Carrier Frequency and Phase Synchronization for Burst-Mode Continuous Phase Modulation

Abstract—**Two carrier frequency and phase synchronization algorithms applicable to burst-mode continuous phase modulation (CPM) are presented. The first algorithm is a data aided (DA) carrier frequency and phase synchronization algorithm only based on the preamble. The second algorithm is a non data aided (NDA) carrier frequency and phase synchronization algorithm applied to the full burst. Simulations show that both algorithms reach the Cramer-Rao bound (CRB) even at low signal to noise ratio (SNR).**

Index Terms—**Continuous Phase Modulation, Synchronization, Frequency Estimation, Phase Estimation**

I. Introduction

Continuous phase modulation (CPM) has the advantages of high spectral efficiency due to the phase continuity and high power efficiency due to the constant envelope. However, CPM has the disadvantage of the high implementation complexity required to build an optimal receiver [1]. CPM has been used in several well-known communications protocols such as GSM and Bluetooth. CPM is also envisioned for the new NATO narrow band waveform (NBWF) [2].

This paper describes two carrier frequency and phase synchronization algorithms for burst-mode CPM. Both algorithms are clock aided, so we assume that the received signal is time synchronized. These algorithms work for full response CPM (pulse length $L = 1$) as well as partial response CPM ($L > 1$) and for any modulation index ($h \leqslant 1/2$). The burst consists of a preamble and a data sequence modulated by CPM. The preamble is a pseudo-random sequence.

The first algorithm is a data aided (DA) carrier frequency and phase synchronization algorithm only based on the preamble. The second algorithm is a non data aided (NDA) carrier frequency and phase synchronization algorithm applied to the full burst. The second algorithm is an extension of the NDA feed forward carrier frequency synchronization algorithm with minimum shift keying (MSK)-type signals as described in [3]. The extended algorithm transforms the received signal into a CPM signal with four phase states by exponentiation, takes the fourth power of the transformed signal, and applies the iterative frequency estimation algorithm [4] on the resulting signal.

The paper is organized as follows. First, we present the signal model. More specifically, the complex baseband representation of a CPM signal and the burst-mode transmission model are presented. Then, the carrier frequency and phase synchronization algorithms are described. Simulations are conducted to show the performance of these algorithms compared to the Cramer-Rao bound (CRB). Finally, a conclusion is given.

II. Signal Model

The complex baseband representation of a CPM signal is given by

$$x(t, \mathbf{a}) = e^{j\psi(t,\mathbf{a})} \tag{1}$$

$$\psi(t, \mathbf{a}) = \pi h \sum_i a_i q(t - iT) \tag{2}$$

with h the modulation index, T the symbol period, $\mathbf{a} = \{a_i\}$ the information belonging to the M-ary alphabet $\{\pm 1, \ldots, \pm M-1\}$, $q(t)$ the phase response of the system with $q(t) = \int_0^t g(u)du$ and satisfying the condition $q(LT) = 1$, L the pulse length, $g(t)$ the frequency pulse, time-limited to the interval $[0, LT]$ and satisfying the condition $g(t) = g(LT-t)$. Full response CPM corresponds to $L = 1$. Partial response CPM corresponds to $L > 1$. MSK-type modulation corresponds to a binary CPM with $h = 1/2$. The most important frequency pulses are the rectangular (LREC), raised-cosine (LRC), spectral raised cosine (LSRC), Gaussian and tamed FM as defined in [1]. Laurent [5] showed that the complex baseband representation of a CPM signal (1) can be written as a sum of $K = 2^{L-1}$ pulse amplitude modulation (PAM) signals

$$x(t, \mathbf{a}) = \sum_i \sum_{k=0}^{K-1} b_{k,i} c_k(t - iT) \tag{3}$$

with $b_{k,i}$ a function of the information sequence $\{a_i\}$ and $c_k(t)$ an equivalent shaping pulse of the k^{th} PAM signal [6]. Laurent also showed that $c_0(t)$, which represents the pulse of longest duration $(L+1)T$, also happens to have the highest energy and is the most important component of the signal [5]. Therefore, the baseband signal (1) can be approximated as

$$x(t, \mathbf{a}) \approx \sum_i b_{0,i} c_0(t - iT) \tag{4}$$

$$b_{0,i} = b_{0,i-1} e^{j\pi h a_i} \tag{5}$$

$$c_0(t) = \prod_{l=0}^{L-1} p(t + lT) \tag{6}$$

$$p(t) = \begin{cases} \dfrac{\sin(\pi h q(t))}{\sin(\pi h)} & 0 \leq t \leq LT \\ \dfrac{\sin(\pi h (1 - q(t - LT)))}{\sin(\pi h)} & LT \leq t \leq 2LT \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

Assuming transmission over an additive white Gaussian noise (AWGN) channel, the complex baseband representation of the received signal can be written as

$$\begin{aligned} y(t, \mathbf{a}) &= A e^{j(2\pi\alpha t + \phi)} x(t - \tau, \mathbf{a}) + n(t) \\ &\approx A \sum_i b_{0,i-1} e^{j(\pi h a_i + 2\pi\alpha t + \phi)} c_0(t - \tau - iT) + n(t) \end{aligned} \tag{8}$$

with A the received signal amplitude, α the carrier frequency offset, ϕ the carrier phase offset, τ the time offset and $n(t)$ the AWGN with variance $N_0/2$ per dimension. The received samples can be written as

$$y(k) = y(t, \mathbf{a}) \Big|_{t = \frac{kT}{F}} \tag{9}$$

with F the oversampling factor. The burst-mode transmission model considers the transmission of independent bursts. Each burst has a known duration and structure as described in Figure 1.

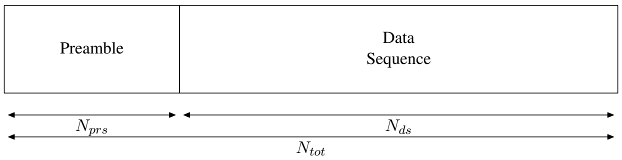

Fig. 1. Structure of the burst

The burst of length N_{tot} consists of a preamble of length N_{prs} and a data sequence signal of length N_{ds} modulated by CPM. The preamble is a pseudo-random sequence. Carrier frequency and phase synchronization is performed on the CPM signal. Two carrier frequency and phase synchronization algorithms are studied in this paper. The first algorithm is a DA carrier frequency and phase synchronization algorithm only based on the preamble. The second algorithm is a NDA carrier frequency and phase synchronization algorithm applied to the full burst.

III. Data aided carrier frequency and phase synchronization

This algorithm applies the iterative frequency estimation algorithm by interpolation on Fourier coefficients described in [4] to the vector corresponding to the point to point multiplication of the received signal and the complex conjugate of the preamble. The algorithm is described in Algorithm 1 with $N = N_{prs}$ and $z = [z(0) \dots z(N-1)$ and

$$z(k) = y(k)x^*(k) \tag{10}$$

Finally, the DA carrier frequency estimate is given by $\hat{\alpha} = \frac{\hat{\beta}+\hat{\delta}_Q}{N}$, and the DA carrier phase estimate by $\hat{\phi} = arg(X_0)$.

IV. Non data aided carrier frequency and phase synchronization

The second algorithm is an extension of the NDA feed forward carrier frequency synchronization algorithm with MSK-type signals as described in [3]. We assume that the received signal is low-pass filtered to eliminate out-of-band noise and sampled at symbol rate $1/T$, leading to $y(i) = y(k)|_{i=Fk}$. The extended algorithm uses exponentiation to transform the CPM signal with modulation index $h < 1/2$ into a CPM signal with modulation index $h = 1/2$. An implementation of the transformation of a CPM signal with small modulation index into a CPM signal with modulation index $h = 1/2$ is described in [7]. Exponentiation can also be described by the following method

$$\tilde{\psi}(i) = \frac{arg(y(i))}{h}\frac{1}{2} \tag{11}$$

The received signal is then reconstructed by the following formula

$$\tilde{y}(i) = e^{j\tilde{\psi}(i)} \tag{12}$$

The reconstructed signal is a CPM signal with four phase states for any frequency pulse whenever the pulse length $L = 1, 2$. For $L \geq 3$, an additional exponentiation taking into account the pulse length L and the frequency pulse might be necessary to obtain a CPM signal with four phase states. In [3], a quadratic non linearity (QNL) is applied on the received signal $z(k) = (-1)^k \tilde{y}(k)^2$ for CPM signals without inter symbol interference (ISI) or pulse length $L = 1$. However, this QNL does not apply for partial response CPM with ISI ($L > 1$). In this case, after the transformation of the received signal into a CPM signal with four phase states by exponentiation, the fourth power non linearity is applied on the transformed signal $z(i) = \tilde{y}(i)^4$. The iterative frequency estimation algorithm [4] described Algorithm 1 is applied to the corresponding vector $z = [z(0) \dots z(N-1)]$ with $N = N_{prs} + N_{ds}$. Finally, the NDA carrier frequency estimate is given by $\hat{\alpha} = \frac{\hat{\beta}+\hat{\delta}_Q}{N}\frac{h}{4}$, and the NDA carrier phase estimate by $\hat{\phi} = arg(X_0)\frac{h}{4}$.

Algorithm 1 Iterative frequency estimation algorithm for DA and NDA carrier frequency and phase synchronization algorithms

Let $Z = FFT(z)$, $E(k) = |Z(k)|^2$, $k = 0 \dots N-1$
Find $\hat{\beta} = \underset{k}{max} \ E(k)$
Set $\hat{\delta}_0 = 0$
Loop : for each i from 1 to Q
$\quad X_p = \sum_{n=0}^{N-1} z(n) e^{-j2\pi n \frac{\hat{\beta}+\hat{\delta}_{i-1}+p}{N}}, p = \pm 0.5$
$\quad \hat{\delta}_i = \hat{\delta}_{i-1} + \frac{1}{2} Re\left\{\frac{X_{0.5}+X_{-0.5}}{X_{0.5}-X_{-0.5}}\right\}$
$X_0 = \sum_{n=0}^{N-1} z(n) e^{-j2\pi n \frac{\hat{\beta}+\hat{\delta}_Q}{N}}$

V. Simulation results

Simulations are conducted to show the performance of the DA and NDA carrier frequency and phase synchronization algorithms compared to the CRB given by [8]

$$CRB = \frac{6}{N(N^2-1)}\frac{1}{\rho} \tag{13}$$

with N the number of samples (N_{prs} for DA, N_{tot} for NDA) and $\rho = A^2/\sigma^2$ the SNR. A total of 10^4 Monte Carlo trials are used to generate the simulation results with carrier frequency offsets in the range $\alpha \in]-0.5h, 0.5h[$ for $L = 1$ with the QNL and $\alpha \in]-0.25h, 0.25h[$ for $L > 1$ with the fourth power non linearity.

Figure 2 shows the mean square error (MSE) performance of the DA and NDA carrier frequency and phase synchronization algorithms with rectangular frequency pulse and pulse length $L = 1$ (1-REC) and $L = 2$ (2-REC), preamble length $N_{prs} = 108$ and CPM data sequence signal length $N_{ds} = 345$ for different modulation indexes $h = 1/2, 1/3, 1/4, 1/6, 1/8$. The DA carrier frequency and phase synchronization algorithm is applied to the preamble N_{prs}. The NDA carrier frequency and phase synchronization algorithm is applied on the full burst $N_{prs} + N_{ds}$. The QNL and the fourth power are applied for $L = 1$ (1-REC) and $L = 2$ (2-REC), respectively. As the NDA carrier frequency and phase synchronization algorithm

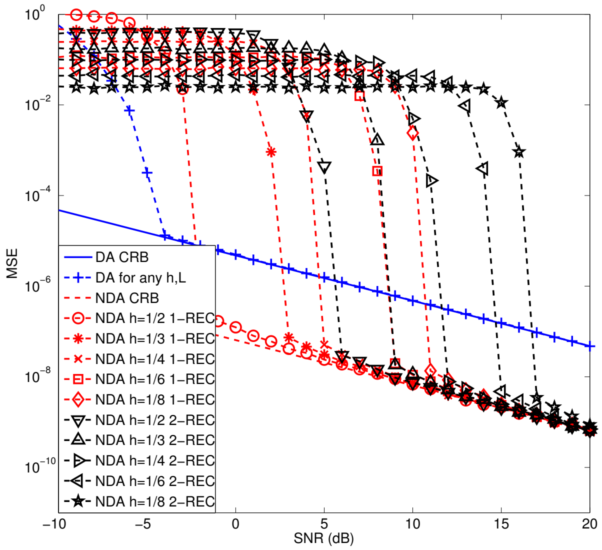

Fig. 2. Influence of the modulation index and the pulse length on the BER performance for the DA and NDA carrier frequency and phase synchronization algorithms

is applied on the full burst, its CRB is lower than the DA carrier frequency and phase synchronization algorithm.

Simulations show that the DA carrier frequency and phase synchronization algorithm reaches the CRB at the same SNR threshold SNR=-4 dB for any modulation index and any pulse length L. The SNR threshold value of the NDA carrier frequency and phase synchronization algorithm increases as the modulation index decreases and as the pulse length increases, i.e. SNR=-1 dB for $h = 1/2, L = 1$ and SNR=9 dB for $h = 1/3, L = 2$. For SNRs higher than the threshold, the MSE performance of the NDA carrier frequency and phase synchronization algorithm is better than the DA carrier frequency and phase synchronization algorithm because of the lower CRB.

VI. Conclusion

Two carrier frequency and phase synchronization algorithms applicable to burst-mode CPM were presented. The first algorithm is a DA carrier frequency and phase synchronization algorithm only based on the preamble. The second algorithm is a NDA carrier frequency and phase synchronization algorithm applied to the full burst. Simulations have shown that both algorithms reach the Cramer-Rao bound even at low SNRs. The DA carrier frequency and phase synchronization algorithm has the advantage of having the same MSE performance for any modulation index and any pulse length. The NDA carrier frequency and phase synchronization algorithm has the advantage of having a lower CRB due to the exploitation of the full burst.

References

[1] J. Anderson, T. Aulin and C. Sundberg, *Digital Phase Modulation*, New York: Plenum, 1986.

[2] *Narrowband Waveform for VHF/UHF Radio - Physical Layer Standard and Propagation Models*, STANAG 5631/AComP-5631, Edition 1.0 Ratification Draft, NATO Unclassified, Janary 2015.

[3] M. Morelli and U. Mengali, *Feedforward Carrier Frequency Estimation with MSK-Type Signals*, IEEE Communications Letters, Vol. 2, No. 8, August 1998.

[4] E. Aboutanios and B. Mulgrew, *Iterative Frequency Estimation by Interpolation on Fourier Coefficients*, IEEE Transactions on Signal Processing, Vol. 53 , No. 4, April 2005.

[5] P. Laurent, *Exact and approximate construction of digital phase modulations by superposition of amplitude modulated pulses (AMP)*, IEEE Transactions on Communications, Vol. COM-34, pp. 150-160, February 1986.

[6] G. Kaleh, *Simple Coherent Receivers for Partial Response Continuous Phase Modulation*, IEEE Journal on Selected Areas in Communications, Vol. 7, No.9, pp. 1427-1436, December 1989.

[7] Z. Xi, J. Zhu and Y. Fu, *Low-Complexity Detection of Binary CPM With Small Modulation Index*, IEEE Communications Letters, Vol. 20, No. 1, January 2016.

[8] S. Kay, *A fast and accurate single frequency estimator*, IEEE Transactions on Acoustics, Speech, Signal Processing, 1989, **37**, pp. 1987-1990.