Robust Blind Carrier Frequency Synchronization for Direct Sequence Spread Spectrum Systems

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A robust blind carrier frequency synchronization technique applicable to direct sequence spread spectrum systems is presented. The proposed method extends a blind timing synchronization technique based on the autocorrelation matrix by applying open-loop single frequency estimation algorithms to the eigenvector of the time synchronized autocorrelation matrix largest eigenvalue. A comparison of different single frequency estimation algorithms is given. Simulation results show that some algorithms attains the Cramer-Rao bound even at low SNRs.

Introduction: Direct sequence spread spectrum (DSSS) is a very attractive modulation technique for military and civilian communication systems, mainly due to its resistance to narrowband interference and low probability of detection. DSSS also allows the sharing of the same frequency band among multiple users by means of code division multiple access (CDMA). These nice properties make DSSS a popular spreading technique which is used in a lot of commercial wireless communications systems like wireless LAN (IEEE 802.11b), wireless PAN (IEEE 802.15.4), global navigation satellite systems (GPS, Galilelo), 3G mobile telecommunications (UMTS).

In DSSS, the data bits are modulated by M-ary phase shift keying (M-PSK). DSSS multiplies the modulated data bits by a spreading code which is generally a pseudo-random bit sequence (PRBS), with a chip rate much higher than the symbol rate of the modulated data bits, thereby spreading the energy of the original modulated data into a much wider band. This operation can hide the signal into the noise, thus providing very low SNRs. Frequency estimation in complex additive white Gaussian noise (AWGN) is a common problem and many techniques have been proposed over the years [1, 2, 3, 4]. Frequency estimation of M-PSK signals in AWGN applies the previous techniques to the Mth power received signal [5, 6]. However, these frequency estimation techniques do not provide accurate frequency estimates at low SNRs.

Blind techniques have been proposed for DSSS detection [8], estimation of the spreading sequence [7, 9], and robust estimation of the timing offset [10]. This paper presents a robust blind carrier frequency synchronization technique applicable to DSSS systems. The proposed method extends the blind timing synchronization technique based on the autocorrelation matrix [10] by applying open-loop single frequency estimation algorithms to the Mth power eigenvector of the time synchronized autocorrelation matrix that corresponds to the largest eigenvalue. A comparison of different single frequency estimation algorithms is given. Simulation results show that some algorithms attains the Cramer-Rao bound even at low SNRs.

Proposed method: The transmitted signal consists of a spreading code $\mathbf{x} = \{x(i)\}$ of length N multiplied by the M-PSK modulated data $\mathbf{d} = \{d(k)\}$ of length K with symbol rate $1/N$. The received signal $\mathbf{y} = \{y(i)\}$ of length $L = KN$ can be modeled as

$$y(i) = Ae^{j(2\pi\alpha i+\phi)}d(k)x(i) + n(i) \tag{1}$$

with $k = \lfloor i/N \rfloor$, A the received signal amplitude, α the frequency offset, ϕ the phase offset and $n(i)$ the AWGN with variance $N_0/2$ per dimension.

The blind detection of DSSS consists of dividing the received signal into P blocks of length T and calculating the fluctuations of autocorrelation estimators $\hat{\mathbf{r}} = \{\hat{r}(j)\}$ of length T given by [8]

$$\hat{r}(j) = E[|r(j)|^2] = \frac{1}{P}\sum_{p=0}^{P-1}|r(j,p)|^2 \tag{2}$$

with

$$r(j,p) = \frac{1}{T}\sum_{t=0}^{T-1} y(pN+t)y^*(pN+t+j) \tag{3}$$

If N is known, a simple scheme to detect the presence of a DSSS signal is to divide the received signal into P blocks of size T with $N < T < 2N$ and to verify if the position of the largest peak of the fluctuations of autocorrelation estimators is equal to N. If N is unknown, a simple scheme to detect the presence of a DSSS signal is to divide the received signal into P blocks of size T and to verify if the positions of the largest peaks of the fluctuations of autocorrelation estimators are multiple of the first peak position. However, several trials might be needed to fall in the case $2N < T < 3N$ in the case of the detection of DSSS by two largest peaks whose positions are multiple of the first peak position.

A robust estimation of the timing offset operating independently of carrier frequency offset requires the maximization of the Frobenius norm of the autocorrelation matrix given by [10]

$$\theta_{opt} = \underset{\theta=\{0,\ldots,N-1\}}{argmax} ||\mathbf{R}_y(\theta)||^2 \tag{4}$$

with

$$\mathbf{R}_y(\theta) = E[\mathbf{y}(\theta,p)\mathbf{y}^H(\theta,p)] = \frac{1}{P}\sum_{p=0}^{P-1}\mathbf{y}(\theta,p)\mathbf{y}^H(\theta,p) \tag{5}$$

and $\mathbf{y}(\theta,p) = [y(pN+\theta),\ldots,y((p+1)N+\theta-1)]^T$. The eigenvalue decomposition of the time synchronized autocorrelation matrix is given by

$$\mathbf{R}_y(\theta_{opt}) = \mathbf{V}_y\mathbf{\Lambda}_y\mathbf{V}_y^H \tag{6}$$

with $\mathbf{\Lambda}_y = diag(\lambda_y(0),\ldots,\lambda_y(N-1)$ the matrix of eigenvalues and $\mathbf{V}_y$ the unitary matrix whose columns contain the eigenvectors of the corresponding eigenvalues. In ideal conditions (flat fading channel without carrier frequency offset), the eigenvector of the time synchronized autocorrelation matrix of the largest eigenvalue corresponds to the spreading sequence [9]

$$\hat{\mathbf{v}}_y = [v_y(\beta_{opt},0),\ldots,v_y(\beta_{opt},N-1)] \tag{7}$$

with

$$\beta_{opt} = \underset{\beta=\{0,\ldots,N-1\}}{argmax} \lambda(\beta) \tag{8}$$

While the eigenvector of the largest eigenvalue can be a good estimate of the spreading sequence in a flat fading channel without carrier frequency offset, this is no longer true in multipath channels with carrier frequency offset. In multipath channels, the eigenvector of the largest eigenvalue corresponds to the convolution of the spreading sequence and the channel response [11]. There is the same correspondence for the eigenvector of the smallest eigenvalue in the noise subspace method presented in [7]. The case of multipath channels is out of scope of this letter and we focus on carrier frequency offset. In the following, we show that the eigenvector of the largest eigenvalue contains information about the spreading sequence and the carrier frequency offset. We describe a robust blind carrier frequency synchronization technique applicable to DSSS systems with carrier frequency offset.

Theorem The eigenvector of the largest eigenvalue in the presence of a carrier frequency offset is the eigenvector of the largest eigenvalue without carrier frequency offset multiplied point to point by the carrier frequency offset vector

$$\mathbf{f} = [1, e^{j2\pi\alpha},\ldots,e^{j2\pi\alpha(N-1)}]^T \tag{9}$$

Proof Equation (1) can be represented by the following model

$$\mathbf{y}(p) = diag(\mathbf{s}(p))\mathbf{g}(p) + \mathbf{n}(p) \tag{10}$$

with

$$\mathbf{g}(p) = [e^{j(2\pi\alpha(pN)},\ldots,e^{j(2\pi\alpha((p+1)N-1)}]^T$$
$$\mathbf{s}(p) = [Ad(p)x(pN),\ldots,Ad(p)x((p+1)N-1)]^T$$
$$\mathbf{n}(p) = [n(pN),\ldots,n((p+1)N-1)]^T$$

Then we have

$$\begin{aligned}\mathbf{y}(p)\mathbf{y}^H(p) = \; & diag(\mathbf{s}(p))\mathbf{g}(p)\mathbf{g}^H(p)diag(\mathbf{s}(p))^H \\ & +diag(\mathbf{s}(p))\mathbf{g}(p)\mathbf{n}^H(p) + \mathbf{g}^H(p)diag(\mathbf{s}(p))^H\mathbf{n}(p) \\ & +\mathbf{n}(p)\mathbf{n}^H(p)\end{aligned} \tag{11}$$

In the first term of equation (11) we have $\mathbf{g}(p)\mathbf{g}^H(p) = \mathbf{f}\mathbf{f}^H$. The first term of equation (11) can be rewritten as

$$diag(\mathbf{s}(p))\mathbf{f}\mathbf{f}^H diag(\mathbf{s}(p))^H = diag(\mathbf{f})\mathbf{s}(p)\mathbf{s}^H(p)diag(\mathbf{f})^H \quad (12)$$

The autocorrelation matrix can be written as

$$\begin{aligned}\mathbf{R}_y = \ & E[\mathbf{y}(p)\mathbf{y}^H(p)] = diag(\mathbf{f})\mathbf{R}_s diag(\mathbf{f})^H + \sigma^2\mathbf{I} \\ = \ & diag(\mathbf{f})(\mathbf{R}_s + \sigma^2\mathbf{I})diag(\mathbf{f})^H\end{aligned} \quad (13)$$

with σ^2 the variance of the noise. The eigenvalue decomposition can be written as

$$(\mathbf{R}_s + \sigma^2\mathbf{I}) = \mathbf{V}_s\mathbf{\Lambda}_s\mathbf{V}_s^H \quad (14)$$

which gives

$$\mathbf{R}_y = diag(\mathbf{f})\mathbf{V}_s\mathbf{\Lambda}_s\mathbf{V}_s^H diag(\mathbf{f})^H \quad (15)$$

The eigenvector of the largest eigenvalue of the autocorrelation matrix

$$\hat{\mathbf{v}}_y = diag(\mathbf{f})\hat{\mathbf{v}}_s \quad (16)$$

with $\hat{\mathbf{v}}_s = [v_s(\beta_{opt}, 0), \ldots, v_s(\beta_{opt}, N-1)]$ completes the proof. Therefore, the algorithms for estimating a single frequency in a noise environment [1, 2, 3, 4] can be applied to the Mth power eigenvector of the time synchronized autocorrelation matrix that corresponds to the largest eigenvalue.

Simulation results: The mean square error (MSE) performance versus SNR of the carrier offset estimators for DSSS systems is shown on Figure 1. The spreading code is a Gold code of degree 7 with length N=127. The number of data bits is set to K=60 and the data bits are modulated by BPSK (M=2). The same algorithms [1, 2, 3, 4] are applied to the square power of the received signal (red dashed lines) and the square power of the eigenvector corresponding to the time synchronized autocorrelation matrix largest eigenvalue (blue solid lines). Carrier frequency offsets can be estimated in the range of $\alpha \in [-0.5/M \;\; 0.5/M]$. Due to the square power operation to remove the BPSK modulation, the algorithms can provide good estimates of the carrier frequency offset in the range of $\alpha \in [-0.25 \;\; 0.25]$. The frequency offset is set to $\alpha = 1.0e^{-2}$. A total of 100 Monte Carlo trials are used to generate the simulation results. The Cramer-Rao bound for the algorithms applied to the square power of the received signal for a fixed length $L = KN$ is given by [3]

$$CRB(\rho) = \frac{6}{\rho L((L)^2 - 1)} \quad (17)$$

with $\rho = A^2/\sigma^2$ the SNR. It can easily be shown that the Cramer-Rao bound for the algorithms applied to the square power of the eigenvector is given by

$$CRB(\rho) = \frac{6}{\rho L((N)^2 - 1)} \quad (18)$$

The Cramer-Rao bound for the algorithms applied to the square power of the eigenvector has worse MSE performance than the Cramer-Rao bound for the algorithms applied to the square power of the received signal. The algorithm with the addition marker (+) corresponds to the unweighted phase averager (UPA) by Kay [3]. The algorithm with the multiplication marker (x) corresponds to the Lank-Reed-Pollon (LRP) [1] which is equivalent to the unweighted linear predictor by Kay [3]. The algorithm with the circle marker (o) corresponds to the linear regression (LR) by Tretter [2] which is equivalent to the weighted phase averager (WPA) by Kay [3]. The algorithm with the square marker (□) corresponds to the weighted linear predictor (WLP) by Kay [3]. The algorithm with the triangle marker (▽) corresponds to the parabolic smoothed central finite difference (PSCFD) by Lovell [4].

It can be observed from Figure 1 that the algorithms applied to the square power of the received signal require higher SNRs than algorithms applied to the square power of the eigenvector. The best performance is obtained by the linear regression (LR) algorithm by Tretter [2] which is equivalent to the weighted phase averager (WPA) by Kay [3]. The Cramer-Rao bound is attained by these algorithms at around 12 dB on the square power of the received signal and around -8 dB on the square power of the eigenvector. Therefore, the proposed method can provide good estimates of the carrier frequency offset at low SNRs.

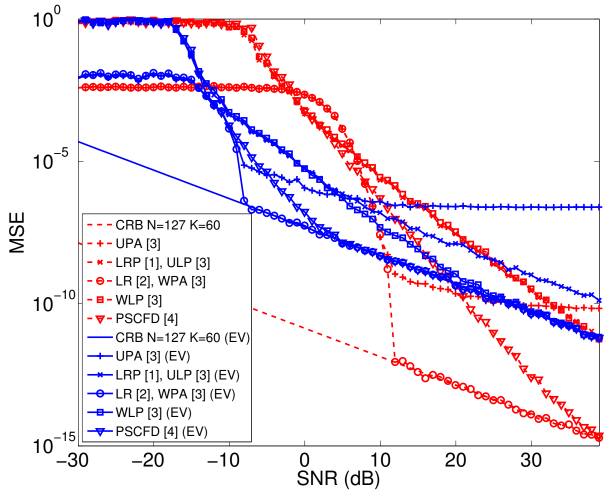

Fig. 1 *Performance of the carrier frequency offset estimators for DSSS systems*

Conclusion: A robust blind carrier frequency synchronization technique applicable to direct sequence spread spectrum systems has been presented. The proposed method extends a blind timing synchronization technique based on the autocorrelation matrix by applying open-loop single frequency estimation algorithms to the eigenvector of the time synchronized autocorrelation matrix largest eigenvalue. A comparison of different single frequency estimation algorithms were given. Simulation results have shown that some algorithms attains the Cramer-Rao bound even at low SNRs. A possible direction for future work is to study carrier frequency synchronization of DSSS systems in multipath channels.

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References

1 Lank, G. W. and Reed, I. S. and Pollon G. E.: 'A semicoherent detection and doppler estimation statistic', *IEEE Transactions on Aerospace and Electronic Systems*, 1973, **9**, pp. 151-165

2 Tretter, S. A.: 'Estimating the frequency of a noisy sinusoid by linear regression', *IEEE Transactions on Information Theory*, 1985, **31**, pp. 832-835

3 Kay, S. M.: 'A fast and accurate single frequency estimator', *IEEE Transactions on Acoustics, Speech, Signal Processing*, 1989, **37**, pp. 1987-1990

4 Lovell, B. C. and Williamson, R. C.: 'The Statistical Performance of Some Instantaneous Frequency Estimators', *IEEE Transactions on Signal Processing*, 1992, **40(7)**, pp. 1708-1723

5 Viterbi A. J and Viterbi A. M.: 'Nonlinear Estimation of PSK-Modulated Carrier Phase with Application to Burst Digital Transmission', *IEEE Transactions on Information Theory*, 1983 **29**, pp. 543-551

6 Meyr, H. and Moeneclaey, M and Fechtel S. A.: 'Digital Communication Receivers: Synchronization, Channel Estimation, and Signal Processing', *John Wiley & Sons*, 1998

7 Tsatsanis, N. K and Giannakis, G. B.: 'Blind Estimation of direct sequence spread spectrum signals in multipath', *IEEE Transactions on Signal Processing*, 1997, **40(12)**, pp. 1241-1251

8 Burel, G.: 'Detection of Spread Spectrum Transmissions using fluctuations of correlation estimators', *IEEE International Symposium on Intelligent Signal Processing and Communication Systems (ISPACS)*, 2000, November, Hawaii, USA

9 Burel, G. and Bouder, C.: 'Blind Estimation of the Pseudo-random Sequence of a Direct Sequence Spread Spectrum Signal', *IEEE 21st Century Military Communications Conference (MILCOM)*, 2000, october, Los Angeles, USA

10 Bouder, C. and Azou, S. and Burel, G.: 'A robust synchronisation procedure for blind estimation of the symbol period and the timing offset in spread spectrum transmissions', *IEEE International Symposium on Spread Spectrum Techniques and Applications (ISSSTA)*, 2002, septembre, Prague, Czech Republic

11 Bouder, C. and Azou, S. and Burel, G.: 'Performance Analysis of a Spreading Sequence Estimator for Spread Spectrum Transmissions', *Journal of The Franklin Institute*, 2004, **341(7)**, pp. 595-614