviation are -1.89 and 1.13 dBW respectively. Therefore, we can draw from these simulations that an adequate number of spectrum sensing devices and an adequate spacing between spectrum sensing devices should be chosen depending on the jammer transmission powers in order to have a better chance of having a well-condition matrix for inversion and for convergence. Indeed, an ill-condition matrix is obtained in the case of an insufficient number of spectrum sensing devices assigned to a jammer. Such scenarios exist for instance if a

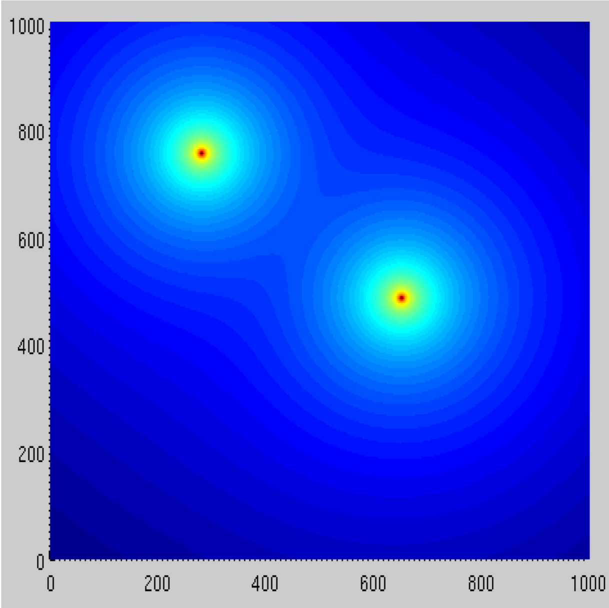

Fig. 2. Results of algorithm 1 for two jammers

jammer is at the border of the square area or if a jammer has much more power than another jammer, therefore taking all the assigments of the spectrum sensing devices.

Future work will study the impact of path loss disturbances like shadowing on the method, the estimation of parameters to obtain a full blind method (number of jammers, path loss exponent, assignment of the dominant jammer to each spectrum sensing device), and the determination of the optimal number of spectrum sensing devices and optimal spacing between spectrum sensing devices. For instance, to take into account path loss disturbances like shadowing effect, the presented problem could be extended to a weighted least square constrained optimization problem to minimize the mean square error for the localization and transmission power estimation of multiple jammers in the presence of known transmitters.

IV. Conclusion

The problem of jammer localization is an important problem in a tactical context. This paper has described a method for jammer localization and transmission power estimation using only received signal strength (RSS) from spectrum sensing devices for radio environment map (REM). This method is able to localize multiple jammers and to estimate their transmission powers in the presence of known transmitters. Simulations have shown the efficiency of the method compared to existing methods in the literature such as inverse distance weighting (IDW), kriging, or LiveREM. Monte Carlo trials have shown that an adequate spacing between spectrum sensing devices should be chosen depending on the jammer transmission powers in order to have a better chance of having a well-condition matrix for inversion and for convergence.

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