Improved DADS Performance with Noise Reduction by Averaging

Vincent Le Nir and Bart Scheers

***Abstract*—In this paper, a method is proposed to improve the noise performance of the delay and add direct sequence (DADS) modulation scheme by modifying the structure of the reference signal and the demodulation algorithm. It is shown that noise reduction by averaging can enhance the bit error rate (BER) performance of DADS considerably. As the number of noisy chips exploited at the receiver increases, the signal to noise ratio (SNR) gap between the BER performance of the proposed modulation scheme and the binary phase-shift keying (BPSK) modulation scheme approaches 3 dB. The theoretical results derived for the BER are verified by simulations in additive white Gaussian noise (AWGN) and frequency selective Rayleigh channels.**

***Index Terms*—Spread spectrum modulation scheme, transmit reference, noncoherent detection**

I. Introduction

Impulse radio ultrawideband (IR-UWB) systems uses very short pulses of less than a nanosecond in conjunction either with time-hopping (TH) or direct sequence (DS) for multiple access, and either pulse position modulation (PPM) or pulse amplitude modulation (PAM) for data transmission. In transmit-reference (TR) systems, a reference signal is included in the transmitted signal and can be separated either in time/frequency as long as the time/frequency separation is less than the coherence time/bandwidth. The receiver can simply be an autocorrelation receiver and exploits the inherent multipath diversity with a low complexity receiver [1]. As a full digital implementation is extremely power consuming since GHz Analog to Digital (A/D) converters are needed, wideband analog delay lines are preferred for an integrated receiver design despite their inherent difficulties [2].

In chaotic-based systems, a chaotic sequence is used as a reference signal instead of very short pulses. For instance, in the differential chaos shift keying (DCSK) introduced by Kolumbàn et al., the chaotic sequence is used to modulate the data signal. Then, the reference signal is transmitted sequentially with the data-modulated signal [3]. In the correlation delay sift keying (CDSK) introduced by Sushchik et al. in 2000, the reference and the data-modulated signals are not transmitted sequentially, but added together with a predefined delay [4], [5].

Recently, delay and add direct sequence (DADS) has been introduced by Scheers and Le Nir [6]. DADS is proposed as a full digital modulation scheme and does not require wideband analog delay lines, thus it is more suited to spread-spectrum (SS) than UWB applications. DADS is a digital modulation scheme in which a pseudo-noise (PN) sequence is used on one hand as an embedded reference signal, and on the other hand for modulating the data information. This modulation scheme provides a processing gain and therefore inherits the advantage of conventional spread-spectrum communications such as multipath mitigation, anti-jamming, and multiple access capabilities. DADS is inspired on the CDSK scheme, in which a PN sequence is used for the spreading instead of a multilevel chaotic source. DADS offers some interesting properties compared to the CDSK, mainly the possibility to select some PN sequence which improves the BER performance without any additional complexity at the receiver. Indeed, theoretical analysis and simulation results have shown that the BER performance of CDSK is comparable to DADS with an arbitrary PN sequence, however the DADS performance can be improved by 3 dB with PN sequence selection [7]. Moreover, the DADS modulation scheme has a very simple receiver structure and is particularly suited for short burst transmissions. However, it can be observed that the bit error rate (BER) performance degrades as the length of the PN sequence increases. This effect can also be observed for CDSK and DCSK systems. Enhanced versions of the DCSK systems have been proposed by transmitting several information-modulated signals after the reference signal and by performing noise reduction by averaging and non-redundant error correction techniques [8].

In this paper, a method is proposed to improve the noise performance of the DADS modulation scheme for arbitrary PN sequence and PN sequence selection. For PN sequence selection, the structure of the reference signal is modified according the criterion developed in [7]. The reference signal is generated from the cyclic repetition of the same PN sequence. This PN sequence has two properties. First, the length of the PN sequence is twice the delay (in chips) used in the modulation scheme. Second, the correlation between the PN sequence and its shifted version by the delay used in the modulation scheme is zero. The demodulation algorithm exploits the redundancy of the PN sequence by averaging the received noisy chips in the same bit and/or between multiple bits. Since the receiver is fully implemented in digital, it does not require multiple wideband analog delay lines. It is shown that noise reduction by averaging can enhance the bit error rate (BER) performance of DADS considerably. As the number of noisy chips exploited at the receive side increases, the signal to noise ratio (SNR) gap between the BER performance of the proposed modulation scheme and the binary phase-shift keying (BPSK) modulation scheme approaches 3 dB. The theoretical results derived for the BER are verified by simulations in additive white Gaussian noise (AWGN) and frequency selective Rayleigh channels.

The remainder of this paper is organized as follows. In Section II, we recall the system model for arbitrary PN sequence and PN sequence selection. In Section III, the noise reduction by averaging with PN sequence selection and arbitrary PN sequence is presented in AWGN channels. In section IV, mathematical derivations are also derived for noise reduction by averaging with PN sequence selection and arbitrary PN sequence in frequency selective channels. Finally, simulation results are given in AWGN and frequency selective Rayleigh channels in Section V.

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This research work was carried out in the frame of the Belgian Defense Scientific Research & Technology Study C4/19 funded by the Ministry of Defense (MoD). The scientific responsibility is assumed by its authors.

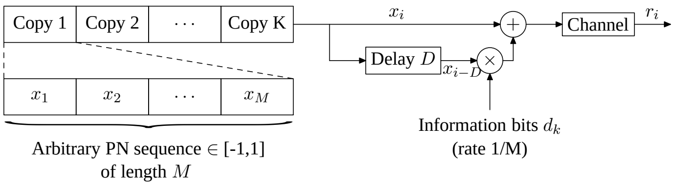

Fig. 1. Transmission chain of the DADS modulation scheme with an arbitrary PN sequence

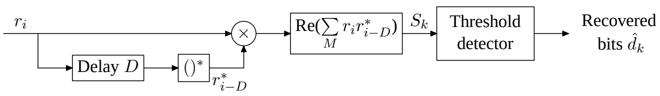

Fig. 2. Reception chain of the DADS modulation scheme

II. System Model

A. Arbitrary Pseudo Noise Sequence for DADS

The transmission chain of the DADS modulation scheme with an arbitrary PN sequence is shown on Figure 1. Assuming that K bits have to be transmitted, the PN sequence of length M is repeated K times to form the reference signal. The transmitted signal is the sum of two signals, namely the reference signal and its delayed version multiplied by the information signal. Considering an AWGN channel, the received signal r_i can be modeled as

$$r_i = d_k x_{i-D} + x_i + n_i \qquad (1)$$

with D the delay (in chips), d_k the information bits taking values in {-1,1} with data rate $1/M$, x_i the transmitted chip of the PN sequence and n_i the AWGN with variance $N_0/2$ per dimension. The reception chain of the DADS modulation scheme is shown on Figure 2. The correlator output is given by

$$S_k = \mathrm{Re}\left(\sum_{i=(k-1)M+D+1}^{kM+D} r_i r^*_{i-D}\right) \qquad (2)$$

with

$$\begin{aligned} r_i r^*_{i-D} = & \underbrace{d_k x^2_{i-D}}_{\text{useful part } a_i} \\ & \underbrace{+d_k x_i x_{i-2D} + x_i x_{i-D} + x_{i-D} x_{i-2D}}_{\text{interference part } b_i} \\ & \underbrace{+(d_k x_{i-D} + x_i) n^*_{i-D} + n_i (d_k x_{i-2D} + x_{i-D}) + n_i n^*_{i-D}}_{\text{noise part } c_i} \end{aligned} \qquad (3)$$

The first line of the equation corresponds to the useful part a_i. The second and third lines corresponds to the interference part b_i, which has 3 terms due to the cross-correlation between two delayed time intervals of the reference signal. The remaining lines correspond to the noise part c_i, which has 5 terms due to the cross-correlation between the reference signal and the noise (the last component being the cross-correlation of the noise with its delayed version). As the correlator output approaches a Gaussian distribution as M increases, the bit error rate (BER) performance can be expressed analytically [9]

$$\begin{aligned} BER = & \frac{1}{2}\Big[\mathrm{Prob}(S_k < 0 | d_k = +1) \\ & + \mathrm{Prob}(S_k \geq 0 | d_k = -1)\Big] \\ = & \frac{1}{4}\,\mathrm{erfc}\left(\frac{E[S_k|d_k=+1]}{\sqrt{2(var[S_k|d_k=+1])}}\right) \\ & + \frac{1}{4}\,\mathrm{erfc}\left(\frac{-E[S_k|d_k=-1]}{\sqrt{2(var[S_k|d_k=-1])}}\right) \end{aligned} \qquad (4)$$

with erfc(.) the complementary error function. Because x_i is statistically independent from x_j or n_j for any $i \neq j$, and n_i is statistically independent from n_j for any $i \neq j$, the cross-correlations between the useful, interference and noise parts are zero. Therefore, the correlator output approaches a Gaussian distribution as M increases with

$$\begin{aligned} E[S_k] &= E[A_k] + E[B_k] + E[C_k] \\ var[S_k] &= var[A_k] + var[B_k] + var[C_k] \end{aligned} \qquad (5)$$

and

$$\{A_k, B_k, C_k\} = \mathrm{Re}\left(\sum_{i=(k-1)M+D+1}^{kM+D} \{a_i, b_i, c_i\}\right) \qquad (6)$$

The integrated useful part A_k, interference part B_k and noise part C_k approach Gaussian distributions as M increases with the following means

$$\begin{aligned} E[A_k] &= d_k M P_s \\ E[B_k] &= 0 \\ E[C_k] &= 0 \end{aligned} \qquad (7)$$

with P_s the energy per chip and variances

$$\begin{aligned} var[A_k] &= 0 \\ var[B_k] &= 3MP_s^2 \\ var[C_k] &= 4MP_s\tfrac{N_0}{2} + M\tfrac{N_0^2}{2} \end{aligned} . \qquad (8)$$

Knowing that a transmitted data bit is the sum of two sequences of length M, the energy per bit E_b can be written as $E_b = 2MP_s$, the derivation of the BER formula leads to the following expression [6]

$$BER = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{8N_0\left(1 + \frac{3E_b}{4MN_0} + \frac{MN_0}{2E_b}\right)}}\right) \qquad (9)$$

As the length of the spreading code M increases, the BER converges towards the formula

$$BER \underset{M \nearrow}{\longrightarrow} \frac{1}{2}\,\mathrm{erfc}\left(\frac{E_b}{2N_0\sqrt{M}}\right) \qquad (10)$$

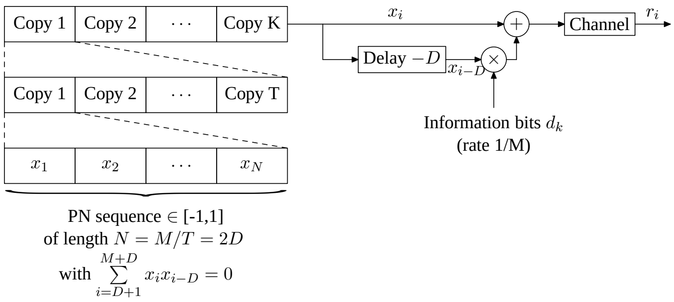

Fig. 3. Transmission chain of the DADS modulation scheme with PN sequence selection

B. *Pseudo Noise Sequence Selection for DADS*

The transmission chain of the DADS modulation scheme with PN sequence selection is shown on Figure 3. The structure of the reference signal is modified according the criterion introduced in [7] for PN sequence selection

$$\text{select } \{x_i\} = \begin{cases} x_i = x_{i-2D} & \forall i \\ \sum_{i=D+1}^{M+D} x_i x_{i-D} = 0 & \end{cases} \tag{11}$$

As shown on Figure 3, a PN sequence satisfying this criterion can be easily generated from the repetition of the same PN sequence whose length N is twice the delay D used in the modulation scheme ($N = 2D$). Moreover, the autocorrelation between the PN sequence and its shifted version by the delay D should be zero. Therefore, a PN sequence satisfying this condition has to be selected from the 2^N possible codes. The ratio between the number of codes satisfying this autocorrelation property and the total number of codes 2^N for delays $D = 2, 4, 6, 8$ are 0.5, 0.375, 0.3125, and 0.2734 respectively. The reception chain of the DADS modulation scheme shown on Figure 2 can also be used with PN sequence selection [7]. The received signal multiplied by its delayed conjugate version is given by

$$\begin{aligned} r_i r^*_{i-D} = & \underbrace{d_k(x^2_{i-D} + x^2_i)}_{\text{useful part } a_i} + \underbrace{2x_i x_{i-D}}_{\text{interference part } b_i} \\ & \underbrace{+(d_k x_{i-D} + x_i)n^*_{i-D} + n_i(d_k x_i + x_{i-D}) + n_i n^*_{i-D}}_{\text{noise part } c_i} \end{aligned} \tag{12}$$

Because the cross-correlations between the useful, interference and noise parts are zero, the correlator output approaches a Gaussian distribution as M increases according to the formulas (5) and (6). The integrated useful part A_k, interference part B_k and noise part C_k approach Gaussian distributions as M increases with means

$$\begin{aligned} E[A_k] &= 2d_k M P_s \\ E[B_k] &= 0 \\ E[C_k] &= 0 \end{aligned} \tag{13}$$

and variances

$$\begin{aligned} var[A_k] &= 0 \\ var[B_k] &= 0 \\ var[C_k] &= 4MP_s\tfrac{N_0}{2} + M\tfrac{N_0^2}{2} \end{aligned} \tag{14}$$

Therefore, a 3 dB gain on the useful part is achieved while nulling the interference part. The derivations of the BER formula give the following expression

$$BER = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{2N_0\left(1 + \dfrac{MN_0}{2E_b}\right)}}\right) \tag{15}$$

As the length of the spreading code M increases, the BER converges towards the formula

$$BER \underset{M\nearrow}{\longrightarrow} \frac{1}{2}\,\text{erfc}\left(\frac{E_b}{N_0\sqrt{M}}\right) \tag{16}$$

confirming the 3 dB gain with formula (10).

III. Noise reduction by averaging in DADS with Pseudo Noise Sequence Selection in Additive White Gaussian Noise Channels

A. *Averaging in the same bit and between multiple bits*

The demodulation algorithm can exploit the redundancy of the PN sequence by averaging the received noisy chips in the same bit and between multiple bits as shown on Figure 4. The idea is then to correlate the delayed version of the received signal with its enhanced version. Considering Q bits and assuming that the PN sequence of length N has been repeated T times, the enhanced received signal can be generated by averaging the T chips and Q bits

$$\begin{aligned} \hat{r}_i &= \frac{1}{QT}\sum_{q=1}^{Q}\sum_{t=1}^{T} r_{j+(t-1)N+(q-1)M} \\ \hat{r}_i &= \frac{1}{QT}\sum_{q=1}^{Q}\sum_{t=1}^{T} (d_q x_{j+(t-1)N+(q-1)M-D} \\ & + x_{j+(t-1)N+(q-1)M} + n_{j+(t-1)N+(q-1)M}) \end{aligned} \tag{17}$$

with $j = i \mod N$. Knowing that $x_i = x_{i-2D}$, $N = 2D$, and $M = TN$

$$\hat{r}_i = (\frac{1}{Q}\sum_{q=1}^{Q} d_q)x_{i-D} + x_i + \frac{1}{QT}\sum_{q=1}^{Q}\sum_{t=1}^{T} n_{j+(t-1)N+(q-1)M} \tag{18}$$

The correlator output becomes

$$S_k = \text{Re}\left(\sum_{i=(k-1)M+D+1}^{kM+D} \hat{r}_i r^*_{i-D}\right) \tag{19}$$

with

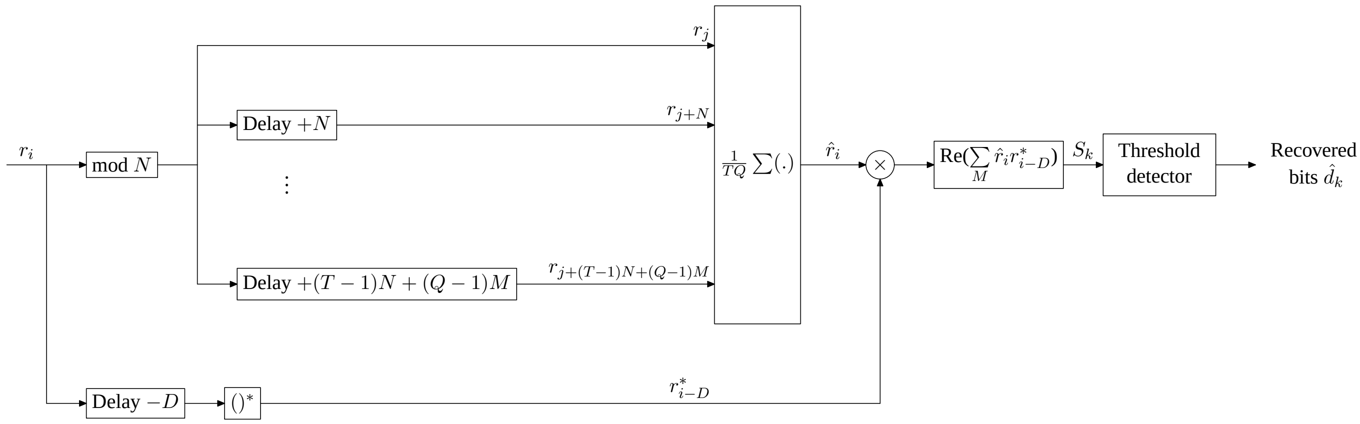

Fig. 4. Generic receiver exploiting noise reduction by averaging in DADS

$$
\begin{aligned}
\hat{r}_i r^*_{i-D} = & \underbrace{d_k x_i^2}_{\text{useful part } a_i} \\
& \underbrace{+(\frac{1}{Q}\sum_{q=1}^{Q} d_q)x^2_{i-D} +(\frac{d_k}{Q}\sum_{q=1}^{Q} d_q)x_{i-D}x_i + x_i x_{i-D}}_{\text{interference part } b_i} \\
& \underbrace{\begin{aligned} &+(\frac{1}{Q}\sum_{q=1}^{Q} d_q)x_{i-D}n^*_{i-D} + x_i n^*_{i-D} \\ &+\frac{1}{QT}\sum_{q=1}^{Q}\sum_{t=1}^{T} n_{j+(t-1)N+(q-1)M}(d_k x_i + x_{i-D}) \\ &+\frac{1}{QT}\sum_{q=1}^{Q}\sum_{t=1}^{T} n_{j+(t-1)N+(q-1)M}n^*_{i-D}\end{aligned}}_{\text{noise part } c_i}
\end{aligned}
\tag{20}
$$

Because the cross-correlations between the useful, interference and noise parts are zero, the correlator output approaches a Gaussian distribution as M increases according to the formulas (5) and (6). The integrated useful part A_k, interference part B_k and noise part C_k approach Gaussian distributions as M increases with means

$$
\begin{aligned}
E[A_k] &= d_k M P_s \\
E[B_k] &= (\frac{1}{Q}\sum_{q=1}^{Q} d_q) M P_s \\
E[C_k] &= 0
\end{aligned}
\tag{21}
$$

and variances

$$
\begin{aligned}
var[A_k] &= 0 \\
var[B_k] &= 0 \\
var[C_k] &= (1+(\frac{1}{Q}\sum_{q=1}^{Q} d_q)^2) M P_s \tfrac{N_0}{2} + 2\frac{M}{Q}P_s\tfrac{N_0}{2} + 2\frac{N}{Q}\tfrac{N_0^2}{2}
\end{aligned}
\tag{22}
$$

leading to the following BER

$$
\begin{aligned}
BER = & \frac{1}{4}\,\text{erfc}\left(\sqrt{E_b\left(1+\frac{1}{Q}+\frac{1}{Q}\sum_{\substack{q=1\\ q\neq k}}^{Q} d_q\right)^2 / 2N_0\Gamma}\right) \\
& +\frac{1}{4}\,\text{erfc}\left(\sqrt{E_b\left(-1-\frac{1}{Q}+\frac{1}{Q}\sum_{\substack{q=1\\ q\neq k}}^{Q} d_q\right)^2 / 2N_0\Gamma}\right)
\end{aligned}
\tag{23}
$$

with

$$
\Gamma = 1 + (\frac{1}{Q}\sum_{q=1}^{Q} d_q)^2 + \frac{2}{Q} + \frac{4NN_0}{QE_b} \tag{24}
$$

One can notice that (23)(24) are semi-analytical formulas, since they depend on the data bits which are concerned for noise averaging. In the simulation results, Monte Carlo methods will be used to generate the theoretical average BER performance [10]. The same approach will be used through the paper whenever noise averaging involves several data bits. As the number of bits Q increases, the BER converges towards the BER of the BPSK system in AWGN channels shifted by 3 dB

$$
BER = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right) \tag{25}
$$

B. Averaging in the same bit only

As a special case, the demodulation algorithm can exploit the redundancy of the PN sequence by averaging the received noisy chips in the same bit only ($q = k$). Following the same derivations, the averaged received signal (17) can be rewritten as

$$
\hat{r}_i = d_k x_{i-D} + x_i + \frac{1}{T}\sum_{t=1}^{T} n_{j+(t-1)N} \tag{26}
$$

and (19) can be rewritten as

$$\begin{aligned}\hat{r}_i r^*_{i-D} &= \underbrace{d_k(x^2_{i-D}+x^2_i)}_{\text{useful part } a_i} + \underbrace{2x_i x_{i-D}}_{\text{interference part } b_i} \\ &\underbrace{\begin{aligned}&+d_k x_{i-D} n^*_{i-D} + x_i n^*_{i-D} \\ &+\frac{1}{T}\sum_{t=1}^{T} n_{j+(t-1)N}(d_k x_i + x_{i-D}) \\ &+\frac{1}{T}\sum_{t=1}^{T} n_{j+(t-1)N} n^*_{i-D}\end{aligned}}_{\text{noise part } c_i}\end{aligned} \tag{27}$$

Because the cross-correlations between the useful, interference and noise parts are zero, the correlator output approaches a Gaussian distribution as M increases according to the formulas (5) and (6). The integrated useful part A_k, interference part B_k and noise part C_k approach Gaussian distributions as M increases with means

$$\begin{aligned}E[A_k] &= 2d_k M P_s \\ E[B_k] &= 0 \\ E[C_k] &= 0\end{aligned} \tag{28}$$

and variances

$$\begin{aligned}var[A_k] &= 0 \\ var[B_k] &= 0 \\ var[C_k] &= 4MP_s\tfrac{N_0}{2} + 2N\tfrac{N_0^2}{2}\end{aligned}\ . \tag{29}$$

leading to the following BER

$$BER = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{2N_0\left(1+\frac{NN_0}{E_b}\right)}}\right) \tag{30}$$

C. Remark

With an arbitrary PN sequence, there is no redundancy to be exploited in the same bit. However, the demodulation algorithm can exploit the redundancy of the PN sequence by averaging the received noisy chips between multiple bits. This is also a special case of the generic reception chain shown in Figure 4 with $N = M$ and $T = 1$, except that the noise averaging has to be done on the delayed version of the received signal. Considering Q bits, the enhanced received signal can be generated by averaging the Q bits

$$\hat{r}^*_{i-D} = \left(\frac{1}{Q}\sum_{q=1}^{Q} d_q\right)x_{i-2D} + x_{i-D} + \frac{1}{Q}\sum_{q=1}^{Q} n^*_{j-D+(q-1)M} \tag{31}$$

The correlator output can be rewritten as

$$\begin{aligned}r_i\hat{r}^*_{i-D} &= \underbrace{d_k x^2_{i-D}}_{\text{useful part } a_i} \\ &\underbrace{\begin{aligned}&+\left(\frac{1}{Q}\sum_{q=1}^{Q} d_q\right)x_i x_{i-2D} \\ &+\left(\frac{1}{Q}\sum_{q=1}^{Q} d_q\right)d_k x_{i-D}x_{i-2D} + x_i x_{i-D}\end{aligned}}_{\text{interference part } b_i} \\ &\underbrace{\begin{aligned}&+\left(\frac{1}{Q}\sum_{q=1}^{Q} d_q\right)n_i x_{i-2D} + n_i x_{i-D} \\ &+(d_k x_{i-D} + x_i)\left(\frac{1}{Q}\sum_{q=1}^{Q} n^*_{j-D+(q-1)M}\right) \\ &+n_i\left(\frac{1}{Q}\sum_{q=1}^{Q} n^*_{j-D+(q-1)M}\right)\end{aligned}}_{\text{noise part } c_i}\end{aligned} \tag{32}$$

Because the cross-correlations between the useful, interference and noise parts are zero, the correlator output approaches a Gaussian distribution as M increases according to the formulas (5) and (6). The integrated useful part A_k, interference part B_k and noise part C_k approach Gaussian distributions as M increases with means

$$\begin{aligned}E[A_k] &= d_k M P_s \\ E[B_k] &= 0 \\ E[C_k] &= 0\end{aligned} \tag{33}$$

with P_s the energy per chip and variances

$$\begin{aligned}var[A_k] &= 0 \\ var[B_k] &= \left(1+2\left(\frac{1}{Q}\sum_{q=1}^{Q} d_q\right)^2\right)MP_s^2 \\ var[C_k] &= \left(1+\left(\frac{1}{Q}\sum_{q=1}^{Q} d_q\right)^2\right)MP_s\tfrac{N_0}{2} + 2\frac{M}{Q}P_s\tfrac{N_0}{2} + \frac{M}{Q}\tfrac{N_0^2}{2}\end{aligned} \tag{34}$$

leading to the following BER

$$BER = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{2N_0\Gamma}}\right) \tag{35}$$

with

$$\Gamma = 1+\left(\frac{1}{Q}\sum_{q=1}^{Q} d_q\right)^2 + \frac{2}{Q} + \frac{E_b}{MN_0}\left(1+2\left(\frac{1}{Q}\sum_{q=1}^{Q} d_q\right)^2\right) + \frac{2MN_0}{QE_b} \tag{36}$$

As the number of bits Q and the length of the PN sequence M increases with a ratio $M/Q << 1$, the BER converges towards the BER of the BPSK system in AWGN channels shifted by 3 dB (25).

IV. Noise reduction by averaging in DADS with Pseudo Noise Sequence Selection in Frequency Selective Channels

A. Averaging in the same bit and between multiple bits

We consider a frequency selective channel with AWGN. The received signal r_i can be modeled as

$$r_i = \sum_{l=0}^{L-1} h_l \left(d_k x_{i-l-D} + x_{i-l}\right) + n_i \tag{37}$$

with L the number of taps and h_l the complex-valued channel attenuation for the l^{th} tap. Considering Q bits and assuming that the PN sequence of length N has been repeated T times, the enhanced received signal can be generated by averaging the T chips and Q bits

$$\begin{aligned}\hat{r}_i = &\sum_{l=0}^{L-1} h_l((\frac{1}{Q}\sum_{q=1}^{Q} d_q)x_{i-l-D} + x_{i-l}) \\ &+\frac{1}{QT}\sum_{q=1}^{Q}\sum_{t=1}^{T} n_{j+(t-1)N+(q-1)M}\end{aligned} \tag{38}$$

The enhanced received signal multiplied by the delayed version of the received signal gives

$$\begin{aligned}\hat{r}_i r^*_{i-D} = &\underbrace{d_k \sum_{l=0}^{L-1} |h_l|^2 x^2_{i-l}}_{\text{useful part } a_i} \\ &\underbrace{\begin{aligned}&+(\frac{1}{Q}\sum_{q=1}^{Q} d_q)\sum_{l=0}^{L-1}|h_l|^2 x^2_{i-l-D} \\ &+(\frac{d_k}{Q}\sum_{q=1}^{Q} d_q)\sum_{l=0}^{L-1}|h_l|^2 x_{i-l-D}x_{i-l} + \sum_{l=0}^{L-1}|h_l|^2 x_{i-l}x_{i-l-D} \\ &+\sum_{l=0}^{L-1}\sum_{l'\neq l} h_l h^*_{l'}((\frac{1}{Q}\sum_{q=1}^{Q} d_q)x_{i-l-D} + x_{i-l})(d_k x_{i-l'} + x_{i-l'-D})\end{aligned}}_{\text{interference part } b_i} \\ &\underbrace{\begin{aligned}&+\sum_{l=0}^{L-1} h_l((\frac{1}{Q}\sum_{q=1}^{Q} d_q)x_{i-l-D} + x_{i-l})n^*_{i-D} \\ &+\frac{1}{QT}\sum_{q=1}^{Q}\sum_{t=1}^{T}\sum_{l=0}^{L-1} h^*_l n_{j+(t-1)N+(q-1)M}(d_k x_{i-l} + x_{i-l-D}) \\ &+\frac{1}{QT}\sum_{q=1}^{Q}\sum_{t=1}^{T} n_{j+(t-1)N+(q-1)M} n^*_{i-D}\end{aligned}}_{\text{noise part } c_i}\end{aligned} \tag{39}$$

Assuming that the delay $\tau_{max} \leq D \ll M$, the cross-correlations between the useful, interference and noise parts are zero. The correlator output approaches a Gaussian distribution as M increases according to the formulas (5) and (6). The useful part A_k, the interference part B_k and the noise part C_k can be approximated as Gaussian variables as M increases with means

$$\begin{aligned}E[A_k] &= d_k \sum_{l=0}^{L-1} |h_l|^2 M P_s \\ E[B_k] &= (\frac{1}{Q}\sum_{q=1}^{Q} d_q)\sum_{l=0}^{L-1}|h_l|^2 M P_s \\ E[C_k] &= 0\end{aligned} \tag{40}$$

and variances

$$\begin{aligned}var[A_k] &= 0 \\ var[B_k] &= 2(1+(\frac{1}{Q}\sum_{q=1}^{Q} d_q)^2)\sum_{l=0}^{L-1}\sum_{l'\neq l}|h_l|^2|h_{l'}|^2 M P_s^2 \\ var[C_k] &= (1+(\frac{1}{Q}\sum_{q=1}^{Q} d_q)^2)\sum_{l=0}^{L-1}|h_l|^2 M P_s \tfrac{N_0}{2} \\ &\quad +2\frac{M}{Q}\sum_{l=0}^{L-1}|h_l|^2 P_s \tfrac{N_0}{2} + 2\frac{N}{Q}\tfrac{N_0^2}{2}\end{aligned} \tag{41}$$

leading to the following BER

$$\begin{aligned}BER = &\frac{1}{4} \underset{h_l}{E}\left[\text{erfc}\left(\sqrt{\sum_{l=0}^{L-1}|h_l|^2 E_b\left(1+\frac{1}{Q}+\frac{1}{Q}\sum_{\substack{q=1\\ q\neq k}}^{Q} d_q\right)^2 / 2N_0\Gamma}\right)\right] \\ &+\frac{1}{4} \underset{h_l}{E}\left[\text{erfc}\left(\sqrt{\sum_{l=0}^{L-1}|h_l|^2 E_b\left(-1-\frac{1}{Q}+\frac{1}{Q}\sum_{\substack{q=1\\ q\neq k}}^{Q} d_q\right)^2 / 2N_0\Gamma}\right)\right]\end{aligned} \tag{42}$$

with $\underset{h_l}{E}$ the expected value over the channel realizations [10] and

$$\begin{aligned}\Gamma = &1 + (\frac{1}{Q}\sum_{q=1}^{Q} d_q)^2 + \frac{2}{Q} \\ &+\frac{2(1+(\frac{1}{Q}\sum_{q=1}^{Q} d_q)^2)\sum_{l=0}^{L-1}\sum_{l'\neq l}|h_l|^2|h_{l'}|^2 E_b}{\sum_{l=0}^{L-1}|h_l|^2 M N_0} + \frac{4NN_0}{Q\sum_{l=0}^{L-1}|h_l|^2 E_b}\end{aligned} \tag{43}$$

B. Averaging in the same bit only

The enhanced received signal can be generated by averaging the T chips

$$\begin{aligned}\hat{r}_i = &\sum_{l=0}^{L-1} h_l(d_k x_{i-l-D} + x_{i-l}) \\ &+\frac{1}{T}\sum_{t=1}^{T} n_{j+(t-1)N}\end{aligned} \tag{44}$$

The enhanced received signal multiplied by the conjugate delayed version of the received signal gives

$$\begin{aligned}\hat{r}_i r^*_{i-D} = &\underbrace{d_k \sum_{l=0}^{L-1}|h_l|^2(x^2_{i-l} + x^2_{i-l-D})}_{\text{useful part } a_i} \\ &\underbrace{\begin{aligned}&+2\sum_{l=0}^{L-1}|h_l|^2 x_{i-l}x_{i-l-D} \\ &+\sum_{l=0}^{L-1}\sum_{l'\neq l} h_l h^*_{l'}(d_k x_{i-l-D} + x_{i-l})(d_k x_{i-l'} + x_{i-l'-D})\end{aligned}}_{\text{interference part } b_i} \\ &\underbrace{\begin{aligned}&+\sum_{l=0}^{L-1} h_l(d_k x_{i-l-D} + x_{i-l})n^*_{i-D} \\ &+\frac{1}{T}\sum_{t=1}^{T}\sum_{l=0}^{L-1} h^*_l n_{j+(t-1)N}(d_k x_{i-l} + x_{i-l-D}) \\ &+\frac{1}{T}\sum_{t=1}^{T} n_{j+(t-1)N} n^*_{i-D}\end{aligned}}_{\text{noise part } c_i}\end{aligned} \tag{45}$$

Assuming that the delay $\tau_{max} \leq D \ll M$, the cross-correlations between the useful, interference and noise parts are zero. The correlator output approaches a Gaussian distribution as M increases according to the formulas (5) and (6). The useful part A_k, the interference part B_k and the noise part C_k can be approximated as Gaussian variables as M increases with means

$$\begin{aligned} E[A_k] &= 2d_k \sum_{l=0}^{L-1} |h_l|^2 M P_s \\ E[B_k] &= 0 \\ E[C_k] &= 0 \end{aligned} \tag{46}$$

and variances

$$\begin{aligned} var[A_k] &= 0 \\ var[B_k] &= 4 \sum_{l=0}^{L-1} \sum_{l' \neq l} |h_l|^2 |h_{l'}|^2 M P_s^2 \\ var[C_k] &= 4 \sum_{l=0}^{L-1} |h_l|^2 M P_s \tfrac{N_0}{2} + 2N \tfrac{N_0^2}{2} \end{aligned} \tag{47}$$

leading to the following BER

$$BER = \frac{1}{2} \underset{h_l}{E} \left[\text{erfc} \left(\sqrt{\frac{E_b}{2N_0 \Gamma}} \right) \right] \tag{48}$$

with

$$\begin{aligned} \Gamma &= 1 + \frac{\sum_{l=0}^{L-1} \sum_{l' \neq l} |h_l|^2 |h_{l'}|^2 E_b}{\sum_{l=0}^{L-1} |h_l|^2 M N_0} \\ &+ \frac{N N_0}{\sum_{l=0}^{L-1} |h_l|^2 E_b} \end{aligned} \tag{49}$$

C. Remark

With an arbitrary PN sequence and considering Q bits, the enhanced received signal can be generated by averaging the Q bits

$$\begin{aligned} \hat{r}_{i-D}^* &= \sum_{l=0}^{L-1} h_l^* \big((\frac{1}{Q} \sum_{q=1}^{Q} d_q) x_{i-l-2D} + x_{i-l-D} \big) \\ &+ \frac{1}{Q} \sum_{q=1}^{Q} n_{j-D+(q-1)M}^* \end{aligned} \tag{50}$$

The received signal multiplied by the conjugate delayed version of the enhanced received signal gives

$$\begin{aligned} r_i \hat{r}_{i-D}^* = & \underbrace{d_k \sum_{l=0}^{L-1} |h_l|^2 x_{i-l-D}^2}_{\text{useful part } a_i} \\ & \underbrace{\begin{aligned} &+ (\frac{1}{Q} \sum_{q=1}^{Q} d_q) \sum_{l=0}^{L-1} |h_l|^2 x_{i-l} x_{i-l-2D} \\ &+ d_k (\frac{1}{Q} \sum_{q=1}^{Q} d_q) \sum_{l=0}^{L-1} |h_l|^2 x_{i-l-D} x_{i-l-2D} + \sum_{l=0}^{L-1} |h_l|^2 x_{i-l} x_{i-l-D} \\ &+ \sum_{l=0}^{L-1} \sum_{l' \neq l} h_l h_{l'}^* (d_k x_{i-l-D} + x_{i-l}) ((\frac{1}{Q} \sum_{q=1}^{Q} d_q) x_{i-l'-2D} + x_{i-l'-D}) \end{aligned}}_{\text{interference part } b_i} \\ & \underbrace{\begin{aligned} &+ n_i \sum_{l=0}^{L-1} h_l^* ((\frac{1}{Q} \sum_{q=1}^{Q} d_q) x_{i-l-2D} + x_{i-l-D}) \\ &+ \sum_{l=0}^{L-1} h_l (d_k x_{i-l-D} + x_{i-l}) (\frac{1}{Q} \sum_{q=1}^{Q} n_{j-D+(q-1)M}^*) \\ &+ n_i (\frac{1}{Q} \sum_{q=1}^{Q} n_{j-D+(q-1)M}^*) \end{aligned}}_{\text{noise part } c_i} \end{aligned} \tag{51}$$

Because the cross-correlations between the useful, interference and noise parts are zero, the correlator output approaches a Gaussian distribution as M increases according to the formulas (5) and (6). The integrated useful part A_k, interference part B_k and noise part C_k approach Gaussian distributions as M increases with means

$$\begin{aligned} E[A_k] &= d_k \sum_{l=0}^{L-1} |h_l|^2 M P_s \\ E[B_k] &= 0 \\ E[C_k] &= 0 \end{aligned} \tag{52}$$

and variances

$$\begin{aligned} var[A_k] &= 0 \\ var[B_k] &= (1 + 2(\frac{1}{Q} \sum_{q=1}^{Q} d_q)^2) \sum_{l=0}^{L-1} |h_l|^4 M P_s^2 \\ &\quad + 2(1 + (\frac{1}{Q} \sum_{q=1}^{Q} d_q)^2) \sum_{l=0}^{L-1} \sum_{l' \neq l} |h_l|^2 |h_{l'}|^2 M P_s^2 \\ var[C_k] &= (1 + (\frac{1}{Q} \sum_{q=1}^{Q} d_q)^2) \sum_{l=0}^{L-1} |h_l|^2 M P_s \tfrac{N_0}{2} \\ &\quad + 2\frac{M}{Q} \sum_{l=0}^{L-1} |h_l|^2 P_s \tfrac{N_0}{2} + \frac{M}{Q} \tfrac{N_0^2}{2} \end{aligned} \tag{53}$$

leading to the following BER

$$BER = \frac{1}{2} \underset{h_l}{E} \left[\text{erfc} \left(\sqrt{\sum_{l=0}^{L-1} |h_l|^2 E_b / 2N_0 \Gamma} \right) \right] \tag{54}$$

with

$$\begin{aligned} \Gamma = 1 + \frac{2}{Q} + (\frac{1}{Q} \sum_{q=1}^{Q} d_q)^2 + \frac{(1 + 2(\frac{1}{Q} \sum_{q=1}^{Q} d_q)^2) \sum_{l=0}^{L-1} |h_l|^4 E_b}{\sum_{l=0}^{L-1} |h_l|^2 M N_0} \\ + \frac{2(1 + (\frac{1}{Q} \sum_{q=1}^{Q} d_q)^2) \sum_{l=0}^{L-1} \sum_{l' \neq l} |h_l|^2 |h_{l'}|^2 E_b}{\sum_{l=0}^{L-1} |h_l|^2 M N_0} + \frac{2 M N_0}{Q \sum_{l=0}^{L-1} |h_l|^2 E_b} \end{aligned} \tag{55}$$

V. SIMULATION RESULTS

In this Section, simulation results are given to verify the formulas of Sections II, III and IV for arbitrary PN sequence and PN sequence selection with/without noise averaging within a single bit and between multiple bits in AWGN and frequency selective Rayleigh channels.

Figure 5 shows the BER performance of arbitrary PN sequence and PN sequence selection with/without noise averaging within a single bit and between multiple bits Q, delay $D = 2$, and length of the spreading code $M = 4096$ in AWGN channels. The difference between the theoretical and simulated BER performance is very close, thus proving the relevance of the Gaussian approximation for all the proposed transmitter and receiver schemes. As indicated by the formulas (10) and (16), a gain of about 3 dB at 10^{-3} can be obtained by PN selection without noise averaging compared to arbitrary

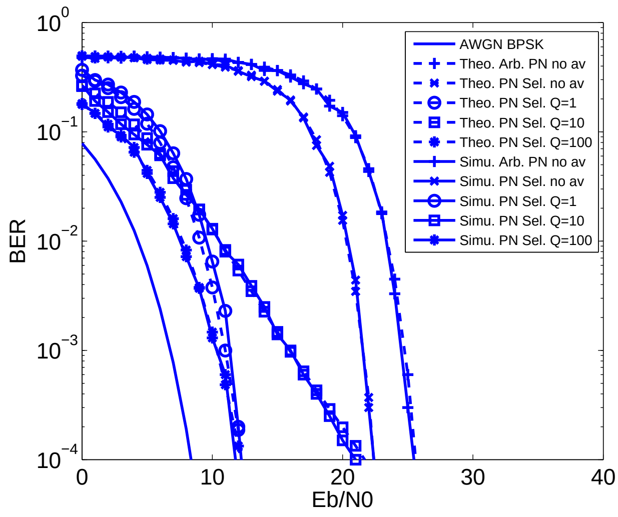

Fig. 5. Simulation results for arbitrary PN sequence and PN sequence selection with/without noise averaging within a single bit and between multiple bits Q, delay $D = 2$, and length of the spreading code $M = 4096$ in AWGN channels

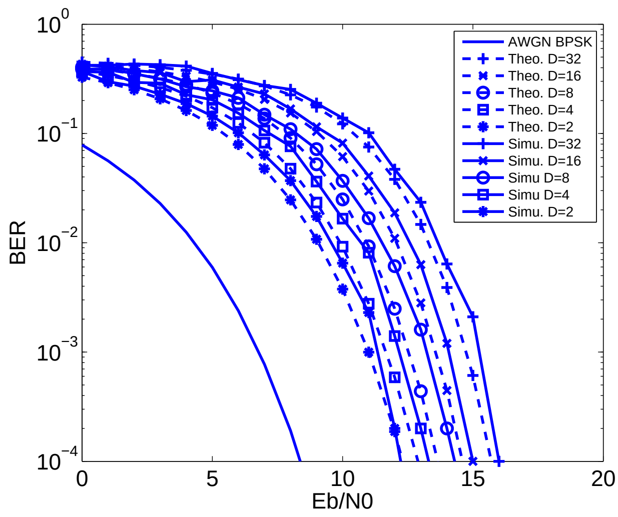

Fig. 6. Simulation results for PN sequence selection with noise averaging within a single bit $Q = 1$, various delays D, and length of the spreading code $M = 4096$ in AWGN channels

PN sequence without noise averaging. When noise averaging within a single bit $Q = 1$ is performed at the receive side with PN selection, a gain of about 10 dB at 10^{-3} can be obtained compared to PN selection without noise averaging. The BER performance can further be improved if a sufficient number of bits $Q > 1$ is used for PN selection with noise averaging within a single bit and between multiple bits. Indeed, the BER performance degrades when only few bits are used for noise averaging (loss of about 5 dB at 10^{-3} with $Q = 10$). However, a further gain of about 1 dB can be observed when $Q = 100$. As Q increases, the BER performance converges towards the BER performance of the BPSK AWGN performance shifted by 3 dB as indicated by formulas (25).

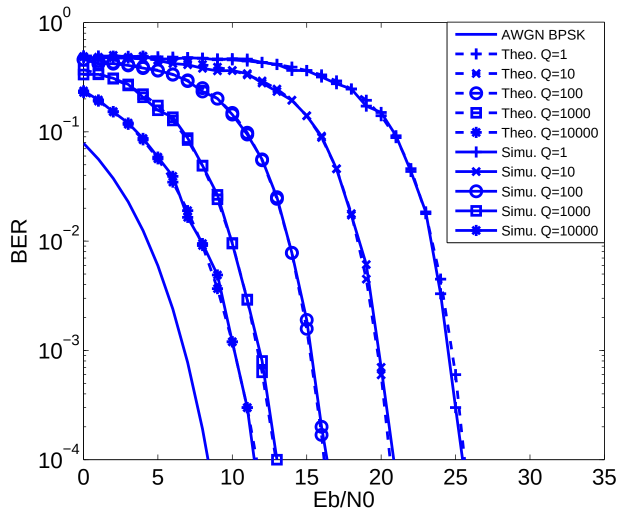

Fig. 7. Simulation results for arbitrary PN sequence with/without noise averaging between multiple bits Q, delay $D = 2$, and length of the spreading code $M = 4096$ in AWGN channels

Figure 6 shows the BER performance of PN sequence selection with noise averaging within a single bit $Q = 1$, various delays D, and length of the spreading code $M = 4096$ in AWGN channels. The difference between the theoretical and simulated BER performance is also very close. With noise averaging, it can be observed that the BER performance no longer degrades as M increases, but still degrades as N increases, N being the length of the pseudo code of length $N = 2D$ which is repeated T times to form the spreading sequence of length M. Indeed, a loss of about 1 dB at 10^{-3} occurs when the delay D doubles (when the length of the pseudo code N doubles as well). Therefore, the delay D should be kept as small as possible when considering PN sequence selection with noise averaging within a single bit.

Figure 7 shows the BER performance of arbitrary PN sequence with/without noise averaging between multiple bits Q, delay $D = 2$, and length of the spreading code $M = 4096$ in AWGN channels. The Gaussian approximated BER formulas (35) and (36) match well the simulated BER performance. It can be observed that a gain of about 5 dB, 9 dB, 13 dB and 14 dB can be obtained at 10^{-3} for arbitrary PN sequence with noise averaging between respectively $Q = 10$, $Q = 100$, $Q = 1000$, and $Q = 10000$ bits compared to arbitrary PN sequence without noise averaging. As indicated by the formulas (35) and (36), the BER performance also converges towards the BER performance of the BPSK AWGN performance shifted by 3 dB as Q increases, but also requires more bits than PN sequence selection with noise averaging within a single bit and between multiple bits.

Figure 8 shows the BER performance of PN sequence selection with noise averaging within a single bit $Q = 1$, various number of taps L, delay $D = 32$, and length of the spreading code $M = 4096$ in frequency selective Rayleigh channels. The maximum delay spread $\tau_{max} \leq D$ and the channel taps are modeled as independent and identically distributed (i.i.d.) zero-mean complex Gaussian variables normalized by

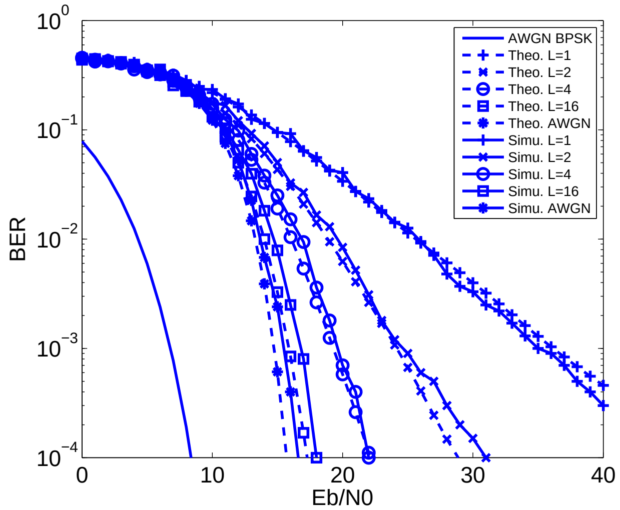

Fig. 8. Simulation results for PN sequence selection with noise averaging within a single bit $Q = 1$, various number of taps L, delay $D = 32$, and length of the spreading code $M = 4096$ in frequency selective Rayleigh channels

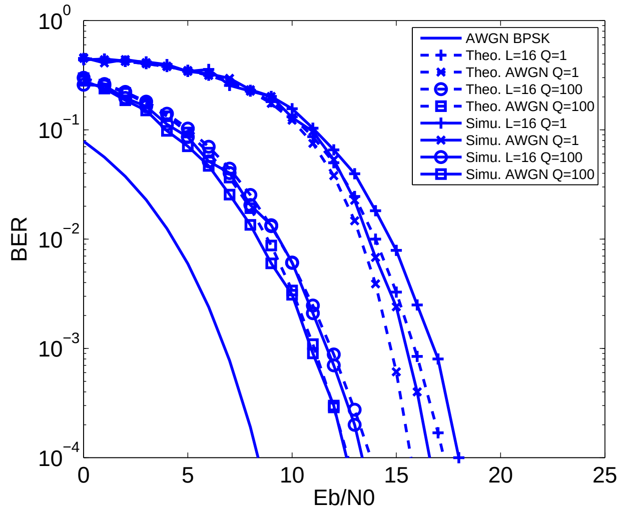

Fig. 9. Simulation results for PN sequence selection with noise averaging within a single bit and between multiple bits Q, delay $D = 32$, and length of the spreading code $M = 4096$ in AWGN and frequency selective Rayleigh channels with number of taps $L = 16$

the square root of the number of taps $1/\sqrt{L}$. The channel taps are assumed to be constant within the duration of a single bit. The theoretical formulas (48) and (49) for frequency selective Rayleigh channels coincide with the simulated BER performance. A loss of about 19.5 dB at 10^{-3} can be observed between the BER performance in AWGN and single tap $L = 1$ Rayleigh channels. This loss is reduced to a value of about 9 dB, 4 dB, and 1 dB at 10^{-3} in respectively two taps $L = 2$, four taps $L = 4$, and sixteen taps $L = 16$ Rayleigh channels. Therefore, the diversity order offered by frequency selective Rayleigh channels is exploited without adding some extra complexity to the receiver, which is a huge advantage compared to coherent DSSS modulations in which a large number of Rake fingers would be needed.

Figure 9 shows the comparison between the BER performance of PN sequence selection with noise averaging within a single bit and between multiple bits Q, delay $D = 32$, and length of the spreading code $M = 4096$ in AWGN and frequency selective Rayleigh channels with number of taps $L = 16$. The theoretical formulas (42) and (43) for frequency selective Rayleigh channels coincide with the simulated BER performance. As observed previously, a loss of about 1.3 dB at 10^{-3} can be observed between the BER performance in AWGN and sixteen tap $L = 16$ Rayleigh channels for PN sequence selection with noise averaging within a single bit $Q = 1$. The BER performance can further be improved if a sufficient number of bits $Q > 1$ is used for PN selection with noise averaging within a single bit and between multiple bits. Indeed, a further gain of about 4.5 dB can be observed in AWGN channels when $Q = 100$. Moreover, a loss of about 0.8 dB can be observed between the BER performance in AWGN and sixteen tap $L = 16$ Rayleigh channels for PN sequence selection with noise averaging within a single bit and multiple bits $Q = 100$. Therefore, the BER performance of this scheme is very close to the BPSK AWGN performance shifted by 3 dB as predicted by the formulas (42) and (43).

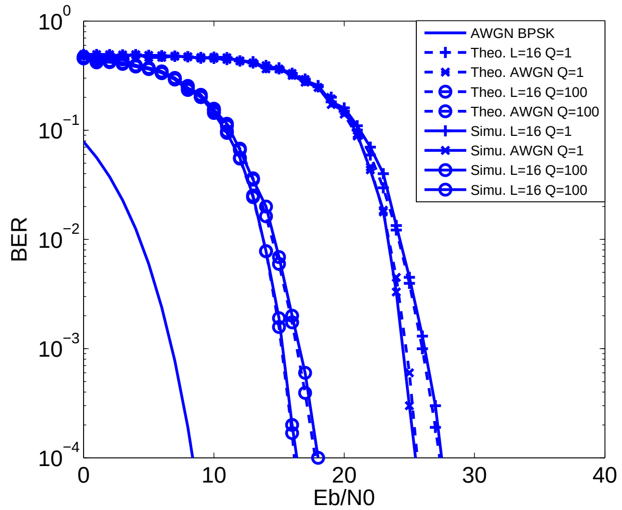

Fig. 10. Simulation results for arbitrary PN sequence with/without noise averaging between multiple bits Q, delay $D = 32$, and length of the spreading code $M = 4096$ in AWGN and frequency selective Rayleigh channels with number of taps $L = 16$

Figure 10 shows the comparison between the BER performance of arbitrary PN sequence with/without noise averaging between multiple bits Q, delay $D = 32$, and length of the spreading code $M = 4096$ in AWGN and frequency selective Rayleigh channels with number of taps $L = 16$. The theoretical formulas (54) and (55) for frequency selective Rayleigh channels coincide with the simulated BER performance. A loss of about 1.7 dB at 10^{-3} can be observed between the BER performance in AWGN and sixteen tap $L = 16$ Rayleigh channels for arbitrary PN sequence without noise averaging. The BER performance can be improved if a sufficient number of bits $Q > 1$ is used for arbitrary PN sequence with noise averaging between multiple bits. Indeed, a gain of about 9.2 dB can be observed in AWGN channels when $Q = 100$. In this case, the loss between the BER performance in AWGN and sixteen tap $L = 16$ Rayleigh channels is reduced to 1.3 dB at 10^{-3}. However, in order to improve the BER performance, one has to consider even more bits for noise averaging contrary to PN sequence selection with noise averaging within a single bit in which $Q = 100$ is sufficient to obtain the best BER performance represented by the BPSK AWGN performance shifted by 3 dB.

VI. Conclusion

In this paper, a method has been proposed to improve the noise performance of the DADS modulation scheme by modifying the structure of the reference signal and the demodulation algorithm. It has been shown that noise reduction by averaging can enhance the bit error rate (BER) performance of DADS considerably. The theoretical results derived for the BER were verified by simulations in AWGN and frequency selective Rayleigh channels. When noise averaging within a single bit and between multiple bits is performed at the receive side, gains in the order of tens of dB can be achieved. As the number of noisy chips exploited at the receive side increases, the SNR gap between the BER performance of the proposed modulation scheme and the binary phase-shift keying (BPSK) modulation scheme approaches 3 dB. Moreover, the diversity order offered by frequency selective Rayleigh channels is exploited without adding some extra complexity to the receiver.

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