Distributed Power Allocation for Parallel Broadcast Channels with Only Common Information in Cognitive Tactical Radio Networks

Vincent Le Nir, Bart Scheers

***Abstract*—A tactical radio network is a radio network in which a transmitter broadcasts the same information to its receivers. In this paper, dynamic spectrum management is studied for multiple cognitive tactical radio networks coexisting in the same area. First, we consider the problem of common rate maximization subject to a total power constraint for a single tactical radio network having multiple receivers and using parallel sub-channels (parallel multicast channels). Mathematical derivations show that the optimal power allocation can be found in closed form under multiple hypothesis testing. An outer loop can be used to minimize the power subject to a common rate constraint. Then, we extend the iterative waterfilling algorithm to the coexistence of multiple cognitive tactical radio networks without requiring any cooperation between the different networks. The power allocation is performed autonomously at the transmit side assuming knowledge of the noise variances and channel variations of the network. Simulation results show that the proposed algorithm is very robust in satisfying these constraints while minimizing the overall power in various scenarios.**

***Index Terms*—Cognitive tactical radio networks, broadcast channels, distributed power allocation, iterative water-filling, constrained optimization methods.**

I. Introduction

Tactical radio networks are networks in which information (voice and packet based data) are conveyed from one transmitter to multiple receivers. When several coalition nations coexist in the same area, current technologies do not permit reconfigurability, interoperability nor coexistence of the radio terminals. Software defined radio has been developed for reconfigurability of the terminals with software upgrades and for portability of the waveforms. Cognitive radio has been introduced by Mitola in 1999 as an extension to software defined radio [1]. Cognitive radio has been developed for spectrum availability recognition, reconfigurability, interoperability and coexistence between terminals by means of software defined radio technology, intelligence, awareness and learning [1], [2]. The fundamental principles of cognitive radio are on one hand to identify other radios in the environment that might use the same spectral resources by means of spectrum sensing and on the other hand to design a transmission strategy that minimizes interference to and from these radios by means of dynamic spectrum management. The major goals of cognitive radio are to provide a high utilization of the radio spectrum and reliable communications whenever and wherever needed [2]. Applications of cognitive radio include, but are not limited to, tactical radio networks, emergency networks, and wireless local area networks with high throughput and range.

The broadcast channel has been introduced by Cover in 1972 as a communication channel in which there is one transmitter and two or more receivers [3]. The broadcast channel in which independent messages are sent to the receivers (unicast channel) belongs to the class of degraded channels in which one user's signal is a degraded version of the other signals. Its capacity region is fully characterized and can be achieved by superposition coding [3], [4]. Contrary to a single unicast channel, the sum of unicast channels as well as MIMO broadcast channels are non-degraded [5], [6].

Previous studies on parallel broadcast channels have focused on scenarios in which independent messages are sent to the receivers (parallel unicast channels) [7], [8], [9], [10]. The optimal power allocation can be achieved by a multilevel water-filling over the parallel channels, which is an extension of Gallager's 1968 water-filling strategy for single-user parallel Gaussian channels [11]. Some other studies have considered parallel broadcast channels in which simultaneous common and independent messages are sent to the receivers [5], [12], [13], or simultaneous common and confidential messages are sent to the receivers [14].

Contrary to an unicast channel, a tactical radio network can be thought as a multicast channel with only common information. The capacity of a single multicast channel is limited by the capacity of the *worst receiver* [4], [15]. However, less work has been done on parallel multicast channels [16].

In the first part of the paper (Section II), we extend the waterfilling strategy [11] to multiple receivers considering parallel multicast channels with perfect channel state information (CSI) at the transmit side. In this case, the extended waterfilling strategy maximizes the common rate subject to a power constraint (inner loop) or minimizes the power subject to a common rate constraint (outer loop). Mathematical derivations show that the optimal power allocation can be found in closed form under multiple hypothesis testing [14], [17], [18].

Distributed multi-user power control has been studied for parallel interference channels, leading to a common strategy known as iterative water-filling [19], [20], [21]. Distributed algorithms, although sub-optimal, are preferred to centralized algorithms in practical scenarios because of their scalability. In the iterative waterfilling algorithm, each network considers the interference of all other networks as noise and iteratively performs a waterfilling strategy. At each iteration, the power spectrum of each network modifies the interference caused to all other networks. This process is performed iteratively until the power spectra of all networks converge.

In the second part of the paper (Section III), capitalizing on the previous results, we introduce an autonomous dynamic spectrum management algorithm based on iterative waterfilling [19] for multiple cognitive tactical radio networks coexisting in a given area and willing to broadcast a common information (voice, data...) to their group. The problem can be modeled as N networks, each network j with a single transmitter willing to send a common message to its corresponding T_j receivers over N_c parallel scalar Gaussian sub-channels. It is assumed that each transmitter has the knowledge of the channel variations and noise variances in its own network and iteratively updates its power spectrum until a common rate constraint for all receivers is satisfied. Although this paper focuses on multiple cognitive radio networks for tactical communications, the proposed algorithm can be applied to any application requiring spectrum management between multiple cognitive radio networks for parallel multicast channels with only common information. In Section IV, simulation results are given for multiple scenarios and compare the proposed algorithm with the *worst sub-channel* strategy. Finally, Section V concludes this paper.

II. Single Tactical Radio Network

Consider a T-receiver N_c parallel Gaussian broadcast channel as shown in Figure 1

$$y_{it} = h_{it}x_i + n_{it} \quad t = 1 \dots T, i = 1 \dots N_c \tag{1}$$

where x_i is the transmitted signal, n_{it} represents a complex noise with variance σ_{it}^2 and h_{it} corresponds to the channel seen by receiver

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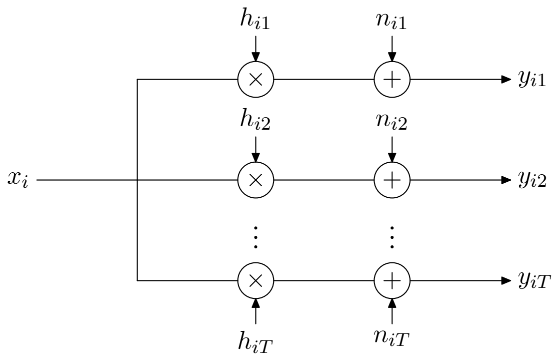

Fig. 1. T-receiver N_c parallel Gaussian broadcast channel

t on tone i. The maximum common information rate that can be supported by the channel is given by:

$$\begin{aligned} &\max_{\underline{\phi}} \min_{t} \sum_{i=1}^{N_c} \log_2(1+\frac{|h_{it}|^2\phi_i}{\Gamma\sigma_{it}^2}) \\ &\text{subject to} \sum_{i=1}^{N_c} \phi_i = P^{tot} \end{aligned} \tag{2}$$

with $\phi_i = E[|x_i|^2]$ the variance of the input signal on channel i, $\underline{\phi}$ the power allocation among all sub-channels, P^{tot} the total power constraint, and Γ the SNR gap which measures the loss with respect to theoretically optimum performance [22]. To achieve the maximum common information rate, the common message codebook cannot be broken into different codebooks for each channel, i.e. joint encoding and joint decoding must be performed across all sub-channels [23]. This transmission scheme is referred to as "single codebook, variable power" transmission [24].

The expression in (2) is the maximization of the minimum of a set of sums of concave functions of ϕ_i. Since the sum and the minimum operations preserve concavity, the objective is concave, and maximizing a concave function yields a convex optimization problem. This max-min optimization problem can be efficiently solved by the approach based on minimax hypothesis testing given in [14], [17], [18]. For two receivers, the optimal power allocation algorithm is given by three steps:

- Step 1: Find $\underline{\phi}^{(1)}$ given by

$$\begin{aligned} &\max_{\underline{\phi}} R_{01}(\underline{\phi}) \\ &\text{subject to} \sum_{i=1}^{N_c} \phi_i = P^{tot} \end{aligned} \tag{3}$$

  with

$$R_{0t}(\underline{\phi}) = \sum_{i=1}^{N_c} \log_2(1+\frac{|h_{it}|^2\phi_i}{\Gamma\sigma_{it}^2}) \qquad t=1,2 \tag{4}$$

  If $R_{01}(\underline{\phi}^{(1)}) < R_{02}(\underline{\phi}_i^{(1)})$ then the optimal power allocation is $\underline{\phi}^{opt} = \underline{\phi}^{(1)}$ and finish.
- Step 2: Find $\underline{\phi}^{(2)}$ given by

$$\begin{aligned} &\max_{\underline{\phi}} R_{02}(\underline{\phi}) \\ &\text{subject to} \sum_{i=1}^{N_c} \phi_i = P^{tot} \end{aligned} \tag{5}$$

  If $R_{02}(\underline{\phi}^{(2)}) < R_{01}(\underline{\phi}^{(2)})$ then the optimal power allocation is $\underline{\phi}^{opt} = \underline{\phi}^{(2)}$ and finish.
- Step 3: For a given set of weights $\{w_t\}$ corresponding to the index n with $\sum_{t=1}^{2} w_t = 1$, find $\underline{\phi}^{(n)}$ given by

$$\begin{aligned} &\max_{\underline{\phi}} \sum_{t=1}^{2} w_t R_{0t}(\underline{\phi}) \\ &\text{subject to} \sum_{i=1}^{N_c} \phi_i = P^{tot} \end{aligned} \tag{6}$$

  Search over all n to find n^{opt} that satisfies $R_{01}(\underline{\phi}^{(n)}) = R_{02}(\underline{\phi}^{(n)})$, then the optimal power allocation is $\underline{\phi}^{opt} = \underline{\phi}^{(n^{opt})}$ and finish.

First consider the optimization problem of step 1 and 2. As the objective function is concave, the power allocation can be derived by the standard Karush-Kuhn-Tucker (KKT) conditions [25]. The modified Lagrangian function for step 1 and 2 is given by

$$L(\lambda,\underline{\phi}) = \sum_{i=1}^{N_c} \left(\log_2(1+\frac{|h_{it}|^2\phi_i}{\Gamma\sigma_{it}^2}) - \lambda\phi_i \right) + \lambda P^{tot} \quad t=1,2 \tag{7}$$

with λ the Lagrange multiplier associated with the total power constraint. By taking the derivative of the modified Lagrangian function with respect to ϕ_i, we can solve the KKT system of the optimization problem. The derivative with respect to ϕ_i is given by

$$\frac{\partial L(\lambda,\underline{\phi})}{\partial \phi_i} = \frac{1}{\ln 2}\frac{1}{\frac{\Gamma\sigma_{i1}^2}{|h_{it}|^2}+\phi_i} - \lambda \quad t=1,2 \tag{8}$$

Nulling the derivative gives

$$\frac{\partial L(\lambda,\underline{\phi})}{\partial \phi_i} = 0 \Rightarrow \frac{1}{\frac{\Gamma\sigma_{it}^2}{|h_{it}|^2}+\phi_i} = \underbrace{\lambda \ln 2}_{\tilde{\lambda}} \quad t=1,2 \quad . \tag{9}$$

The optimal power allocation corresponds to Gallager's water-filling strategy for single-user parallel Gaussian channels [11]:

- Step 1:

$$\phi_i^{(1)} = \left[\frac{1}{\tilde{\lambda}} - \frac{\Gamma\sigma_{i1}^2}{|h_{i1}|^2} \right]^+ \tag{10}$$

- Step 2:

$$\phi_i^{(2)} = \left[\frac{1}{\tilde{\lambda}} - \frac{\Gamma\sigma_{i2}^2}{|h_{i2}|^2} \right]^+ \tag{11}$$

We now consider the optimization problem of step 3. As the objective is a weighted sum of concave functions, the power allocation can also be derived by the standard KKT conditions. The modified Lagrangian function for step 3 is given by

$$L(\lambda,\underline{\phi}) = \sum_{i=1}^{N_c} \left(\sum_{t=1}^{2} w_t \log_2(1+\frac{|h_{it}|^2\phi_i}{\Gamma\sigma_{it}^2}) - \lambda\phi_i \right) + \lambda P^{tot} \tag{12}$$

with λ the Lagrange multiplier associated with the total power constraint. By taking the derivative of the modified Lagrangian function with respect to ϕ_i, we can solve the KKT system of the optimization problem. The derivative with respect to ϕ_i is given by

$$\frac{\partial L(\lambda,\underline{\phi})}{\partial \phi_i} = \frac{1}{\ln 2}\sum_{t=1}^{2} \frac{w_t}{\frac{\Gamma\sigma_{it}^2}{|h_{it}|^2}+\phi_i} - \lambda \tag{13}$$

Nulling the derivative gives

$$\frac{\partial L(\lambda,\underline{\phi})}{\partial \phi_i}=0 \Rightarrow \frac{w_1}{\underbrace{\frac{\Gamma\sigma_{i1}^2}{|h_{i1}|^2}}_{a_i}+\phi_i}+\frac{w_2}{\underbrace{\frac{\Gamma\sigma_{i2}^2}{|h_{i2}|^2}}_{b_i}+\phi_i}=\underbrace{\lambda \ln 2}_{\tilde{\lambda}} \quad . \tag{14}$$

The quadratic equation to be solved is

$$\begin{array}{l}\tilde{\lambda}\phi_i^2+(\tilde{\lambda}(a_i+b_i)-(w_1+w_2))\phi_i \\ +\tilde{\lambda}a_ib_i-(w_1b_i+w_2a_i)=0\end{array} \quad . \tag{15}$$

The discriminant is given by

$$\begin{array}{l}\Delta=\tilde{\lambda}^2(a_i-b_i)^2+(w_1+w_2)^2 \\ -2\tilde{\lambda}(a_i-b_i)(w_1-w_2)\end{array} \quad . \tag{16}$$

The power allocation is given by the positive root

$$\phi_i=\left[\frac{1}{2\tilde{\lambda}}+ \sqrt{\frac{(w_1+w_2)^2}{4\tilde{\lambda}^2}-\frac{(a_i-b_i)(w_1-w_2)}{2\tilde{\lambda}}+\frac{(a_i-b_i)^2}{4}}-\frac{a_i+b_i}{2}\right]^+ . \tag{17}$$

In this formula, the optimal power allocation takes into account the difference between the water-fill functions and the weights of the different receivers.

For more than two receivers, the optimal power allocation algorithm is driven by the solutions of higher degree polynomials and involves more steps under multiple hypothesis testing. For instance, with three receivers $T=3$, the optimal power allocation algorithm is given by seven steps involving the solutions of three linear equations, three quadratic equations and a cubic equation [26]. Therefore, for three receivers $T=3$ and four receivers $T=4$, the optimal power allocation is a type of water-filling strategy given by the solutions up to a cubic and a quartic equation respectively. The optimal power allocation can also be found analytically (the solution is not given in this paper due to space limitations). With $T>4$, the optimal power allocation is given by the solutions of polynomial equations up to degree T from the formula

$$\sum_{t=1}^{T}\frac{w_t}{\frac{\Gamma\sigma_{it}^2}{|h_{it}|^2}+\phi_i}=\tilde{\lambda}. \tag{18}$$

In general, the roots for polynomials with higher degree than four can not be expressed analytically but can be solved numerically. Note that to reduce the complexity in a practical algorithm, the weights w_t are taken from a given data set in interval [0 1] with N_s samples, leading to a possible exhaustive search over $S=TN_s^{T-1}$ possibilities. Therefore, to satisfy the conditions requiring the rates of the different receivers to be equal, the optimal value s^{opt} should minimize the dispersion of the rates

$$s^{opt}=\min_s\frac{\sqrt{\frac{1}{T}\sum_{t=1}^{T}[(R_{0t}(\underline{\phi}^{(s)})-\frac{1}{T}\sum_{t=1}^{T}R_{0t}(\underline{\phi}^{(s)}))^2]}}{\frac{1}{T}\sum_{t=1}^{T}R_{0t}(\underline{\phi}^{(s)})} \tag{19}$$

Figure 2 shows the power control for a single tactical radio network. An inner loop determines the power allocation maximizing the common rate subject to a total power constraint. Then, an outer loop minimizes the power such that a common rate constraint R^{com} is achieved. In the Annex, Algorithm 1 provides the proposed power allocation for power minimization subject to a common rate constraint. The inner loop and the outer loop correspond to lines 13-21 and 6-30 respectively. Note that if all the steps in the multiple hypothesis testing are needed, the complexity of the algorithm increases exponentially with the number of receivers $O(TN_s^{T-1})$.

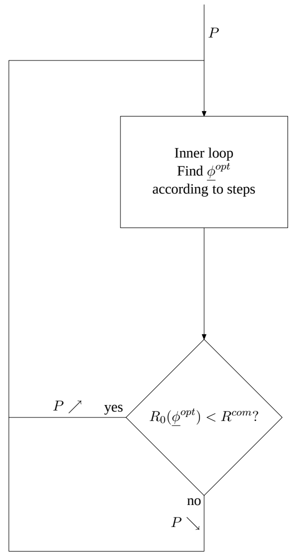

Fig. 2. Power control for a single tactical radio network

III. Multiple cognitive tactical radio networks

The coexistence of multiple cognitive tactical radio networks is shown on Figure 3. In each network j, the T_j receivers are within the transmission range of the transmitter which broadcasts a common information. The transmission range is represented by the gray area around the transmitter. The different networks can interfere with each other, causing transmission losses if dynamic spectrum management techniques are not implemented. Our goal is to alleviate this problem by equipping each terminal with an algorithm which gives the possibility to optimize its transmission power for each sub-channel. We assume that the links between the transmitter and the receivers of each network exhibit quasi-static fading channels, i.e. in which the coherence times of the fading channels are larger than the time necessary to compute the algorithm. Such an assumption is motivated by the fact that tactical radio networks using VHF and low UHF bands exhibit long coherence times for low mobility patterns. The received signals $y_{j,it}$ can be modeled as

$$y_{j,it}=h_{jj,it}x_{ij}+\sum_{k\neq j}^{N}h_{jk,it}x_{ik}+n_{j,it} \quad \begin{array}{l} i=1\ldots N_c, \\ j=1\ldots N, \\ t=1\ldots T_j\end{array} \tag{20}$$

where $n_{j,it}$ represents a complex noise with variance $\sigma_{j,it}^2$ and $h_{jk,it}$ corresponds to the channel from network k to j seen by receiver t and tone i. We consider the maximization of the aggregate common rate subject to a total power constraint per network

$$\begin{array}{l}\max\limits_{\underline{\underline{\phi}}}\sum\limits_{j=1}^{N}\min\limits_{t}\sum\limits_{i=1}^{N_c}\log_2(1+\frac{|h_{jj,it}|^2\phi_{ij}}{\Gamma(\sigma_{j,it}^2+\sum\limits_{k\neq j}|h_{jk,it}|^2\phi_{ik})}) \\ \text{subject to} \sum\limits_{i=1}^{N_c}\phi_{ij}=P_j^{tot}\ \forall j\end{array} \tag{21}$$

with $\underline{\underline{\phi}}$ the power allocation among all sub-channels and networks. Similarly to a single tactical radio network, multiple hypothesis

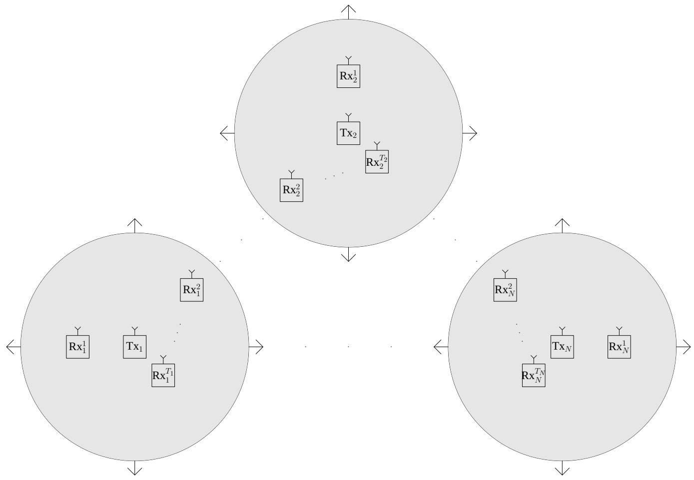

Fig. 3. Multiple cognitive radio networks for tactical communications

testing can be used to transform the above problem into different steps according to different values of w_{jt}, with $\sum_{t=1}^{T_j} w_{jt} = 1, \forall j$. Note that the number of steps under multiple hypothesis testing increases exponentially with the number of networks N. In the following, we omit the steps under multiple hypothesis testing for clarity. Therefore, (21) reduces to the following problem

$$\begin{aligned} &\max_{\underline{\underline{\phi}}} \sum_{j=1}^{N} \sum_{t=1}^{T_j} w_{jt} R_{0jt}(\underline{\underline{\phi}}) \\ &\text{subject to} \sum_{i=1}^{N_c} \phi_{ij} = P_j^{tot} \ \forall j \end{aligned} \tag{22}$$

with multiple conditions according to the weights w_{jt}, $\forall t, j$ on the rates

$$R_{0jt}(\underline{\underline{\phi}}) = \sum_{i=1}^{N_c} \log_2\Big(1 + \frac{|h_{jj,it}|^2 \phi_{ij}}{\Gamma(\sigma_{j,it}^2 + \sum_{k \neq j} |h_{jk,it}|^2 \phi_{ik})}\Big) \quad \forall t, j \tag{23}$$

Considering jointly the maximization of the aggregate common rate subject to a total power constraint per network in a centralized algorithm is an extensive task, since it would require the knowledge of the channel variations of all the interference terms $h_{jk,it}$ $\forall i, j, t, k$. This knowledge can be acquired through a feedback channel from the receivers to the transmitter of each network assuming that the acquisition time is much lower than the coherence time of the channel fading. To this end, each terminal must be equipped with a spectrum sensing function to estimate the noise variances and a channel estimation function to estimate its channel variations. This information can be further transmitted to a centralized unit. Moreover, even if a centralized cognitive manager was able to collect all the channel state information (CSI) within and between the different networks, solving (22) would require an exhaustive search over all possible ϕ_{ij}'s, or a more efficient genetic algorithm.

Distributed algorithms, although sub-optimal, are preferred to centralized algorithms for the coexistence between several tactical radio networks because of their scalability. Therefore, it is assumed that each transmitter has the knowledge of the channel variations in its own network j ($h_{jk,it}$, $\forall k = j, i, t$). We propose a sub-optimal distributed algorithm for power minimization subject to a common rate constraint based on the iterative water-filling algorithm initially derived for dynamic spectrum management in digital subscriber line (DSL) [19]. Note that a more robust iterative water-filling algorithm such as [20], [21] can also be applied in case of imperfect channel and noise variance information. Each update of one network's water-filling affects the interference of the other networks and this process is repeated iteratively between the networks until the power allocation of all networks converge and reach a Nash equilibrium. As the power updates between networks can be performed asynchronously, an iterative water-filling based algorithm is very attractive when multiple tactical radio networks coexist in the same area. Let us derive the modified Lagrangian function of (22)

$$\begin{aligned} &L(\underline{\lambda}, \underline{\underline{\phi}}) = \\ &\sum_{i=1}^{N_c} \left(\sum_{j=1}^{N} \sum_{t=1}^{T_j} w_{jt} \log_2\Big(1 + \frac{|h_{jj,it}|^2 \phi_{ij}}{\Gamma(\sigma_{j,it}^2 + \sum_{k \neq j} |h_{jk,it}|^2 \phi_{ik})}\Big) - \sum_{j=1}^{N} \lambda_j \phi_{ij} \right) \\ &+ \sum_{j=1}^{N} \lambda_j P_j^{tot} \end{aligned} \tag{24}$$

in which $\underline{\lambda}$ are the Lagrange multipliers for all networks. Assuming that the noise variances and the channel variations have been estimated by the receivers and given to their transmitter, we can solve the KKT system of the optimization problem by taking the derivative of the modified Lagrangian function with respect to ϕ_{ij}

$$\frac{\partial L(\underline{\lambda}, \underline{\underline{\phi}})}{\partial \phi_{ij}} = \frac{1}{\ln 2} \sum_{t=1}^{T_j} \frac{w_{jt}}{\Gamma\big(\frac{\sigma_{j,it}^2}{|h_{jj,it}|^2} + \sum_{k \neq j} \frac{|h_{jk,it}|^2}{|h_{jj,it}|^2} \phi_{ik}\big) + \phi_{ij}} - \lambda_j \quad . \tag{25}$$

Therefore, after collecting the noise variances and the channel variations of its network, each transmitter has to apply Algorithm 1 autonomously and to update its power allocation regularly to reach an equilibrium between the different networks. As shown on Figure 4, within each network, an inner loop determines the power allocation maximizing the common rate subject to a total power constraint. This process is updated regularly between all the different networks until they reach a Nash equilibrium. Finally, an outer loop minimizes the power such that a common rate constraint is achieved for each network. The algorithm for the coexistence of multiple tactical radio networks is presented in the Annex (Algorithm 2).

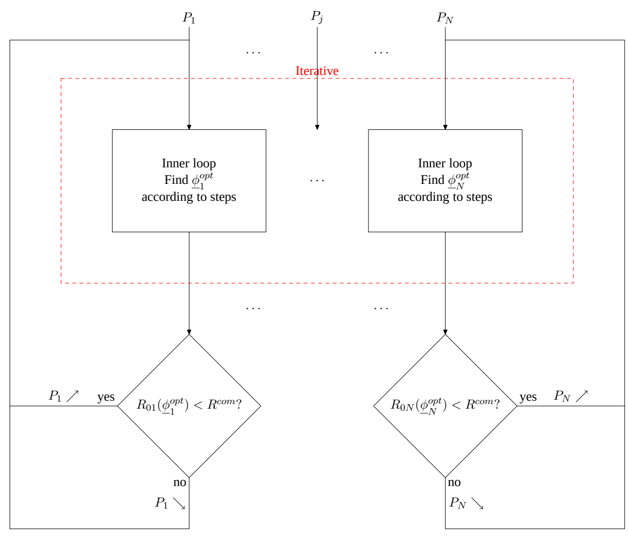

Fig. 4. Distributed power control for multiple tactical radio networks

IV. Simulation Results

For the simulations, the log-distance path loss model is used to measure the path loss between the transmitter and the receivers [27]:

$$PL(dB) = PL(d_0) + 10n\log_{10}(\frac{d}{d_0}) \quad (26)$$

with n the path loss exponent, d is the distance between the transmitter and the receiver, and d_0 the close-in reference distance. The reference path loss is calculated using the free space path loss formula:

$$PL(d_0) = -32.44 - 20\log_{10}(f_c) - 20\log_{10}(d_0) \quad (27)$$

where f_c is the carrier frequency in MHz and d_0 the reference distance in kilometers. The transmitter and the receivers are placed randomly in a square area of 1 km^2. The carrier frequency is chosen to be in the very high frequency (VHF) band ($f_c = 80$ MHz). The SNR gap for an uncoded quadrature amplitude modulation (QAM) to operate at a symbol error rate 10^{-7} is $\Gamma = 9.8$ dB. The sub-channel bandwidth is $\Delta f = 25$ kHz, the path loss exponent is $n = 4$, reference distance $d_0 = 20$ meters and thermal noise with the following expression:

$$\sigma_n^2 = -204\text{dB/Hz} + 10\log_{10}(\Delta f) \quad (28)$$

which gives a noise variance per sub-channel of approximately $\sigma_n^2 = 10^{-16}$.

A. Single Tactical Radio Network

Simulation results for a single tactical radio network are performed using Monte Carlo trials for the locations of the transmitters and the receivers with $T = 2$ receivers and $N_c = 4$ sub-channels and 2 particular scenarios. The maximum available power at the transmitter is $P^{tot} = 1W$. In the first scenario (left part of Figure 5), the first receiver sees a small noise on the first three sub-channels and a very strong noise on the 4$^{\text{th}}$ sub-channel, while the second receiver sees a very strong noise on the 1$^{\text{st}}$ sub-channel and a small noise on the last three sub-channels. In the second scenario (right part of Figure 5), we take an extreme situation in which the first receiver sees a very strong noise on the 3$^{\text{rd}}$ and 4$^{\text{th}}$ sub-channels and the second receiver sees a very strong noise on the 1$^{\text{st}}$ and 2$^{\text{nd}}$ sub-channels. The different noises seen by the different receivers can be thought as sub-channel variations depending on the location, a sub-channel occupied by a primary transmitter, a jammer, or different channel characteristics.

In the first set of simulations, we compare Algorithm 1 with the *worst sub-channel* strategy for the minimization of the power subject to a common rate constraint. The *worst sub-channel* strategy corresponds to the strategy in which the common message codebook is broken into different codebooks for each sub-channel, therefore the common rate is limited by the weakest receiver in each sub-channel [23]. This transmission scheme is referred to as "multiple codebook, variable power" transmission [24]. In the *worst sub-channel* strategy, the water-filling is performed on the worst sub-channel conditions considering both receivers. More precisely, the values a_i and b_i are compared for each sub-channel and the greatest value is selected for the water-filling. For the *worst sub-channel* strategy, the inner loop maximizes the rate of the superposition of the receiver's worst sub-channels given by

$$\begin{aligned} &\max_{\underline{\phi}} \sum_{i=1}^{N_c} \min_t \log_2(1 + \frac{|h_{it}|^2\phi_i}{\Gamma\sigma_{it}^2}) \\ &\text{subject to} \sum_{i=1}^{N_c} \phi_i = P^{tot} \end{aligned} \quad . \quad (29)$$

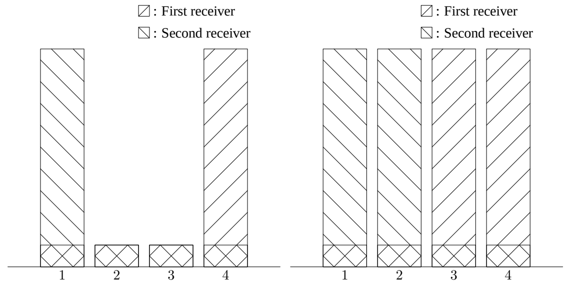

Fig. 5. Water-fill functions for two scenarios over four sub-channels

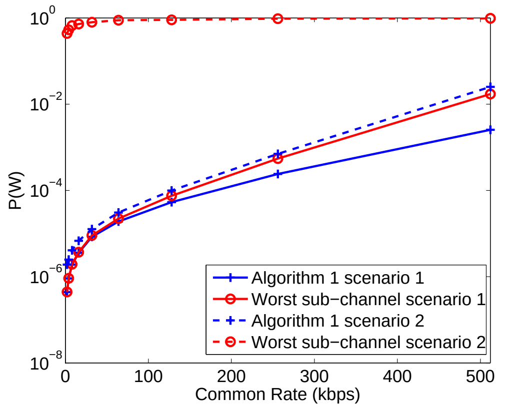

Fig. 6. Results on the power minimization subject to a common rate constraint for both scenarios

Figure 6 shows the results of the power minimization subject to a common rate constraint ranging from $R^{com} = 2$ kbps to $R^{com} = 512$ kbps over 10^3 Monte Carlo trials for both scenarios. Algorithm 1 provides the minimum power for all scenarios compared to the *worst sub-channel* strategy as this is the optimal strategy. Moreover, for the scenario 2, the *worst sub-channel* strategy uses the maximum power for all common rate constraints.

B. Multiple cognitive tactical radio networks

In the second set of simulations, we compare Algorithm 2 with the *worst sub-channel* strategy for the minimization of the power subject to a common rate constraint with $N = 2$ networks whose transmitters and receivers are in a same square area of 1 km^2. To highlight the robustness of our algorithm, we take an extreme scenario in which the receivers see a different noise σ_n^2 on their $N_c = 4$ sub-channels (as shown in Figure 7). In the first network, a very strong noise ($\sigma_n^2 = 10^{-9}$) is seen on the 4th sub-channel by the first receiver and the 1st sub-channel by the second receiver. In the second network, a very strong noise ($\sigma_n^2 = 10^{-9}$) is seen on the 3th sub-channel by the first receiver and the 2nd sub-channel by the second receiver. Figure 8 shows the results of the power minimization subject to a common rate constraint ranging from $R^{com} = 2$ kbps to $R^{com} = 512$ kbps over 10^3 Monte Carlo trials. The results are averaged for both networks. In this scenario, it can be seen that Algorithm 2 outperforms the *worst sub-channel* strategy. Therefore, in practical scenarios in which the interference temperature varies along the sub-channel and the receiver locations, Algorithm 2 provides a novel distributed strategy to find the power allocation minimizing the power subject to a common rate constraint. Since it is based on closed-form expressions, the algorithm has reasonable complexity for a low number of receivers. However, the search for the best set of weights require an exhaustive search

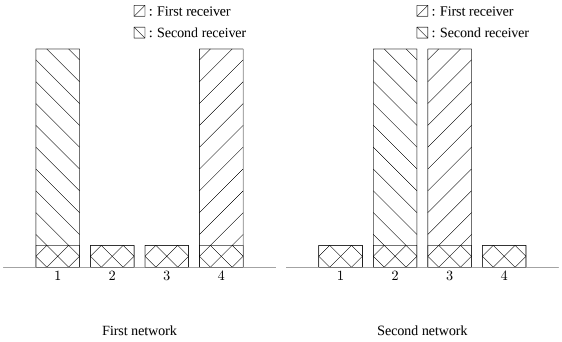

Fig. 7. Water-fill functions for two networks over four sub-channels

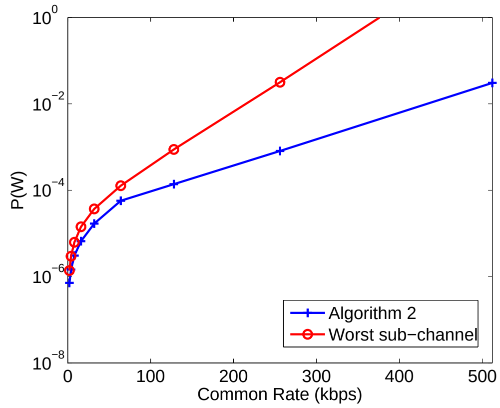

Fig. 8. Results on the power minimization subject to a common rate constraint averaged for two networks

over all possible weights. Therefore, to reduce the complexity in a practical algorithm, the weights w_t are taken from a given data set in interval [0 1].

V. Conclusion

In this paper, dynamic spectrum management was studied for multiple cognitive tactical radio networks coexisting in the same area. First, we have considered the problem of power minimization subject to a common rate constraint for a single tactical radio network with multiple receivers over parallel channels (parallel multicast channels). Then, we have extended the iterative waterfilling algorithm to multiple receivers for the coexistence of multiple cognitive tactical radio networks assuming knowledge of the noise variances and channel variations of the network. Simulation results have shown that the proposed algorithm is very robust in satisfying these constraints while minimizing the overall power in various scenarios.

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ANNEX

Algorithm 1 Minimization of the power subject to a common rate constraint

init $P = 10^{-9}$
init $c = 2$
init $p = 0$
init $R_t = 0\ \forall t$
while $|R_0(\underline{\phi}^{opt}) - R^{com}| > \epsilon$
for all steps
init $w_t\ \forall t$ according to step
init $\lambda = 10^{-9}$
init $d = 2$
init $b = 0$
init $\phi_i = 0\ \forall i$
while $|\sum_{i=1}^{N_c} \phi_i - P| > \epsilon$
Calculate $\underline{\phi}$ according to the roots of (18)
if $\sum_{i=1}^{N_c} \phi_i - P < 0$
$b = b + 1$
$\lambda = \lambda / d$
$d = d - 1/2^b$
end if
$\lambda = \lambda \times d$
end while
If condition satisfied on $R_{0t}(\underline{\phi}^{opt})\ \forall t$ exit step loop
end for
if $R_0(\underline{\phi}^{opt}) - R^{com} > 0$
$p = p + 1$
$P = P/c$
$c = c - 1/2^p$
end if
$P = P \times c$
end while

Algorithm 2 Distributed power allocation for minimization of the power subject to a common rate constraint

init $P_j = 10^{-9}\ \forall j$
init $c_j = 2\ \forall j$
init $p_j = 0\ \forall j$
init $R_{jt} = 0\ \forall t, j$
while $|R_{0j}(\underline{\phi}_j^{opt}) - R^{com}| > \epsilon\ \forall j$
for iteration=1 to 20
for j=1 to N
for all steps
init $w_{jt}\ \forall t$ according to step
init $\lambda = 10^{-9}$
init $d = 2$
init $b = 0$
init $\phi_{ij} = 0\ \forall i$
while $|\sum_{i=1}^{N_c} \phi_{ij} - P_j| > \epsilon$
Calculate $\underline{\phi}_j$ according to the roots of (18)
if $\sum_{i=1}^{N_c} \phi_{ij} - P_j < 0$
$b = b + 1$
$\lambda = \lambda / d$
$d = d - 1/2^b$
end if
$\lambda = \lambda \times d$
end while
If condition satisfied on $R_{0jt}(\underline{\phi}_j^{opt})\ \forall t$ exit step loop
end for
end for
end for
for j=1 to N
if $R_{0j}(\underline{\phi}_j^{opt}) - R^{com} > 0$
$p_j = p_j + 1$
$P_j = P_j / c_j$
$c_j = c_j - 1/2^{p_j}$
end if
$P_j = P_j \times c_j$
end for
end while