Multiple Input Multiple Output Iterative Water-Filling Algorithm for Multiple Broadcast Networks Distributing Only Common Information

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Abstract

This paper considers the multiple input multiple output (MIMO) iterative water-filling algorithm (IWFA) for the coexistence of multiple broadcast networks distributing only common information. In the first part of the paper, we propose a power allocation for a MIMO broadcast network using parallel sub-channels and distributing only common information either to maximize the common rate subject to a total power constraint or to minimize the power subject to a common rate constraint. Assuming that the number of receive antennas is larger or equal to the number of transmit antennas, mathematical derivations show that the power allocation can be expressed in closed-form for two receivers. The second part of the paper focuses on the MIMO IWFA for the coexistence of multiple broadcast networks distributing only common information. Simulation results show that the proposed strategy has better performance than the current state of the art strategy which takes into account the worst sub-channels of all receivers. Moreover, a performance gain of about 6 dBW can be achieved with MIMO 2x2 systems as opposed to single input single output (SISO) systems.

Keywords: MIMO, IWFA, Broadcast channels, Common information only

1. Introduction

Tactical voice and data radio networks are often networks in which the same information is conveyed from a transmitter to multiple receivers, e.g. push-to-talk voice or position updates. Therefore, a tactical radio network can be modeled as a broadcast channel with only common information [4]. Making these types of networks cognitive [16, 10], can have some interesting operational advantages. For instance, in a crisis situation, several radio networks belonging to different coalition nations or non-governmental organization (NGO) will be deployed in the same small area. However, as the geographical location of the crisis nor the coalition partners are known in advance and the reaction time is limited, it is impossible to do a proper frequency planning in advance. In these situations, a cognitive tactical radio network, implementing dynamic spectrum access by changing its operating parameters (e.g. transmit power, carrier frequency, modulation strategy etc.) in an autonomous way, can provide reliable communications and efficient utilization of the radio spectrum without a priori spectral information from the other radio networks nor the environment. Furthermore, it is not inconceivable that future tactical networks will be equipped with multiple antennas for multiple input multiple output (MIMO) operations and will be able to transmit on multiple channels at the same time.

Most studies on broadcast channels have focused on the power allocation for parallel Gaussian single input single output (SISO) and multiple input single output (MISO) broadcast channels with common and independent information [6, 11, 34, 9, 13, 32, 30, 31, 8, 19]. For instance, in broadcast channels with independent information, the optimal power allocation is obtained by performing a multi-level water-filling over the parallel Gaussian channels. The capacity region of the Gaussian MIMO broadcast channels with or without common information has been studied for a single channel in [12, 14, 35]. With only common information, the capacity of the Gaussian multiple input multiple output (MIMO) broadcast channel is obtained by maximizing the common information rate. Without common information, the MIMO Gaussian broadcast channel is non-degraded and its capacity is obtained by maximizing the dirty paper coding (DPC) sum rate. The extension to parallel Gaussian MIMO broadcast channels without common information is studied in [38, 18]. However, to our knowledge, less studies have been done on the extension to parallel Gaussian MIMO broadcast channels with only common information. This scenario is of particular interest in a tactical radio network composed of a transmitter which broadcasts the same information to its multiple

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**This research work was carried out in the frame of the Belgian Defense Scientific Research Technology Study C4/19 funded by the Belgian Ministry of Defense (BE MoD). The scientific responsibility is assumed by its authors.

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receivers.

The iterative water-filling algorithm (IWFA) has been introduced as a distributed power control algorithm for parallel Gaussian interference channels [37]. Its convergence has been studied in [23]. In the IWFA, each user iteratively updates its own transmit power while optimizing its own utility function by considering the interference coming from the other users as noise, converging towards a Nash equilibrium in which any change in a player's own strategy would result in a rate loss. The IWFA has also been extended to parallel Gaussian MIMO interference channels in [36, 29]. In this paper, we extend the MIMO IWFA to the coexistence of multiple broadcast networks distributing only common information. It is assumed that the transmitter in each tactical radio network knows the MIMO channel state information (CSI) and the noise covariance matrices of its receivers. This knowledge can be acquired by a feedback channel from the receivers to the transmitter of each network assuming that the acquisition time is much lower than the coherence time of the channel fading. We assume that the links between the transmitter and the receivers of each network exhibit quasi-static fading channels, i.e. in which the coherence times of the fading channels are larger than the time necessary to compute the algorithm. Such an assumption is motivated by the fact that tactical radio networks using very high frequency (VHF) and low ultra high frequency (UHF) bands exhibit long coherence times for low mobility patterns. The parallel channels are represented by multiple orthogonal sub-carriers as used in orthogonal frequency division multiplexing (OFDM) or multiple non-overlapping narrowband sub-channels. Therefore, each parallel sub-channel is considered as a flat quasi-static fading channel (no intersymbol interference).

In Section II, we propose a power allocation for a MIMO broadcast network using parallel sub-channels and distributing only common information either to maximize the common rate subject to a total power constraint or to minimize the power subject to a common rate constraint. Assuming that the number of receive antennas is larger or equal to the number of transmit antennas, mathematical derivations show that the power allocation can be expressed in closed-form for two receivers. In Section III, we extend the MIMO IWFA to the coexistence of multiple broadcast networks distributing only common information. In Section IV, simulation results compare the proposed algorithms with the current state of the art strategy which takes into account the worst sub-channels of all receivers.

2. Optimal power allocation for parallel Gaussian MIMO broadcast channels with only common information

Consider a T-receiver N_c-parallel Gaussian MIMO broadcast channel with only common information as shown in Figure 1. The transmitter has N_t antennas and each receiver t has $N_r^t \geq N_t$ antennas. This condition is necessary

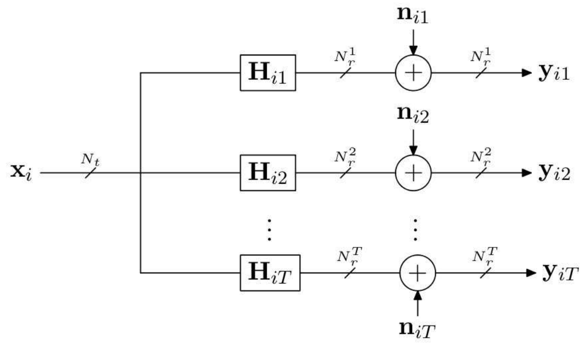

Figure 1: T-receiver MIMO Gaussian broadcast channel

for the existence of the matrices $\mathbf{A}_i$ and $\mathbf{B}_i$ later defined in this paper. The received signal can be modeled as

$$\mathbf{y}_{it} = \mathbf{H}_{it}\mathbf{x}_i + \mathbf{n}_{it} \quad \begin{array}{l} t = 1 \dots T \\ i = 1 \dots N_c \end{array} \tag{1}$$

where $\mathbf{n}_{it}$ are i.i.d. complex Gaussian random vectors of length N_r^t with autocorrelation matrix $\mathbf{R}_{it}$ and $\mathbf{H}_{it}$ corresponds to the $N_r^t \times N_t$ channel matrix seen by receiver t on tone i. The maximum common information rate that can be supported by the system is given by [12, 5]

$$\begin{aligned} &\max_{\underline{\mathbf{\Phi}}} \min_{t=1\dots T} R_{0t}(\underline{\mathbf{\Phi}}) \\ &\text{subject to } \sum_{i=1}^{N_c} \mathrm{Tr}(\mathbf{\Phi}_i) = P^{tot} \\ &\qquad\qquad \mathbf{\Phi}_i \succeq 0 \ \forall i \\ &\text{with } R_{0t}(\underline{\mathbf{\Phi}}) = \Delta f \sum_{i=1}^{N_c} \log_2 |\mathbf{I} + \mathbf{H}_{it}\mathbf{\Phi}_i\mathbf{H}_{it}^H\mathbf{R}_{it}^{-1}| \end{aligned} \tag{2}$$

with $\mathbf{\Phi}_i = E[\mathbf{x}_i\mathbf{x}_i^H]$ the variance of the input signal on sub-channel i, Δf the sub-channel bandwidth, P^{tot} the total power constraint, $|.|$ the determinant operator, Tr(.) the trace operator, $(.)^H$ the Hermitian operator and $\underline{\mathbf{\Phi}} = (\mathbf{\Phi}_1, \dots, \mathbf{\Phi}_{N_c})$ the power allocation among all sub-channels. With a single sub-channel $N_c = 1$, the optimal solution can be solved by standard numerical techniques since this is a concave maximization [12]. To achieve the maximum common information rate for multiple sub-channels $N_c > 1$, the common message codebook cannot be broken into different codebooks for each channel, i.e. joint encoding and joint decoding must be performed across all sub-channels [13]. This transmission scheme is referred to as "single codebook, variable power" transmission [2]. In the following, we derive a framework for parallel Gaussian MIMO broadcast channels which leads to the optimal solution for the maximization of the common rate subject to a total power constraint and its dual form, i.e minimization of power subject to a common rate constraint.

The expression in (2) is the maximization of the minimum of a set of sums of concave functions of $\mathbf{\Phi}_i$. Since the sum and the minimum operations preserve concavity, the objective is concave, and maximizing a concave function yields a convex optimization problem. Moreover, by introducing weight values $\underline{w} = (w_1, \ldots, w_T)$, with $\sum_{t=1}^{T} w_t = 1$, (2) can be transformed into the following problem

Find $\underline{\mathbf{\Phi}}^{\underline{w}^{opt}}$ given by

$$\begin{aligned} &\max_{\underline{\mathbf{\Phi}}} \sum_{t=1}^{T} w_t R_{0t}(\underline{\mathbf{\Phi}}) \\ &\text{subject to } \sum_{i=1}^{N_c} \mathrm{Tr}(\mathbf{\Phi}_i) = P^{tot} \\ &\qquad\qquad\quad \mathbf{\Phi}_i \succeq 0 \ \forall i \end{aligned} \tag{3}$$

with

$$\underline{w}^{opt} = \min_{\underline{w}} \frac{\sqrt{\frac{1}{T}\sum_{t=1}^{T}[(R_{0t}(\underline{\mathbf{\Phi}}^{\underline{w}}) - \frac{1}{T}\sum_{t=1}^{T} R_{0t}(\underline{\mathbf{\Phi}}^{\underline{w}}))^2]}}{\frac{1}{T}\sum_{t=1}^{T} R_{0t}(\underline{\mathbf{\Phi}}^{\underline{w}})} \tag{4}$$

Equation (2) is equivalent to equation (3) and (4) owing to the multiple hypothesis testing approach as proposed in [21, 22, 26]. In the two receiver case, three steps for multiple hypothesis testing are followed. The first step performs the power allocation considering the first receiver only ($w_1 = 1$,$w_2 = 0$). Then, if the rate of the first receiver is lower that the rate of the second receiver, this power allocation is optimal since any weight given to the second receiver would reduce the rate of the first receiver. The second step performs the power allocation considering the second receiver only ($w_1 = 0$,$w_2 = 1$). Then, if the rate of the second receiver is lower that the rate of the first receiver, this power allocation is optimal since any weight given to the first receiver would reduce the rate of the second receiver. If neither of the above cases are true, it means that there is an optimal weight vector w_1 and w_2 such that the rates of the first receiver and the second receiver are equal. As performed in [20], these three steps can be merged into one considering the optimal weight vector w_1 and w_2 such that the difference between the rates of the first receiver and the second receiver is minimized. If step 1 is true, any weight given to the second receiver would increase the rate of the second receiver and decrease the rate of the first receiver, therefore the difference between the two rates would be increased. If step 2 is true, any weight given to the first receiver would increase the rate of the first receiver and decrease the rate of the second receiver, therefore the difference between the two rates would be increased. In the last step, the difference between the rates are already minimized. The extension to multiple receivers is straightforward, an example of multiple hypothesis testing in the three receiver case is given in [25]. Equations (3) and (4) allows to formulate multiple hypothesis testing for the multiple receiver case in a simple manner. However, to avoid an exhaustive search on all the different combination of weights, the different steps of multiple hypothesis testing can be employed in a practical algorithm [21, 22, 26]. As pointed out previously, for two receivers, multiple hypothesis testing involves the three following steps

- Step 1: Find $\underline{\mathbf{\Phi}}^{(1,0)}$ given by

$$\begin{aligned} &\max_{\underline{\mathbf{\Phi}}} R_{01}(\underline{\mathbf{\Phi}}) \\ &\text{subject to } \sum_{i=1}^{N_c} \mathrm{Tr}(\mathbf{\Phi}_i) = P^{tot} \\ &\qquad\qquad\quad \mathbf{\Phi}_i \succeq 0 \ \forall i \end{aligned} \tag{5}$$

  If $R_{01}(\underline{\mathbf{\Phi}}^{(1,0)}) < R_{02}(\underline{\mathbf{\Phi}}^{(1,0)})$ then the optimal power allocation is $\underline{\mathbf{\Phi}}^{\underline{w}^{opt}} = \underline{\mathbf{\Phi}}^{(1,0)}$ and finish. If this condition is true, we do not need to calculate additional set of weights since the rate of the first receiver is lower than the rate of the second receiver while the power allocation considers only the first receiver. Considering the two receivers for the power allocation will reduce the rate of the first receiver and consequently the common rate.

- Step 2: Find $\underline{\mathbf{\Phi}}^{(0,1)}$ given by

$$\begin{aligned} &\max_{\underline{\mathbf{\Phi}}} R_{02}(\underline{\mathbf{\Phi}}) \\ &\text{subject to } \sum_{i=1}^{N_c} \mathrm{Tr}(\mathbf{\Phi}_i) = P^{tot} \\ &\qquad\qquad\quad \mathbf{\Phi}_i \succeq 0 \ \forall i \end{aligned} \tag{6}$$

  If $R_{02}(\underline{\mathbf{\Phi}}^{(0,1)}) < R_{01}(\underline{\mathbf{\Phi}}^{(0,1)})$ then the optimal power allocation is $\underline{\mathbf{\Phi}}^{\underline{w}^{opt}} = \underline{\mathbf{\Phi}}^{(0,1)}$ and finish. Similarly to step 1, if this condition holds, we do not need to calculate additional set of weights since the rate of the second receiver is lower than the rate of the first receiver while the power allocation considers only the second receiver. Considering the two receivers for the power allocation will reduce the rate of the second receiver and consequently the common rate.

If none of the previous conditions are true, this means that $R_{01}(\underline{\mathbf{\Phi}}^{(1,0)}) > R_{02}(\underline{\mathbf{\Phi}}^{(1,0)})$ and $R_{02}(\underline{\mathbf{\Phi}}^{(0,1)}) > R_{01}(\underline{\mathbf{\Phi}}^{(0,1)})$. Therefore, the two receivers should be considered for the power allocation and there exists a set of weights such that the rates of the two receivers are equal. The search of the optimal set of weights is summarized in the following step.

- Step 3: For all (w_1, w_2) with $\sum_{t=1}^{2} w_t = 1$, find $\underline{\mathbf{\Phi}}^{(w_1,w_2)}$ given by

$$\begin{aligned} &\max_{\underline{\mathbf{\Phi}}} \sum_{t=1}^{2} w_t R_{0t}(\underline{\mathbf{\Phi}}) \\ &\text{subject to } \sum_{i=1}^{N_c} \mathrm{Tr}(\mathbf{\Phi}_i) = P^{tot} \\ &\qquad\qquad\quad \mathbf{\Phi}_i \succeq 0 \ \forall i \end{aligned} \tag{7}$$

Find $(w_1,w_2)^{opt}$ that satisfies $R_{01}(\underline{\mathbf{\Phi}}^{(w_1,w_2)^{opt}}) = R_{02}(\underline{\mathbf{\Phi}}^{(w_1,w_2)^{opt}})$, then the optimal power allocation is $\underline{\mathbf{\Phi}}^{(w_1,w_2)^{opt}}$ and finish. Note that to reduce the complexity in a practical algorithm, the weights w_t are taken from a given data set in interval [0 1] with N_s samples, leading to an exhaustive search over a maximum of N_s possibilities including step 1 and step 2 of multiple hypothesis testing. Therefore, to satisfy the conditions requiring the rates of the different receivers to be equal, the optimal value $(w_1,w_2)^{opt}$ should minimize the dispersion of the rates

$$(w_1,w_2)^{opt} = \min_{(w_1,w_2)} \frac{\sqrt{\frac{1}{2}\sum_{t=1}^{2}[(R_{0t}(\underline{\mathbf{\Phi}}^{(w_1,w_2)}) - \frac{1}{2}\sum_{t=1}^{2} R_{0t}(\underline{\mathbf{\Phi}}^{(w_1,w_2)}))^2]}}{\frac{1}{2}\sum_{t=1}^{2} R_{0t}(\underline{\mathbf{\Phi}}^{(w_1,w_2)})} \tag{8}$$

First consider the optimization problem of step 1 and 2. As the objective function is concave, the power allocation can be derived by the standard Karush-Kuhn-Tucker (KKT) conditions [1]. The modified Lagrangian function which includes the total power constraint for step 1 and 2 is given by

$$L(\lambda,\underline{\mathbf{\Phi}}) = \sum_{i=1}^{N_c}\left(\Delta f\ \log_2|\mathbf{I}+\mathbf{H}_{it}\mathbf{\Phi}_i\mathbf{H}_{it}^H\mathbf{R}_{it}^{-1}| - \lambda \mathrm{Tr}(\mathbf{\Phi}_i)\right) + \lambda P^{tot} \quad t=1,2 \tag{9}$$

with λ the Lagrange multiplier associated with the total power constraint. As the autocorrelation matrix $\mathbf{R}_{it}$ is Hermitian and positive definite, it can be decomposed into $\mathbf{R}_{it} = \mathbf{L}_{it}\mathbf{L}_{it}^H$ (Cholesky decomposition) with $\mathbf{L}_{it}$ a lower triangular matrix. Therefore the modified Lagrangian can be rewritten as

$$L(\lambda,\underline{\mathbf{\Phi}}) = \sum_{i=1}^{N_c}\left(\Delta f\ \log_2|\mathbf{I}+\tilde{\mathbf{H}}_{it}\mathbf{\Phi}_i\tilde{\mathbf{H}}_{it}^H| - \lambda \mathrm{Tr}(\mathbf{\Phi}_i)\right) + \lambda P^{tot} \quad t=1,2 \tag{10}$$

with $\tilde{\mathbf{H}}_{it} = \mathbf{L}_{it}^{-1}\mathbf{H}_{it}$. By taking the derivative of the modified Lagrangian function with respect to $\mathbf{\Phi}_i$, we can solve the KKT system of the optimization problem. The derivative with respect to $\mathbf{\Phi}_i$ is given by

$$\begin{aligned}\frac{\partial L(\lambda,\underline{\mathbf{\Phi}})}{\partial \mathbf{\Phi}_i} &= \frac{\Delta f}{\ln 2}\tilde{\mathbf{H}}_{it}^H(\mathbf{I}+\tilde{\mathbf{H}}_{it}\mathbf{\Phi}_i\tilde{\mathbf{H}}_{it}^H)^{-1}\tilde{\mathbf{H}}_{it} - \lambda\mathbf{I} \quad t=1,2\\ &= \frac{\Delta f}{\ln 2}((\tilde{\mathbf{H}}_{it}^H\tilde{\mathbf{H}}_{it})^{-1}+\mathbf{\Phi}_i)^{-1} - \lambda\mathbf{I} \quad t=1,2\end{aligned} \tag{11}$$

The optimal power allocation is obtained through the singular value decomposition (SVD) of each channel matrix combined with standard water-filling as described in [7, 33, 17]

$$\begin{aligned}\frac{\partial L(\lambda,\underline{\mathbf{\Phi}})}{\partial \mathbf{\Phi}_i} = 0 \quad &\Rightarrow \mathbf{\Phi}_i = \frac{\Delta f}{\lambda \ln 2}\mathbf{I} - (\tilde{\mathbf{H}}_{it}^H\tilde{\mathbf{H}}_{it})^{-1}\\ &\Rightarrow \mathbf{\Phi}_i = \mathbf{V}_{it}\left[\frac{\Delta f}{\lambda \ln 2}\mathbf{I} - \mathbf{\Sigma}_{it}^{-2}\right]^+ \mathbf{V}_{it}^H\end{aligned} \tag{12}$$

in which the SVD $\tilde{\mathbf{H}}_{it} = \mathbf{U}_{it}\mathbf{\Sigma}_{it}\mathbf{V}_{it}^H$ contains the diagonal matrix of singular values $\mathbf{\Sigma}_{it}$ and the unitary matrices $\mathbf{U}_{it}$ and $\mathbf{V}_{it}$[1]. Hence the optimal power allocation can be written as

- Step 1:

$$\mathbf{\Phi}_i^{(1,0)} = \mathbf{V}_{i1}\left[\frac{\Delta f}{\lambda \ln 2}\mathbf{I} - \mathbf{\Sigma}_{i1}^{-2}\right]^+ \mathbf{V}_{i1}^H \tag{13}$$

- Step 2:

$$\mathbf{\Phi}_i^{(0,1)} = \mathbf{V}_{i2}\left[\frac{\Delta f}{\lambda \ln 2}\mathbf{I} - \mathbf{\Sigma}_{i2}^{-2}\right]^+ \mathbf{V}_{i2}^H \tag{14}$$

The $[.]^+$ operator is inserted to obtain positive semi-definite matrices without loss of optimality [33, 15].

We now consider the optimization problem of step 3. As the objective is a weighted sum of concave functions, the power allocation can also be derived by the standard KKT conditions. The modified Lagrangian function which includes the total power constraint for step 3 is given by

$$L(\lambda,\underline{\mathbf{\Phi}}) = \sum_{i=1}^{N_c}\left(\Delta f \sum_{t=1}^{2} w_t \log_2|\mathbf{I}+\tilde{\mathbf{H}}_{it}\mathbf{\Phi}_i\tilde{\mathbf{H}}_{it}^H| - \lambda \mathrm{Tr}(\mathbf{\Phi}_i)\right) + \lambda P^{tot} \tag{15}$$

with λ the Lagrange multiplier associated with the total power constraint. By taking the derivative of the modified Lagrangian function with respect to $\mathbf{\Phi}_i$, we can solve the KKT system of the optimization problem. The derivative with respect to $\mathbf{\Phi}_i$ is given by

$$\frac{\partial L(\lambda,\underline{\mathbf{\Phi}})}{\partial \mathbf{\Phi}_i} = \frac{\Delta f}{\ln 2}\sum_{t=1}^{2} w_t((\tilde{\mathbf{H}}_{it}^H\tilde{\mathbf{H}}_{it})^{-1}+\mathbf{\Phi}_i)^{-1} - \lambda\mathbf{I} \tag{16}$$

Nulling the derivative gives

$$\begin{aligned}&\frac{\partial L(\lambda,\underline{\mathbf{\Phi}})}{\partial \mathbf{\Phi}_i} = 0\\ &\Rightarrow w_1(\underbrace{(\tilde{\mathbf{H}}_{i1}^H\tilde{\mathbf{H}}_{i1})^{-1}}_{\mathbf{A}_i}+\mathbf{\Phi}_i)^{-1} + w_2(\underbrace{(\tilde{\mathbf{H}}_{i2}^H\tilde{\mathbf{H}}_{i2})^{-1}}_{\mathbf{B}_i}+\mathbf{\Phi}_i)^{-1} = \underbrace{\frac{\lambda \ln 2}{\Delta f}}_{\tilde{\lambda}}\mathbf{I}\\ &\Rightarrow w_1(\mathbf{A}_i+\mathbf{\Phi}_i)^{-1} + w_2(\mathbf{B}_i+\mathbf{\Phi}_i)^{-1} = \tilde{\lambda}\mathbf{I}\end{aligned} \tag{17}$$

[1] For practical implementations, we introduce in (12) the SNR gap Γ which measures the loss with respect to theoretically optimum performance [3], giving

$$\mathbf{\Phi}_i = \mathbf{V}_{it}\left[\frac{\Delta f}{\lambda \ln 2}\mathbf{I} - \Gamma\mathbf{\Sigma}_{it}^{-2}\right]^+ \mathbf{V}_{it}^H$$

with $\mathbf{A}_i$ and $\mathbf{B}_i$ two Hermitian matrices. The existence of the matrices $\mathbf{A}_i$ and $\mathbf{B}_i$ requires $N_r^1 \geq N_t$ and $N_r^2 \geq N_t$. Then, by making the variable change $\tilde{\mathbf{\Phi}}_i = \mathbf{\Phi}_i + \frac{\mathbf{A}_i + \mathbf{B}_i}{2}$ and by defining the Hermitian matrix $\mathbf{C}_i = \frac{\mathbf{A}_i - \mathbf{B}_i}{2}$, this leads to

$$\begin{aligned} & w_1(\tilde{\mathbf{\Phi}}_i + \mathbf{C}_i)^{-1} + w_2(\tilde{\mathbf{\Phi}}_i - \mathbf{C}_i)^{-1} = \tilde{\lambda}\mathbf{I} \\ & \tilde{\mathbf{\Phi}}_i - (w_1 - w_2)\mathbf{C}_i = \tilde{\lambda}(\tilde{\mathbf{\Phi}}_i + \mathbf{C}_i)(\tilde{\mathbf{\Phi}}_i - \mathbf{C}_i) \\ & \tilde{\mathbf{\Phi}}_i - (w_1 - w_2)\mathbf{C}_i = \tilde{\lambda}(\tilde{\mathbf{\Phi}}_i^2 - \mathbf{C}_i^2 + \mathbf{C}_i\tilde{\mathbf{\Phi}}_i - \tilde{\mathbf{\Phi}}_i\mathbf{C}_i) \end{aligned} \tag{18}$$

As $\mathbf{\Phi}_i$ is an Hermitian matrix, $\tilde{\mathbf{\Phi}}_i$ is also an Hermitian matrix. By taking Hermitian conjugates on both sides, we get

$$\begin{aligned} & \tilde{\mathbf{\Phi}}_i - (w_1 - w_2)\mathbf{C}_i = \tilde{\lambda}(\tilde{\mathbf{\Phi}}_i^2 - \mathbf{C}_i^2 + \mathbf{C}_i\tilde{\mathbf{\Phi}}_i - \tilde{\mathbf{\Phi}}_i\mathbf{C}_i) \\ & \overset{(.)^H}{\Longleftrightarrow} \tilde{\mathbf{\Phi}}_i - (w_1 - w_2)\mathbf{C}_i = \tilde{\lambda}(\tilde{\mathbf{\Phi}}_i^2 - \mathbf{C}_i^2 + \tilde{\mathbf{\Phi}}_i\mathbf{C}_i - \mathbf{C}_i\tilde{\mathbf{\Phi}}_i) \end{aligned} \tag{19}$$

leading to the following system of equations

$$\Rightarrow \begin{cases} \mathbf{C}_i\tilde{\mathbf{\Phi}}_i - \tilde{\mathbf{\Phi}}_i\mathbf{C}_i = 0 \\ \tilde{\mathbf{\Phi}}_i - (w_1 - w_2)\mathbf{C}_i = \tilde{\lambda}(\tilde{\mathbf{\Phi}}_i^2 - \mathbf{C}_i^2) \end{cases} \tag{20}$$

Since $\mathbf{C}_i$ is Hermitian, we can work in a basis in which $\mathbf{C}_i$ is diagonalized. Therefore, we use the eigenvalue decomposition (EVD) $\mathbf{C}_i = \mathbf{W}_i\mathbf{D}_i\mathbf{W}_i^H$ which leads to the following developments of the system of equations

$$\Rightarrow \begin{cases} \mathbf{W}_i\mathbf{D}_i\mathbf{W}_i^H\tilde{\mathbf{\Phi}}_i - \tilde{\mathbf{\Phi}}_i\mathbf{W}_i\mathbf{D}_i\mathbf{W}_i^H = 0 \\ \tilde{\mathbf{\Phi}}_i - (w_1 - w_2)\mathbf{W}_i\mathbf{D}_i\mathbf{W}_i^H = \tilde{\lambda}(\tilde{\mathbf{\Phi}}_i^2 - \mathbf{W}_i\mathbf{D}_i^2\mathbf{W}_i^H) \end{cases} \tag{21}$$

$$\Rightarrow \begin{cases} \mathbf{D}_i\mathbf{W}_i^H\tilde{\mathbf{\Phi}}_i\mathbf{W}_i - \mathbf{W}_i^H\tilde{\mathbf{\Phi}}_i\mathbf{W}_i\mathbf{D}_i = 0 \\ \mathbf{W}_i^H\tilde{\mathbf{\Phi}}_i\mathbf{W}_i - (w_1 - w_2)\mathbf{D}_i = \tilde{\lambda}(\mathbf{W}_i^H\tilde{\mathbf{\Phi}}_i^2\mathbf{W}_i - \mathbf{D}_i^2) \end{cases} \tag{22}$$

Making the variable change $\mathbf{Z}_i = \mathbf{W}_i^H\tilde{\mathbf{\Phi}}_i\mathbf{W}_i$, the final system of equations to be solved is

$$\Rightarrow \begin{cases} \mathbf{D}_i\mathbf{Z}_i - \mathbf{Z}_i\mathbf{D}_i = 0 \\ \mathbf{Z}_i - (w_1 - w_2)\mathbf{D}_i = \tilde{\lambda}(\mathbf{Z}_i^2 - \mathbf{D}_i^2) \end{cases} \tag{23}$$

The first line of the system of equations implies $\mathbf{Z}_i$ to be diagonal since $\mathbf{D}_i$ is diagonal. The power allocation is a type of water-filling strategy given by the solution of parallel quadratic equations

$$\tilde{\lambda}\mathbf{Z}_i^2 - \mathbf{Z}_i - \tilde{\lambda}\mathbf{D}_i^2 + (w_1 - w_2)\mathbf{D}_i = 0 \tag{24}$$

with $\mathbf{Z}_i$ diagonal. The discriminant of this quadratic equation is given by

$$\Delta = \mathbf{I} + 4\tilde{\lambda}^2\mathbf{D}_i^2 - 4\tilde{\lambda}(w_1 - w_2)\mathbf{D}_i \tag{25}$$

The power allocation is given by the positive root

$$\mathbf{Z}_i = \frac{\mathbf{I}}{2\tilde{\lambda}} + \left(\frac{\mathbf{I}}{4\tilde{\lambda}^2} - \frac{(w_1 - w_2)\mathbf{D}_i}{\tilde{\lambda}} + \mathbf{D}_i^2\right)^{1/2} \tag{26}$$

Knowing that $\tilde{\mathbf{\Phi}}_i = \mathbf{W}_i\mathbf{Z}_i\mathbf{W}_i^H$ and $\mathbf{\Phi}_i = \tilde{\mathbf{\Phi}}_i - \frac{\mathbf{A}_i + \mathbf{B}_i}{2}$, the power allocation is given by[2]

$$\mathbf{\Phi}_i = \mathbf{W}_i\left[\frac{\mathbf{I}}{2\tilde{\lambda}} + \left(\frac{\mathbf{I}}{4\tilde{\lambda}^2} - \frac{(w_1 - w_2)\mathbf{D}_i}{\tilde{\lambda}} + \mathbf{D}_i^2\right)^{1/2}\right]\mathbf{W}_i^H - \frac{\mathbf{A}_i + \mathbf{B}_i}{2} \tag{27}$$

Transmit covariance matrices should be positive semi-definite for a practical implementation by linear precoding. However, the $[.]^+$ operator cannot be inserted directly to obtain positive semi-definite matrices because of the difference operation. As proposed in [28], a new EVD is performed on $\mathbf{\Phi}_i = \mathbf{T}_i\mathbf{\Theta}_i\mathbf{T}_i^H$ with $\mathbf{T}_i$ the unitary matrix containing the eigenvectors and $\mathbf{\Theta}_i$ the diagonal matrix containing the eigenvalues. The negative eigenvalues of $\mathbf{\Theta}_i$ are dropped (set to zero) such that the transmit covariance matrices are positive semi-definite. Therefore the power allocation becomes

$$\mathbf{\Phi}_i = \mathbf{T}_i[\mathbf{\Theta}_i]^+\mathbf{T}_i^H \tag{28}$$

The projection transforms the optimal solution into a sub-optimal solution. However, this projection allows to find feasible positive semi-definite matrices close to the optimal covariance matrices and is extremely fast compared to other projection methods [28, 24].

For more than two receivers, the power allocation algorithm is driven by the solutions of higher degree matrix polynomials and involves more steps under multiple hypothesis testing. For instance, with three receivers $T = 3$, the optimal power allocation algorithm is given by seven steps involving the solutions of three matrix linear equations, three matrix quadratic equations and a matrix cubic equation (similarly to [25] in the SISO case). More generally, the optimal power allocation is given by the solutions of matrix polynomial equations up to degree T from the formula

$$\sum_{t=1}^{T} w_t(\tilde{\mathbf{H}}_{it}^{-1}\tilde{\mathbf{H}}_{it}^{-H} + \mathbf{\Phi}_i)^{-1} = \tilde{\lambda}\mathbf{I} \tag{29}$$

The power allocation can be solved numerically, for instance by an exhaustive search on the matrix elements of $\mathbf{\Phi}_i$, although computationally intensive compared to a closed-form solution. Moreover, similarly to the two receivers case, the weights w_t can be taken from a given data set in interval [0 1] to reduce the search space with N_s samples, leading to an exhaustive search over a maximum of $(T-1)N_s^{T-1}$ possibilities including all the steps in multiple hypothesis testing.

[2]Note that the SNR gap Γ, which corresponds to a constant increase in the interference temperature, can also be introduced in formula (27) and gives

$$\mathbf{\Phi}_i = \mathbf{W}_i\left[\frac{\mathbf{I}}{2\tilde{\lambda}} + \left(\frac{\mathbf{I}}{4\tilde{\lambda}^2} - \frac{(w_1 - w_2)\Gamma\mathbf{D}_i}{\tilde{\lambda}} + \Gamma^2\mathbf{D}_i^2\right)^{1/2}\right]\mathbf{W}_i^H - \frac{\Gamma(\mathbf{A}_i + \mathbf{B}_i)}{2}$$

The minimization of the power subject to a common rate constraint is the dual form of the maximization of the common rate subject to a total power constraint. The only difference lies in the algorithm where an outer loop is added to find the minimum amount of power that is needed to support the common rate constraint. Algorithm 1 provides the power allocation for power minimization subject to a common rate constraint R^{com} of a T-receiver N_c parallel Gaussian MIMO broadcast channel with only common information. Note that the actual common rate in the network is defined as $R_0(\underline{\boldsymbol{\Phi}}^{opt}) = \min_{t=1\ldots T} R_{0t}(\underline{\boldsymbol{\Phi}}^{opt})$. The pseudocode providing the power allocation for common rate maximization subject to a total power constraint P^{tot} of a T-receiver N_c parallel Gaussian MIMO broadcast channel with only common information is a subset of Algorithm 1 (inner loop of the algorithm) from line 2 to line 9. A fixed step-size can be used to update $\tilde{\lambda}$ and P^{tot} (e.g. 3 dB) as in Algorithm I. However, an adaptive bisection method is used in the simulations to converge rapidly to the optimal values. In this case, the step-sizes are updated whenever the variables have to be decreased [17]. For the initialization, the variables $\tilde{\lambda}$ and P^{tot} are set to extremely low values (e.g. 10^{-11}), the step-sizes are set to 2, and two counters are set to 0. Whenever a variable $\tilde{\lambda}$ or P^{tot} has to be decreased, its counter is incremented, the variable is decreased by its actual step-size, a new step-size is calculated as the difference between the actual step-size and the inverse of 2 powered by its counter, and the variable is increased by the new calculated step-size. If the variable $\tilde{\lambda}$ or P^{tot} has to be incremented, the variable is increased by the actual step-size. The algorithm has a cubic complexity increase with respect to the number of antennas and a linear complexity increase with the number of iterations of the inner loop L_i an the outer loop L_o necessary. If $N_t = N_r^1 = N_r^2$, this gives a complexity order $O(L_i L_o N_t^3 (T-1) N_s^{T-1})$.

Algorithm 1 Minimization of the power subject to a common rate constraint

```
1  repeat
2    repeat
3      for all (w_1, ..., w_T)
4        Calculate Φ_i^{(w_1,...,w_T)} ∀i according to (29)
5        if Σ_{i=1}^{N_c} Φ_i^{(w_1,...,w_T)} < P^tot decrease λ̃
6        if Σ_{i=1}^{N_c} Φ_i^{(w_1,...,w_T)} > P^tot increase λ̃
7      end for
8      Find Φ^{(w_1,...,w_T)^opt} according to (4)
9    until the desired accuracy is reached
10   if R_0(Φ^{(w_1,...,w_T)^opt}) < R^com increase P^tot
11   if R_0(Φ^{(w_1,...,w_T)^opt}) > R^com decrease P^tot
12 until the desired accuracy is reached
```

3. Multiple Input Multiple Output Iterative Water-Filling Algorithm for Multiple Broadcast Networks Distributing Only Common Information

In this Section, we consider the MIMO IWFA for the coexistence of multiple broadcast networks distributing only common information. However, the MIMO IWFA applies to an interference channel in which a transmitter sends data to only one receiver. This Section extends the MIMO IWFA considering an interference channel in which a transmitter sends common data to multiple receivers. The game theory concept of the MIMO IWFA is adapted to this scenario, i.e. players iteratively update their transmit covariance matrices until a Nash equilibrium point is attained.

Assuming N different networks, each network j having T_j receivers, the received signal can be modeled as

$$\mathbf{y}_{j,it} = \mathbf{H}_{jj,it}\mathbf{x}_{ij} + \sum_{k\neq j}^{N} \mathbf{H}_{jk,it}\mathbf{x}_{ik} + \mathbf{n}_{j,it} \quad \begin{array}{l} i = 1\ldots N_c \\ j = 1\ldots N \\ t = 1\ldots T_j \end{array} \tag{30}$$

where $\mathbf{n}_{j,it}$ are i.i.d. complex Gaussian random vectors of length N_r^t with autocorrelation matrix $\mathbf{R}_{j,it}$ and $\mathbf{H}_{jk,it}$ corresponds to the $N_r^{j,t} \times N_t^k$ channel matrix from network k to network j seen by receiver t on tone i. We consider the maximization of the aggregate common rate subject to a total power constraint per network

$$\begin{array}{l} \max\limits_{\underline{\underline{\boldsymbol{\Phi}}}} \sum\limits_{j=1}^{N} R_{0j}(\underline{\underline{\boldsymbol{\Phi}}}) \\ \text{subject to} \sum\limits_{i=1}^{N_c} \mathrm{Tr}(\boldsymbol{\Phi}_{ij}) = P_j^{tot}\ \forall j \end{array} \tag{31}$$

with

$$R_{0j}(\underline{\underline{\boldsymbol{\Phi}}}) = \min_{(1,\ldots,T_j)} R_{0jt}(\underline{\underline{\boldsymbol{\Phi}}}) \tag{32}$$

and

$$R_{0jt}(\underline{\underline{\boldsymbol{\Phi}}}) = \Delta f \sum_{i=1}^{N_c} \log_2 \left| \mathbf{I} + \mathbf{H}_{jj,it}\boldsymbol{\Phi}_{ij}\mathbf{H}_{jj,it}^H \tilde{\mathbf{R}}_{j,it}^{-1} \right| \tag{33}$$

with $\underline{\underline{\boldsymbol{\Phi}}} = (\underline{\boldsymbol{\Phi}}_1, \ldots, \underline{\boldsymbol{\Phi}}_N)$ the power allocation among all sub-channels and networks, $\underline{\boldsymbol{\Phi}}_j = (\boldsymbol{\Phi}_{1j}, \ldots, \boldsymbol{\Phi}_{N_c j})$ the power allocation among all sub-channels for network j and $\tilde{\mathbf{R}}_{j,it} = \mathbf{R}_{j,it} + \sum_{k\neq j} \mathbf{H}_{jk,it}\boldsymbol{\Phi}_{ik}\mathbf{H}_{jk,it}^H$ the covariance matrix seen by the t^{th} receiver of network j on tone i. Similarly to a single tactical radio network, by introducing weight values w_{jt}, with $\sum_{t=1}^{T_j} w_{jt} = 1$, $\forall j$, (31) can be transformed into the following problem

Find $\underline{\underline{\boldsymbol{\Phi}}}^{(\underline{\underline{w}}^{opt})}$ given by

$$\begin{array}{l} \max\limits_{\underline{\underline{\boldsymbol{\Phi}}}} \sum\limits_{j=1}^{N} \sum\limits_{t=1}^{T_j} w_{jt} R_{0jt}(\underline{\underline{\boldsymbol{\Phi}}}) \\ \text{subject to} \sum\limits_{i=1}^{N_c} \mathrm{Tr}(\boldsymbol{\Phi}_{ij}) = P_j^{tot}\ \forall j \end{array} \tag{34}$$

and

$$\underline{w}_j^{opt} = \min_{\underline{w}_j} \frac{\sqrt{\frac{1}{T_j}\sum_{t=1}^{T_j}[(R_{0jt}(\underline{\underline{\mathbf{\Phi}}}^{(\underline{\underline{w}})}) - \frac{1}{T_j}\sum_{t=1}^{T_j} R_{0jt}(\underline{\underline{\mathbf{\Phi}}}^{(\underline{\underline{w}})}))^2]}}{\frac{1}{T_j}\sum_{t=1}^{T_j} R_{0jt}(\underline{\underline{\mathbf{\Phi}}}^{(\underline{\underline{w}})})} \quad \forall j \tag{35}$$

with $\underline{\underline{w}} = (\underline{w}_1, \ldots, \underline{w}_N)$ the weight allocation among all receivers and networks and $\underline{w}_j = (w_{j1}, \ldots, w_{jT_j})$ the weight allocation among all receivers for network j. Maximization of the aggregate common rate subject to a total power constraint per network in a centralized algorithm is an extensive task, since it requires the knowledge of the sub-channel matrix from any transmitter to any receiver $\mathbf{H}_{jk,it}$ $\forall i, j, k, t$ and an exhaustive search on w_{jt} $\forall j, t$. Although suboptimal, a distributed algorithm only requires the knowledge of the sub-channel gains from a transmitter to its own receivers ($\mathbf{H}_{jj,it}$, $\forall i, j, t$), as well as noise covariance matrices of its receivers estimated by spectrum sensing ($\tilde{\mathbf{R}}_{j,it} = \mathbf{R}_{j,it} + \sum_{k \neq j} \mathbf{H}_{jk,it}\mathbf{\Phi}_{ik}\mathbf{H}_{jk,it}^H$). The distributed algorithm called IWFA for MIMO interference channels [36, 29] iteratively updates the power allocation of each network while considering the interference of all other networks as noise. Under this assumption, the expression in (34) is the maximization of a weighted aggregation of common rates, each common rate being the minimum of a sum of concave functions of $\mathbf{\Phi}_{ij}$. Since the sum and the minimum operations preserve concavity, the objective is concave, and maximizing a concave function yields a convex optimization problem. Considering the MIMO IWFA in parallel Gaussian broadcast channels with only common information, the Lagrangian function can be written as

$$\begin{aligned} & L(\underline{\lambda}, \underline{\underline{\mathbf{\Phi}}}) = \\ & \sum_{i=1}^{N_c}\left(\Delta f \sum_{j=1}^{N}\sum_{t=1}^{T_j} w_{jt}\log_2\left|\mathbf{I} + \mathbf{H}_{jj,it}\mathbf{\Phi}_{ij}\mathbf{H}_{jj,it}^H\tilde{\mathbf{R}}_{j,it}^{-1}\right| \right. \\ & \left. - \sum_{j=1}^{N}\lambda_j \mathrm{Tr}(\mathbf{\Phi}_{ij})\right) + \sum_{j=1}^{N}\lambda_j P_j^{tot} \end{aligned} \tag{36}$$

in which $\underline{\lambda} = (\lambda_1, \ldots, \lambda_N)$ are the Lagrange multipliers over all networks. According to [1], the Karush-Kuhn-Tucker (KKT) conditions of the optimization problem can be solved by taking the derivative of the Lagrangian function with respect to $\mathbf{\Phi}_{ij}$

$$\frac{\partial L(\underline{\lambda}, \underline{\underline{\mathbf{\Phi}}})}{\partial \mathbf{\Phi}_{ij}} = \frac{\Delta f}{\ln 2}\sum_{t=1}^{T_j} w_{jt}(\tilde{\mathbf{H}}_{j,it}^{-1}\tilde{\mathbf{H}}_{j,it}^{-H} + \mathbf{\Phi}_{ij})^{-1} - \lambda_j \mathbf{I} \tag{37}$$

with $\tilde{\mathbf{H}}_{j,it} = \mathbf{L}_{j,it}^{-1}\mathbf{H}_{jj,it}$ and $\mathbf{L}_{j,it}$ the lower triangular matrix from the Cholesky decomposition $\mathbf{L}_{j,it}\mathbf{L}_{j,it}^H = \mathbf{R}_{j,it} + \sum_{k \neq j} \mathbf{H}_{jk,it}\mathbf{\Phi}_{ik}\mathbf{H}_{jk,it}^H$. Therefore, (37) leads to the same solution as (29) in Section II. For instance, with two receivers $T_j = 2$, the power allocation within the network j is given by (27). Therefore, a distributed power control will update the N transmit covariance matrices iteratively to minimize the power subject to a common rate constraint R^{com}. Within each network, an inner loop determines the power allocation maximizing the common rate subject to a total power constraint. This process is updated regularly between all the different networks until they reach a Nash equilibrium. Finally, an outer loop minimizes the power such that a common rate constraint is achieved for each network. These power updates are performed asynchronously from one network to another, and the period of the power update for the outer loop is much higher than the period of the power update for the inner loop. The MIMO IWFA for the coexistence of multiple broadcast networks distributing only common information is presented in Algorithm 2.

Algorithm 2 MIMO IWFA for the coexistence of multiple broadcast networks distributing only common information

repeat
  repeat
    repeat
      for $j = 1$ to N
        for all $(w_{j1}, \ldots, w_{jT_j})$
          Calculate $\mathbf{\Phi}_{ij}^{(w_{j1},\ldots,w_{jT_j})}$ $\forall i$ according to (29)
          if $\sum_{i=1}^{N_c}\mathbf{\Phi}_{ij}^{(w_{j1},\ldots,w_{jT_j})} < P_j^{tot}$ decrease $\tilde{\lambda}_j$
          if $\sum_{i=1}^{N_c}\mathbf{\Phi}_{ij}^{(w_{j1},\ldots,w_{jT_j})} > P_j^{tot}$ increase $\tilde{\lambda}_j$
        end for
        Find $\underline{\mathbf{\Phi}}_j^{(w_{j1},\ldots,w_{jT_j})^{opt}}$ according to (4)
      end for
    $\times$ times
  until the desired accuracy is reached
  for $j = 1$ to N
    if $R_{0j}(\underline{\mathbf{\Phi}}_j^{(w_{j1},\ldots,w_{jT_j})^{opt}}) < R^{com}$ increase P_j^{tot}
    if $R_{0j}(\underline{\mathbf{\Phi}}_j^{(w_{j1},\ldots,w_{jT_j})^{opt}}) > R^{com}$ decrease P_j^{tot}
  end for
until the desired accuracy is reached

4. Results

In this Section, the performance of the proposed algorithms is evaluated by simulations of a single or multiple networks, each network having one transmitter, two receivers $T = 2$ and a number of transmit and receive antennas $N_t = N_r^1 = N_r^2 = 1$ (SISO) or $N_t = N_r^1 = N_r^2 = 2$ (MIMO). It is assumed that the transmitter in each tactical radio network knows the MIMO channel state information (CSI) and the noise covariance matrices of its receivers. This knowledge can be acquired by a feedback channel from the receivers to the transmitter of each network assuming that the acquisition time is much lower than the coherence time of the channel fading. We assume that the links between the transmitter and the receivers of each network exhibit quasi-static fading channels, i.e. in which the coherence times of the fading channels are larger than the time necessary to compute the algorithm. Such an assumption is motivated by the fact that tactical radio networks using very high frequency (VHF) and low ultra high frequency (UHF) bands exhibit long coherence times for low mobility patterns. The parallel channels are represented by multiple orthogonal sub-carriers as used in orthogonal frequency division multiplexing (OFDM) or multiple non-overlapping narrowband sub-channels. Therefore, each parallel sub-channel is considered as a flat quasi-static fading channel (no intersymbol interference). The log-distance path loss model is used to measure the path loss between the transmitter and the receivers [27]. The transmitter and the receivers are placed randomly in a square area of 1 km^2. The carrier frequency is chosen to be in the VHF band ($f_c = 80$ MHz). The SNR gap $\Gamma = 9.8$ dB corresponds to an uncoded quadrature amplitude modulation (QAM) at symbol error rate 10^{-7} [3]. The bandwidth is $\Delta f = 25$ kHz, the path loss exponent in the log-distance path loss model is $n = 4$, reference distance $d_0 = 20$ meters and thermal noise with variance $\sigma^2 = -204\text{dB/Hz}+10\log_{10}(\Delta f)$. We assume that the parallel sub-channels are frequentially and spatially uncorrelated, i.e. the frequency separation of the parallel sub-channels is larger than the coherence bandwidth and the antenna separation of the transmitter and the receivers is larger coherence distance. Moreover, they obey a Rayleigh fading distribution, i.e. the received signal at each frequency and antenna is a sum of many contributions coming from different directions. Therefore, the path loss between each transmitter-receiver pair is multiplied by i.i.d. random matrices whose entries are complex Gaussian with zero mean and unit variance. This spatial model allows to have a lower (SISO) and upper bound (fully uncorrelated MIMO), knowing that a more realistic correlated MIMO will have performance results between these two bounds.

In the first simulation, we compare Algorithm 1 presented in Section II with the worst sub-channel strategy for $T = 2$ receivers and $N_c = 4$ sub-channels. In the worst sub-channel strategy, the inner loop maximizes the rate of the superposition of the receiver's worst sub-channels

$$\begin{aligned} &\max_{\underline{\mathbf{\Phi}}} \sum_{i=1}^{N_c} \min_{t=1\ldots T} \log_2|\mathbf{I}+\mathbf{H}_{it}\mathbf{\Phi}_i\mathbf{H}_{it}^H\mathbf{R}_{it}^{-1}| \\ &\text{subject to } \sum_{i=1}^{N_c} \text{Tr}(\mathbf{\Phi}_i) = P^{tot} \\ &\qquad\qquad\quad \mathbf{\Phi}_i \succeq 0 \ \forall i \end{aligned} \tag{38}$$

The worst sub-channel strategy corresponds to the strategy in which the common message codebook is broken into different codebooks for each sub-channel, therefore the common rate transmitted on each sub-channel is limited by the weakest receiver in each sub-channel [13]. This transmission scheme is referred to as "multiple codebook, variable power" transmission [2]. In the worst sub-channel strategy, the water-filling is performed on the worst sub-channel conditions of both receivers. More precisely, the sum of the eigenvalues $\mathbf{A}_i$ and $\mathbf{B}_i$ are compared and the greatest value is selected for the water-filling of each sub-channel. For a scenario in which spectrum sensing gives an estimated noise of 10^{-16} seen by the receivers on all sub-channels before transmission (corresponding to the thermal noise), performance results between Algorithm 1 and the worst sub-channel strategy are similar. However, for a scenario in which spectrum sensing gives en estimated noise which varies across sub-channels and receivers, performance results between Algorithm 1 and the worst sub-channel strategy show some differences. Therefore, we consider a scenario in which spectrum sensing gives a strong estimated noise of 10^{-9} on the 1^{st} and 2^{nd} sub-channels seen by the first receiver and the and 3^{rd} and 4^{th} sub-channels seen by the second receiver (10^{-16} for the other channels). In a realistic environment, the noise variations across sub-channels and receivers correspond to sub-channels occupied by primary users or jammers at different locations.

Multiple hypothesis testing is used for Algorithm 1 with step-size 0.1 in interval [0 1], leading to an exhaustive search over a maximum of $N_s = 11$ possibilities. For the initialization, the variables $\tilde{\lambda}$ and P^{tot} are set to extremely low values 10^{-11}. The desired accuracy is set to 10^{-10} for the inner loop (power) and 10^{-4} for the outer loop (common rate) in order to have precise results for the simulations, requiring approximately 100 iterations for each loop to converge. In practice, these parameters can be adapted carefully to reach the target power and the target common rate at minimum complexity and power consumption. For instance, one can use the previous knowledge of the variables $\tilde{\lambda}$ and P^{tot} for the initialization, allowing to converge to the desired accuracy in a few iterations.

Figure 2 shows the results of the power minimization subject to a common rate constraint presented in Section II and ranging from $R^{com} = 2$ kbps to $R^{com} = 512$ kbps. These results are expressed in Watts and averaged over 10^3 Monte Carlo trials for the locations of the transmitter and the receivers. Algorithm 1 clearly outperforms the worst case sub-channel strategy. Moreover, there is a substantial gain for using a MIMO 2x2 compared to a SISO channel, i.e. around 6 dBW gain for all the range of R^{com}.

In the second simulation, we compare Algorithm 2 presented in Section III with the worst sub-channel strategy for $N = 2$ networks, $T_j = 2$ receivers $\forall j$ and $N_c = 4$ sub-channels. Similarly to the single network case, for a scenario in which only thermal noise 10^{-16} is seen by the receivers on all sub-channels before transmission, performance results between Algorithm 2 and the worst sub-channel strategy are similar. To compare both algorithms in a more realistic environment in which sub-channels can be occupied by primary users or jammers at different locations, we consider a scenario in which a very strong noise

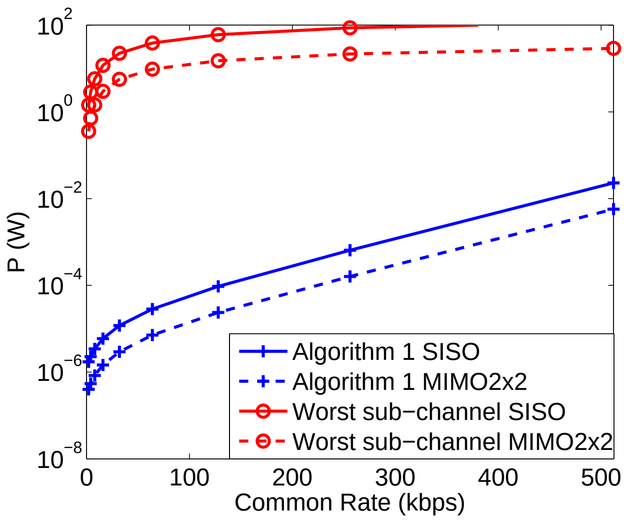

Figure 2: Results on the power minimization subject to a common rate constraint

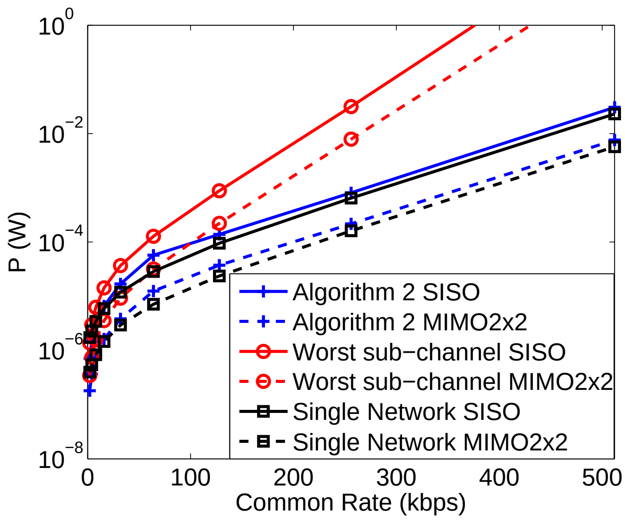

Figure 3: Results on the power minimization subject to a common rate constraint averaged for both networks

of 10^{-9} is seen on the 1^{st} and 4^{th} sub-channels by respectively the first and second receiver of the first network. In the second network, a very strong noise of 10^{-9} is seen on the 2^{nd} and 3^{rd} sub-channels by respectively the first and second receiver (10^{-16} for the other channels).

Figure 3 shows the results of the power minimization subject to a common rate constraint ranging from $R^{com} = 2$ kbps to $R^{com} = 512$ kbps over 10^3 Monte Carlo trials for the locations of the transmitter and the receivers. It can be seen that Algorithm 2 outperforms the worst sub-channel strategy for SISO and MIMO 2x2 channels. Moreover, the SISO and MIMO2x2 algorithms are compared with the single network SISO and MIMO2x2 algorithms to provide a lower bound for the centralized algorithm (31). Figure 3 shows that Algorithm 2 allows to obtain higher common rates and lower power consumption when the number of antennas increases. Although sub-optimal, the SISO and MIMO2x2 algorithms require only a small power augmentation compared to the single network SISO and MIMO2x2 algorithms. Therefore, in practical scenarios in which the interference temperature varies along the sub-channel and the receiver locations, Algorithm 2 provides an efficient distributed strategy to find the power allocation of multiple networks in which each transmitter has to broadcast a common information to its receivers.

5. Conclusion

This paper has considered the multiple input multiple output (MIMO) iterative water-filling algorithm (IWFA) for the coexistence of multiple broadcast networks distributing only common information. In the first part of the paper, we have proposed a power allocation for a MIMO broadcast network using parallel sub-channels and distributing only common information either to maximize the common rate subject to a total power constraint or to minimize the power subject to a common rate constraint. Assuming that the number of receive antennas is larger or equal to the number of transmit antennas, mathematical derivations show that the power allocation can be expressed in closed-form for two receivers. The second part of the paper has focused on the MIMO IWFA for the coexistence of multiple broadcast networks distributing only common information. Simulation results have shown that the proposed strategy has better performance than the current state of the art strategy which takes into account the worst sub-channels of all receivers. Moreover, a performance gain of about 6 dBW can be achieved with MIMO 2x2 systems as opposed to single input single output (SISO) systems. Using the proposed algorithm, it can be seen that the different networks coexisting in the same area will independently and without a priori information converge to a sub-optimal solution that outperforms the worst sub-channel strategy. Although sub-optimal, simulation results have shown that the proposed algorithm allows to obtain higher common rates and substantial gains in power consumption compared to the single network algorithm, and is not far from the optimum solution.

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