Coexistence of tactical cognitive radio networks

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***Abstract*—In this paper, we consider the scenario in which N different cognitive radio networks can not cooperate with each other and wish to broadcast a common information to their network by sharing the same N_c parallel sub-channels. This scenario is particularly adapted to tactical radio networks in which N different networks coexist in a given area and broadcast a common information (voice, data...) to their group. In this context, we propose a novel distributed power allocation for power minimization subject to a minimum rate constraint based on the iterative water-filling principle in which each network updates its power allocation autonomously. The novel algorithm generalizes the iterative waterfilling algorithm to the coexistence of multiple tactical radio networks.**

***Index Terms*—Coexistence of tactical radio networks, broadcast channels, parallel multicast channels, iterative water-filling.**

I. INTRODUCTION

When several coalition nations coexist in the same area, current technologies do not permit reconfigurability, interoperability nor coexistence of the radio terminals. Software defined radio has been developed for reconfigurability of the terminals with software upgrades and for portability of the waveforms. Cognitive radio has been developed for spectrum availability recognition, reconfigurability, interoperability and coexistence between terminals by means of software defined radio technology, intelligence, awareness and learning [1], [2]. Therefore, cognitive radio enables the adaptation of the transmission parameters (transmit power, carrier frequency, modulation strategy) to these scenarios.

Tactical radio networks are networks in which information (mostly voice, but also packet based data) are conveyed from one transmitter to multiple receivers. When several tactical radio networks are set up in the same area and transmit in the same band, the coexistence of these networks is critical. The coexistence of multiple tactical radio networks calls for distributed algorithms implemented in the cognitive terminals. Indeed, although distributed algorithms are suboptimal, they are preferred to centralized algorithms because of their scalability and robustness. Therefore, each terminal must be equipped with spectrum sensing and management functions to detect the spectrum holes and to find the transmit powers improving the performance of the network as a whole (capacity, stability, delay).

The broadcast channel has been introduced by Cover in 1972 as a communication channel in which there is one transmitter and two or more receivers [3]. The broadcast channel with only independent information (unicast channel) belongs to the class of degraded channels in which one user's signal is a degraded version of the other signals. Its capacity region is fully characterized and can be achieved by superposition coding [4]. Contrary to a single unicast channel, the sum of unicast channels as well as MIMO broadcast channels are non-degraded [5], [6]. Previous studies on parallel broadcast channels have focused on scenarios in which independent messages are sent to the receivers (parallel unicast channels) [7], [8], [9], [10], or in which simultaneous common and independent messages are sent to the receivers [5], [11], [12]. Contrary to an unicast channel, a tactical radio network can be thought as a broadcast channel with only common information (also referred to as a multicast channel). The capacity of a single multicast channel is limited by the capacity of the *worst receiver* [4], [13]. However, less work has been done on multicast channels with N_c parallel sub-channels (parallel multicast channels) [14].

In this paper, we propose some solutions for the power allocation of parallel multicast channels, focusing on the power minimization subject to a minimum rate constraint for all receivers. These solutions can be used in a tactical radio network equipped with cognitive terminals, i.e in which the transmitter knows the channel state information (CSI) and noise variances of its receivers. Although the initial problem is difficult to solve, we compare different algorithms inspired from Gallager's water-filling strategy [15] associating an inner loop for rate maximization and an outer loop for power minimization. We then consider the scenario in which N different cognitive radio networks can't cooperate with each other and wish to broadcast a common information to their network by sharing the same N_c parallel sub-channels. In this context, we propose a novel distributed power allocation for power minimization subject to a minimum rate constraint based on the iterative water-filling principle [16] in which each network updates its power allocation autonomously. The novel algorithm generalizes the iterative waterfilling algorithm to the coexistence of multiple tactical radio networks.

II. COEXISTENCE OF MULTIPLE TACTICAL RADIO NETWORKS

The considered scenario is shown on Figure 1, where N different networks coexist in a given area. In each network j, the T_j receivers are within the transmission range of the transmitter which broadcasts a common information. The transmission range is represented by the gray area around the transmitter. Moreover, the transmitter and the receivers of each network are mobile. This mobility is represented by arrows at the cardinal directions of the gray circles. In each network, a receiver can also become a transmitter to broadcast a common information, causing the initial transmitter to be another receiver. Due to this mobility, the different networks can interfere with each other, causing transmission losses if dynamic spectrum management techniques are not implemented. Our goal is to alleviate this problem by equipping each terminal with an algorithm which gives the possibility to optimize its transmission power for each sub-channel. The received signals $y_{j,it}$ can be modeled as

$$y_{j,it} = h_{jj,it}x_{ij} + \sum_{k \neq j}^{N} h_{jk,it}x_{ik} + n_{j,it} \quad \begin{array}{l} i = 1 \ldots N_c, \\ j = 1 \ldots N, \\ t = 1 \ldots T_j \end{array} \quad (1)$$

where $n_{j,it}$ represents a complex noise with variance $\sigma^2_{j,it}$ and $h_{jk,it}$ corresponds to the channel from network k to j on receiver t and tone i. We are interested in the power minimization subject to a minimum rate constraint for each network. In the following, we first derive the power allocation for a single tactical radio network.

A. Single tactical radio network

Considering a single tactical radio network, we assume that each transmitter has knowledge of the fading channels h_{it} in its network. The power minimization subject to a minimum rate constraint R^{min} for all T receivers is given by

$$\begin{array}{l} \min\limits_{(\phi_i)_{i=1\ldots N_c}} \sum\limits_{i=1}^{N_c} \phi_i \\ \text{subject to} \sum\limits_{i=1}^{N_c} \log_2(1 + \frac{|h_{it}|^2 \phi_i}{\Gamma \sigma^2_{it}}) \geq R^{min} \ \forall t \end{array} . \quad (2)$$

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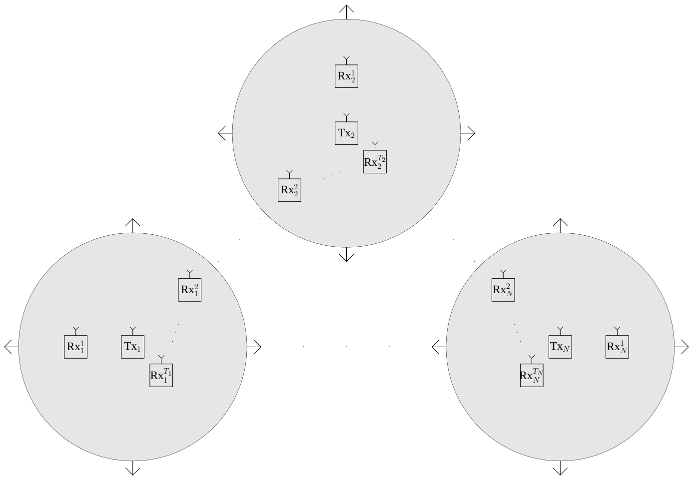

Fig. 1. Multiple cognitive radio networks for tactical communications

with $\phi = E[|x|^2]$ the variance of the transmitted signal and Γ the SNR gap which measures the loss with respect to theoretically optimum performance [17]. The derivation of the modified Lagrangian function leads to a single equation with T Lagrange multipliers. Therefore, the optimal power allocation has an infinite set of solutions and the problem is intractable for $T > 1$. We propose a solution to this problem by defining an utility function which takes into account all the possible achievable rates to the individual receivers. In the following, the weighted sum rate is chosen for this utility function as it allows to consider the possible achievable rates to the individual receivers with a certain flexibility owing to the weighting parameters. Therefore, an inner loop determines the power allocation maximizing the weighted sum rate subject to a total power constraint for a fixed set of weights. The minimum rate is then selected amongst the possible achievable rates to the individual receivers. Then, an outer loop minimizes the power such that a minimum rate constraint is achieved. This process is repeated for all set of weights and the set of weights exhibiting the least power determines the power allocation for power minimization subject to a minimum rate constraint. Let us first define the power allocation of the inner loop. The primal problem for weighted sum rate maximization subject to a power constraint P^{tot} is

$$\begin{aligned} &\max_{(\phi_i)_{i=1\ldots N_c}} \sum_{i=1}^{N_c}\sum_{t=1}^{T} w_t \log_2\left(1+\frac{|h_{it}|^2\phi_i}{\Gamma\sigma_{it}^2}\right) \\ &\text{subject to} \sum_{i=1}^{N_c}\phi_i = P^{tot} \end{aligned} \tag{3}$$

with w_t the weighting factors. The derivative of the modified Lagrangian function [18] with respect to ϕ_i is given by

$$\frac{\partial L(\lambda,(\phi_i)_{i=1\ldots N_c})}{\partial \phi_i} = \frac{1}{\ln 2}\sum_{t=1}^{T}\frac{w_t}{\frac{\Gamma\sigma_{it}^2}{|h_{it}|^2}+\phi_i} - \lambda. \tag{4}$$

We can write the power allocation in closed-form analytical expressions for $1 \le T \le 4$, while numerical algorithms are needed for $T > 4$. Let us take an example with two receivers $T = 2$, the power allocation is given by the solution of a quadratic equation

$$\frac{\partial L(\lambda,(\phi_i)_{i=1\ldots N_c})}{\partial \phi_i} = 0 \Rightarrow \frac{w_1}{\underbrace{\frac{\Gamma\sigma_{i1}^2}{|h_{i1}|^2}}_{a_i}+\phi_i} + \frac{w_2}{\underbrace{\frac{\Gamma\sigma_{i2}^2}{|h_{i2}|^2}}_{b_i}+\phi_i} = \underbrace{\lambda \ln 2}_{\tilde{\lambda}}. \tag{5}$$

Algorithm 1 Minimization of the power subject to a minimum rate constraint

1 n=0
2 for all $w_1,\ldots,w_T$, with $\sum_{t=1}^{T} w_t = 1$
3 n=n+1
4 init $P = 10^{-9}$
5 init $pstep = 2$
6 init $p = 0$
7 init $R_t = 0\ \forall t$
8 while $|min(R_1,\ldots,R_T) - R^{min}| > \epsilon$
9 init $\lambda = 10^{-9}$
10 init $step = 2$
11 init $b = 0$
12 init $\phi_i = 0\ \forall i$
13 while $|\sum_{i=1}^{N_c}\phi_i - P| > \epsilon$
14 Calculate $\phi_i\ \forall i$ according to (4)'s root
15 if $\sum_{i=1}^{N_c}\phi_i - P < 0$
16 $b = b + 1$
17 $\lambda = \lambda/step$
18 $step = step - 1/2^b$
19 end if
20 $\lambda = \lambda * step$
21 end while
22 Individual rates $R_t = \sum_{i=1}^{N_c}\log_2(1+\frac{|h_{it}|^2\phi_i}{\Gamma\sigma_{it}^2})\ \forall t$
23 if $min(R_1,\ldots,R_T) - R^{min} > 0$
24 $p = p + 1$
25 $P = P/pstep$
26 $pstep = pstep - 1/2^p$
27 end if
28 $P = P * pstep$
29 end while
30 $P_n = P$
31 end for
32 $P^{min} = min(P_n)$

The quadratic equation to be solved is

$$\tilde{\lambda}\phi_i^2 + (\tilde{\lambda}(a_i+b_i) - (w_1+w_2))\phi_i + \tilde{\lambda}a_i b_i - (w_1 b_i + w_2 a_i) = 0. \tag{6}$$

The discriminant is given by

$$\Delta = \tilde{\lambda}^2(a_i-b_i)^2 + (w_1+w_2)^2 - 2\tilde{\lambda}(a_i-b_i)(w_1-w_2). \tag{7}$$

The power allocation is given by the positive root

$$\phi_i = \left[\frac{1}{2\tilde{\lambda}} + \sqrt{\frac{(w_1+w_2)^2}{4\tilde{\lambda}^2} - \frac{(a_i-b_i)(w_1-w_2)}{2\tilde{\lambda}} + \frac{(a_i-b_i)^2}{4} - \frac{a_i+b_i}{2}}\right]^+. \tag{8}$$

The algorithm maximizing the weighted sum rate subject to a total power constraint (inner loop) is shown from line 13 to line 21 in Algorithm 1. The outer loop minimizing the power subject to a minimum rate constraint is shown from line 8 to line 29. In the following, we extend the results of a single tactical radio network to the coexistence of multiple tactical radio networks.

B. Multiple tactical radio networks

The results of the precedent Section can be extended to multiple tactical radio networks. We also consider the power minimization for all the different networks subject to a minimum rate constraint

$$\begin{array}{l}\min\limits_{(\phi_{ij})_{i=1\ldots N_c}^{j=1\ldots N}} \sum\limits_{i=1}^{N_c}\sum\limits_{j=1}^{N}\phi_{ij} \\ \text{subject to} \sum\limits_{i=1}^{N_c}\log_2(1+\frac{|h_{jj,it}|^2\phi_{ij}}{\Gamma(\sigma_{j,it}^2+\sum\limits_{k\neq j}|h_{jk,it}|^2\phi_{ik})}) \geq R^{min}\ \forall j,t\end{array} \quad . \tag{9}$$

Similarly to the precedent Section, this problem is intractable for $T_j > 1\ \forall j$. However, as pointed out previously, a solution to this problem can be found by defining an utility function which takes into account all the achievable rates of the receivers and to select the minimum rate in each network. By choosing the weighted rate sum for this utility function, an inner loop determines the power allocation which maximizes the weighted sum rate subject to a total power constraint for all the different networks. This primal problem can be given by

$$\begin{array}{l}\max\limits_{(\phi_{ij})_{i=1\ldots N_c}^{j=1\ldots N}} \sum\limits_{i=1}^{N_c}\sum\limits_{j=1}^{N}\sum\limits_{t=1}^{T_j} w_{jt}\log_2(1+\frac{|h_{jj,it}|^2\phi_{ij}}{\Gamma(\sigma_{j,it}^2+\sum\limits_{k\neq j}|h_{jk,it}|^2\phi_{ik})}) \\ \text{subject to} \sum\limits_{i=1}^{N_c}\phi_{ij} = P_j^{tot}\ \forall j\end{array} \quad . \tag{10}$$

Considering jointly the maximization of the weighted sum rate subject to a total power constraint for all the different networks in a centralized algorithm is an extensive task, since it would require the knowledge of the channel variations of all the interference terms $h_{jk,it}\ \ \forall i,j,t,k$. This knowledge can be acquired through a feedback channel from the receivers to the transmitter of each network assuming that the acquisition time is much lower than the coherence time of the channel fading. To this end, each terminal must be equipped with a spectrum sensing function to estimate the noise variances and a channel estimation function to estimate its channel variations. This information can be further transmitted to a centralized unit.

Distributed algorithms, although sub-optimal, are preferred to centralized algorithms for the coexistence between several tactical radio networks because of their scalability and robustness. Therefore, it is assumed that each transmitter has the knowledge of the channel variations in its own network j ($h_{jk,it}$, $\forall k=j$, $\forall i$, $\forall t$). We propose a sub-optimal distributed algorithm for power minimization subject to a minimum rate constraint based on the iterative water-filling algorithm initially derived for dynamic spectrum management in digital subscriber line (DSL) [16]. Each update of one network's water-filling affects the interference of the other networks and this process is repeated iteratively between the networks until the power allocation of all networks converge and reach a Nash equilibrium. As the power updates between networks can be performed asynchronously, an iterative water-filling based algorithm is very attractive when multiple tactical radio networks coexist in the same area. Let us derive the modified Lagrangian function of (10)

$$\begin{array}{l}L((\lambda_j)_{j=1\ldots N},(\phi_{ij})_{i=1\ldots N_c}^{j=1\ldots N}) = \\ \sum\limits_{i=1}^{N_c}\left(\sum\limits_{j=1}^{N}\sum\limits_{t=1}^{T_j} w_{jt}\log_2(1+\frac{|h_{jj,it}|^2\phi_{ij}}{\Gamma(\sigma_{j,it}^2+\sum\limits_{k\neq j}|h_{jk,it}|^2\phi_{ik})}) - \sum\limits_{j=1}^{N}\lambda_j\phi_{ij}\right) \\ +\sum\limits_{j=1}^{N}\lambda_j P_j^{tot}\end{array} \tag{11}$$

in which the λ_j's are the Lagrange multipliers. Assuming that the noise variances and the channel variations have been estimated by the receivers and given to their transmitter, we can solve the KKT system of the optimization problem by taking the derivative of the modified Lagrangian function with respect to ϕ_{ij}

$$\begin{array}{l}\frac{\partial L((\lambda_j)_{j=1\ldots N},(\phi_{ij})_{i=1\ldots N_c}^{j=1\ldots N})}{\partial\phi_{ij}} = \\ \frac{1}{\ln 2}\sum\limits_{t=1}^{T_j}\frac{w_{jt}}{\Gamma(\frac{\sigma_{j,it}^2}{|h_{it}|^2}+\sum\limits_{k\neq j}\frac{|h_{jk,it}|^2}{|h_{it}|^2}\phi_{ik})+\phi_{ij}} - \lambda_j\end{array} \quad . \tag{12}$$

Therefore, after collecting the noise variances and the channel variations of its network, each transmitter has to apply Algorithm 1 autonomously and to update its power allocation regularly to reach an equilibrium between the different networks. However, in Algorithm 1, the weight loop encompasses the outer loop to find which set of weights corresponds to the global minimum power such that a minimum rate constraint R^{min} is achieved. As the algorithm should be distributed and autonomous, the set of weights minimizing the power have to be determined for each network independently. To this end, we have to shift the weight loop between the inner loop and the outer loop. As the weights have to be chosen in each network, we introduce a rule based on the achievable rates after the computation of the weight loop and the inner loop. An adequate rule on the achievable rates is to choose the weights such that the disparity between the rates within each network is minimized. To this end, we introduce a deviation metric (DM) which measures the dispersion of the rates. The DM for a variable θ is given by:

$$DM = \frac{\sqrt{E[(\theta - E[\theta])^2]}}{E[\theta]} \tag{13}$$

The DM must be computed within each network j for each set of weights n over the T_j receivers. The rule is given by the following formula:

$$DM_j(n) = \frac{\sqrt{\frac{1}{T_j}\sum\limits_{t=1}^{T_j}[(R_{jt}(n) - \frac{1}{T_j}\sum\limits_{t=1}^{T_j}R_{jt}(n))^2]}}{\frac{1}{T_j}\sum\limits_{t=1}^{T_j}R_{jt}(n)} \tag{14}$$

with $R_{jt}(n)$ the rate for the network j, receiver t and the set of weights n. This rule allows to achieve the global minimum power although the decision has to be taken inside the outer loop. It basically means that for a given power the closer the rates of the different receivers within a network, the less power will be needed to achieve the minimum rate constraint. Therefore, it is in the interest of the best receiver to backoff and to give an advantage to the worst receiver to minimize the power globally. The algorithm for the coexistence of multiple tactical radio networks is presented in Algorithm 2.

III. Simulation results

For the simulations, the log-distance path loss model is used to measure the path loss between the transmitter and the receivers [19]:

$$PL(dB) = PL(d_0) + 10n\log_{10}(\frac{d}{d_0}) \tag{15}$$

with n the path loss exponent, d is the distance between the transmitter and the receiver, and d_0 the close-in reference distance. The reference path loss is calculated using the free space path loss formula:

$$PL(d_0) = -32.44 - 20\log_{10}(f_c) - 20\log_{10}(d_0) \tag{16}$$

where f_c is the carrier frequency in MHz and d_0 the reference distance in kilometers. The transmitter and the receivers are placed randomly in a square area of 1 km^2. The carrier frequency is chosen to be in the very high frequency (VHF) band ($f_c = 80$ MHz). The SNR gap for an uncoded quadrature amplitude modulation (QAM) to operate at a symbol error rate 10^{-7} is $\Gamma = 9.8$ dB. The bandwidth

Algorithm 2 Distributed power allocation for minimization of the power subject to a minimum rate constraint

init $P_j = 10^{-9}\ \forall j$
init $pstep_j = 2\ \forall j$
init $p_j = 0\ \forall j$
init $R_{jt} = 0\ \forall t, j$
while $|min(R_{j1}, \ldots, R_{jT_j}) - R^{min}| > \epsilon\ \forall j$
  for iteration=1 to 20
   for j=1 to N
     n=0
     for all $w_{j1},\ldots,w_{jT_j}$, with $\sum_{t=1}^{T_j} w_{jt} = 1$
       n=n+1
       init $\lambda = 10^{-9}$
       init $step = 2$
       init $b = 0$
       init $\phi_{ij} = 0\ \forall i$
       while $|\sum_{i=1}^{N_c} \phi_{ij} - P_j| > \epsilon$
         Calculate $\phi_{ij}\ \forall i$ according to (12)'s root
         if $\sum_{i=1}^{N_c} \phi_{ij} - P_j < 0$
           $b = b + 1$
           $\lambda = \lambda / step$
           $step = step - 1/2^b$
         end if
         $\lambda = \lambda * step$
       end while
       $R_{jt}(n) = \sum_{i=1}^{N_c} \log_2(1 + \frac{|h_{jj,it}|^2 \phi_{ij}}{\Gamma(\sigma_{j,it}^2 + \sum_{k \neq j} |h_{jk,it}|^2 \phi_{ik})})\ \forall t$
       $\phi_{ij}(n) = \phi_{ij}\ \forall i$
     end for
     $n^{opt} = \min_n$ (14)
     $R_{jt} = R_{jt}(n^{opt})\ \forall t$
     $\phi_{ij} = \phi_{ij}(n^{opt})\ \forall i$
    end for
  end for
  for j=1 to N
    if $min(R_{j1}, \ldots, R_{jT_j}) - R^{min} > 0$
      $p_j = p_j + 1$
      $P_j = P_j / pstep_j$
      $pstep_j = pstep_j - 1/2^{p_j}$
    end if
    $P_j = P_j * pstep_j$
  end for
end while

is $\Delta f = 25$ kHz, the path loss exponent is $n = 4$, reference distance $d_0 = 20$ meters and thermal noise with the following expression:

$$\sigma_n^2 = -204\text{dB/Hz} + 10\log_{10}(\Delta f) \tag{17}$$

which gives a noise variance of approximately $\sigma_n^2 = 10^{-16}$. Simulation results are performed using Monte Carlo trials for the locations of the transmitters and the receivers with 2 particular scenarios, $T = 2$ receivers and $N_c = 4$ sub-channels. In the first scenario (left part of Figure 2), the first receiver sees a small noise on the first three sub-channels and a very strong noise on the 4$^{\text{th}}$ sub-channel, while the second receiver sees a very strong noise on the 1$^{\text{st}}$ sub-channel and a small noise on the last three sub-channels. In the second scenario (right part of Figure 2), the first receiver sees a very strong noise on the 3$^{\text{rd}}$ and 4$^{\text{th}}$ sub-channels and the second receiver sees a very strong noise on the 1$^{\text{st}}$ and 2$^{\text{nd}}$ sub-channels.

In the first set of simulations, we compare Algorithm 1 with four other strategies for the minimization of the power subject to a minimum rate constraint. These strategies are the *worst receiver*, *worst case sub-channel*, *best receiver* and *best case sub-channel* strategies. The algorithms for the four strategies are similar to Algorithm 1 except for the inner loop which maximizes a different utility function than the weighted sum rate for the selection of the minimum rate. The maximum available power at the transmitter is $P^{tot} = 1W$. For the *worst receiver* strategy, the inner loop maximizes the minimum rate achieved by the receivers

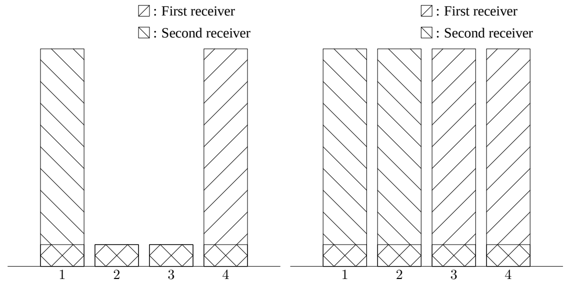

Fig. 2. Water-fill functions for two scenarios over four sub-channels

$$R = \min_t \max_{\sum_{i=1}^{N_c} \phi_i = P^{tot}} \sum_{i=1}^{N_c} \log_2(1 + \frac{|h_{it}|^2 \phi_i}{\Gamma \sigma_{it}^2}). \tag{18}$$

For the *worst case sub-channel* strategy, the inner loop maximizes the minimum rate achieved by the superposition of the receiver's sub-channels

$$R = \max_{\sum_{i=1}^{N_c} \phi_i = P^{tot}} \sum_{i=1}^{N_c} \min_t \log_2(1 + \frac{|h_{it}|^2 \phi_i}{\Gamma \sigma_{it}^2}). \tag{19}$$

For the *best receiver* strategy, the inner loop maximizes the maximum rate achieved by the receivers

$$R = \max_t \max_{\sum_{i=1}^{N_c} \phi_i = P^{tot}} \sum_{i=1}^{N_c} \log_2(1 + \frac{|h_{it}|^2 \phi_i}{\Gamma \sigma_{it}^2}). \tag{20}$$

For the *best case sub-channel* strategy, the inner loop maximizes the maximum rate achieved by the superposition of the receiver's sub-channels

$$R = \max_{\sum_{i=1}^{N_c} \phi_i = P^{tot}} \sum_{i=1}^{N_c} \max_t \log_2(1 + \frac{|h_{it}|^2 \phi_i}{\Gamma \sigma_{it}^2}). \tag{21}$$

Figure 3 shows the results of the power minimization subject to a minimum rate constraint ranging from $R^{min} = 2$ kbps to $R^{min} = 512$ kbps over 10^3 Monte Carlo trials for both scenarios. Algorithm 1 provides always the minimum power for all scenarios compared to the other strategies. For scenario 1, the results of the other strategies highly depend on the minimum rate constraints. For minimum rate constraints $R^{min} < 8$ kbps, the *worst case sub-channel* strategy provides lower power than the *worst receiver*, *best case sub-channel* and *best receiver* strategies. For minimum rate constraints 8 kbps $< R^{min} < 64$ kbps, the *worst receiver* strategy provides lower power than the *worst case sub-channel*, *best case sub-channel* and *best receiver* strategies. For minimum rate constraints 64 kbps $< R^{min} < 256$ kbps, the *worst receiver* strategy provides lower power than the *best case sub-channel*, *worst case sub-channel* and *best receiver* strategies. For minimum rate constraints 256 kbps $< R^{min}$, the *best case sub-channel* strategy provides lower power than the *worst receiver*, *worst case sub-channel* and *best receiver*

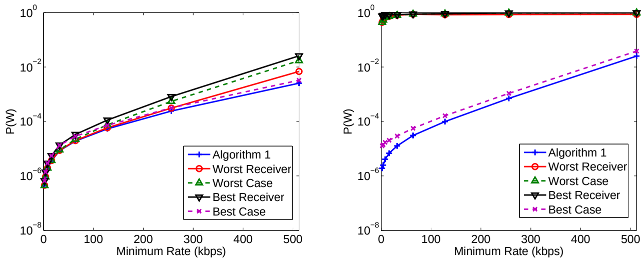

Fig. 3. Results on the power minimization subject to a minimum rate constraint for scenario 1 (left) and 2 (right)

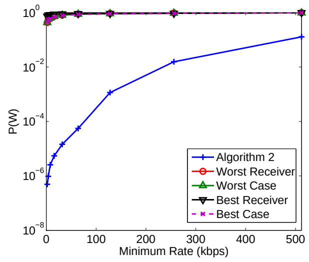

Fig. 4. Results on the power minimization subject to a minimum rate constraint averaged for two networks

strategies. It is hard to give an argument for each ordering between other strategies than Algorithm 1, therefore we keep the conclusion that Algorithm 1 provides always the minimum power while other strategies do not surpass each other at all range of minimum rate constraints. For scenario 2, Algorithm 1 provides the minimum power, followed by the *best case sub-channel* strategy. Moreover, the other strategies, i.e. the *worst receiver*, worst case and *best receiver* strategies use the maximum power for all minimum rate constraints.

In the second set of simulations, we compare Algorithm 2 with the same four other strategies for the minimization of the power subject to a minimum rate constraint with $N = 2$ networks whose transmitters and receivers are in a same square area of 1 km^2. We are interested in a scenario in which the receivers see a different noise σ_n^2 on their $N_c = 4$ sub-channels (similarly to the first set of simulations). In the first network, a very strong noise ($\sigma_n^2 = 10^{-9}$) is seen on the 4^{th} sub-channel by the first receiver and the 1^{st} sub-channel by the second receiver. In the second network, a very strong noise ($\sigma_n^2 = 10^{-9}$) is seen on the 3^{th} sub-channel by the first receiver and the 2^{nd} sub-channel by the second receiver. Figure 4 shows the results of the power minimization subject to a minimum rate constraint ranging from $R^{min} = 2$ kbps to $R^{min} = 512$ kbps over 10^3 Monte Carlo trials. The results are averaged for both networks. In this scenario, Algorithm 2 is the only strategy which provides a viable solution. Therefore, in practical scenarios in which the interference temperature varies along the sub-channel and the receiver locations, Algorithm 2 provides a novel distributed strategy to find the power allocation minimizing the power subject to a minimum rate constraint. Since it is based on closed-form expressions, the algorithm has reasonable complexity for a low number of receivers as the search for the best set of weights require an exhaustive search over all possible weights.

IV. Conclusion

In this paper, we have considered the scenario in which N different cognitive radio networks can not cooperate with each other and wish to broadcast a common information to their network by sharing the same N_c parallel sub-channels. In this context, we have proposed a novel distributed power allocation for power minimization subject to a minimum rate constraint based on the iterative water-filling principle in which each network updates its power allocation autonomously. The novel algorithm generalizes the iterative waterfilling algorithm to the coexistence of multiple tactical radio networks. Simulation results have shown that the proposed strategy provides a viable solution compared to trivial strategies such as the *worst receiver*, *worst case sub-channel*, *best receiver* and *best case sub-channel* strategies, while keeping a reasonable complexity for a low number of receivers since it is based on closed-form expressions.

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