Transmit Covariance Matrices for Broadcast Channels under Per-Modem Total Power Constraints and Non-Zero Signal to Noise Ratio Gap

Vincent Le Nir, Marc Moonen, Jochen Maes, Mamoun Guenach

Abstract— **Finding the capacity region of MIMO BC has been a challenging task during the past few years. The same set of rates can be achieved by duality between the MIMO MAC and the MIMO BC under the same total power constraint and zero SNR gap [1], [2]. However, more practical scenarios for DSL consider per-modem total power constraints as well as a non-zero SNR gap. In this paper, we extend the work done in [3], [4], [5] and we derive a unified framework for the MIMO BC that can cope with per-modem total power constraints and non-zero SNR gap. By properly incorporating the SNR gap in the duality formulas [1] (derived initially for zero SNR gap and total power constraint) and by extending the work of [4] to non-zero SNR gap, the high SNR assumption of [5] can be relaxed and per-modem total power constraints can be met. Simulation results are given for VDSL2 channels, as well as for a bonded VDSL2 scenario exploiting Phantom Mode (PM) transmission with external noise coming from VDSL2 disturbing lines.**

I. Introduction

The multiple input multiple output broadcast channel (MIMO BC) has attracted a lot of attention during the past few years due to the lack of a simple characterization of its capacity region. However, owing to the duality with the MIMO multiple access channel (MAC), the capacity region of the MIMO BC can be analysed based on its dual MIMO MAC, under the same total power constraint [1]. The optimal transmission structure that achieves the MIMO BC capacity region is based on minimum mean square error dirty paper coding (MMSE DPC) and basically consists of a matched filter and a Decision Feedback Equalizer (DFE) whose filters are similar to those of the MIMO MAC MMSE-DFE [6], [7]. Therefore, it is possible to achieve the same set of rates in both the MIMO MAC and the MIMO BC under the same total power constraint and with the same feedforward and feedback filters.

However, more practical scenarios for digital subscriber line (DSL) consider per-modem total power constraints as well as a non-zero so-called SNR gap [2]. Indeed, it is more relevant to consider a constraint on the total power used by each modem separately instead of a constraint on the total power used by all modems together. The capacity region of the MIMO BC under per-antenna total power constraints (in the wireless context) has been shown to be the dual of the MIMO MAC by incorporating an uncertain noise matrix [3]. This leads to a minmax optimization of the uncertain noise matrix and the weighted rate sum over transmit covariance matrices. A less complex algorithm driven only by the maximization of the transmit covariance matrices called BC-optimal spectral balancing (BC-OSB) has been proposed for optimal power allocation under per-modem total power constraints in the xDSL context where the SNR gap is assumed to be zero [4]. Considering a non-zero SNR gap, the BC-OSB becomes sub-optimal since for each tone the decoding orders give different rate sums. However, it has been shown that it is still possible to obtain the same set of rates by the duality between the MIMO MAC and the MIMO BC under the same total power constraint in the context of a practical implementation with a zero forcing-Tomlinson Harashima precoder (ZF-THP), but with a high SNR assumption [5].

In this paper, we extend the work done in [3], [4], [5] and we derive a unified framework that can cope with per-modem total power constraints and a non-zero SNR gap. We generalize the work done in [4] by incorporating the non-zero SNR gap [2] in the duality formulas [1] derived initially for zero SNR gap and total power constraint. The high SNR assumption [5] can be relaxed and the per-modem total power constraints can be met. By the combination of a MIMO MAC algorithm in [8], [9] and the duality between the MIMO MAC and the MIMO BC with modified formulas for the case of non-zero SNR gap, we can compute the transmit covariance matrices for the MIMO BC under per-modem total power constraints achieving the same set of rates as the MIMO MAC.

In section II, we first devise the MIMO MAC and MIMO BC duality transformation formulas under a non-zero SNR gap. Then, in section III, we recall the power allocation algorithm for the MIMO BC under per-modem total power constraints [4] with the incorporation of the non-zero SNR gap [2]. In section IV, simulation results are given for VDSL2 channels, as well as for a bonded very high DSL (VDSL2) scenario exploiting phantom mode (PM) transmission with external noise coming from VDSL2 disturbing lines.

II. Duality under non-zero SNR gap

In this section, we extend the concept of duality between the MIMO MAC and the MIMO BC with zero SNR gap

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initially presented in [1] to the more general duality between the MIMO MAC and the MIMO BC with non-zero SNR gap [2]. We consider a DSL downlink scenario where the Central Office/Remote Terminal (CO/RT) is equipped with T modems serving K users (for instance $T > K$ in a DSL scenario exploiting PM transmission), each user k having a single receiver, and using Discrete Multi-Tone (DMT) modulation with a cyclic prefix longer than the maximum delay spread of the channels. The transmission on tone i can then be modelled as:

$$\mathbf{y}_i = \mathbf{H}_i \mathbf{x}_i + \mathbf{n}_i \quad \text{where} \quad \mathbf{H}_i = \begin{bmatrix} \mathbf{h}_{i1} \\ \vdots \\ \mathbf{h}_{iK} \end{bmatrix} \quad i = 1 \dots N_c \tag{1}$$

- N_c is the number of subcarriers
- $\mathbf{x}_i = \sum_{j=1}^{K} \mathbf{q}_{ij}$ is the $T \times 1$ transmitted vector on tone i
- $\mathbf{q}_{ij}$ is the $T \times 1$ vector of data from the j^{th} user
- $\mathbf{y}_i$ the received signal vector of length $K \times 1$
- $\mathbf{H}_i$ the $K \times T$ MIMO channel matrix
- $\mathbf{h}_{ik}$ the $1 \times T$ MIMO channel matrix for the k^{th} user
- $\mathbf{n}_i$ the vector containing Additive White Gaussian Noise (AWGN)

The dual MIMO MAC can be written as:

$$\mathbf{v}_i = \mathbf{H}_i^H \mathbf{u}_i + \mathbf{w}_i \quad \text{where} \quad \mathbf{H}_i^H = \begin{bmatrix} \mathbf{h}_{i1}^H & \dots & \mathbf{h}_{iK}^H \end{bmatrix} \tag{2}$$

where $\mathbf{u}_i = [u_{i1} \dots u_{iK}]^T$ and u_{ik} is the scalar transmitted on tone i by the k^{th} user. Here $(.)^H$ denotes the hermitian transpose operation and $(.)^T$ the transpose operation. In this paper, we assume $\mathrm{E}[\mathbf{w}_i \mathbf{w}_i^H] = \mathrm{E}[\mathbf{n}_i \mathbf{n}_i^H] = \mathbf{I}$, which can always be obtained after a whitening step. We consider the maximization of the weighted rate sum for queue stability and fair scheduling between users [9]. The weighted rate sum function for the MIMO MAC with a given decoding order $1, \dots, K-1, K$ (i.e. user K is decoded last) and with non-zero SNR gap (assuming the same SNR gap for all the different users) is given by:

$$R^{MAC} = \sum_{k=1}^{K} w_k \sum_{i=1}^{N_c} \log_2 \left[\det \left(\mathbf{I} + \frac{1}{\Gamma_M} \mathbf{B}_{ik}^{-1} \mathbf{h}_{ik}^H \phi_{ik} \mathbf{h}_{ik} \right) \right] \tag{3}$$

- Γ_M the MIMO MAC SNR gap required to achieve a target probability of error for a given modulation and coding scheme
- $\mathrm{E}[u_{ik} u_{ik}^*] = \phi_{ik}$ is the transmit power for user k and tone i
- $\mathbf{B}_{ik} = \mathbf{I} + \sum_{j=k+1}^{K} \mathbf{h}_{ij}^H \phi_{ij} \mathbf{h}_{ij}$

The w_k's are the weights assigned to the different users. In the MAC with non-zero SNR gap, finding the optimal decoding order is a computationally extensive task since for each tone the decoding orders give different rate-sums. A possible approximation is to choose a decoding order defined by the weights, i.e. the user with the largest weight is decoded last [8]. Without loss of generality, we will assume $w_1 < \dots < w_K$. The MAC-BC duality dictates a reversal of the decoding/encoding order, hence in the BC the user with the largest weight has to be encoded first. The weighted rate sum function for the MIMO BC with a given encoding order $K, K-1, \dots, 1$ (i.e. user K is encoded first) and with non-zero SNR gap is then given by:

$$R^{BC} = \sum_{k=1}^{K} w_k \sum_{i=1}^{N_c} \log_2 \left[1 + \frac{1}{\Gamma_B} a_{ik}^{-1} \mathbf{h}_{ik} \mathbf{Q}_{ik} \mathbf{h}_{ik}^H \right] \tag{4}$$

- Γ_B the MIMO BC SNR gap required to achieve a target probability of error for a given modulation and coding scheme
- $\mathrm{E}[\mathbf{q}_{ik} \mathbf{q}_{ik}^H] = \mathbf{Q}_{ik}$, $\mathrm{E}[\mathbf{x}_i \mathbf{x}_i^H] = \sum_{k=1}^{K} \mathbf{Q}_{ik}$ the sum of K transmit covariance matrices for the different users over tone i
- $a_{ik} = 1 + \sum_{j=1}^{k-1} \mathbf{h}_{ik} \mathbf{Q}_{ij} \mathbf{h}_{ik}^H$

We can rewrite the weighted rate sum for the MIMO BC as:

$$R^{BC} = \sum_{k=1}^{K} w_k \sum_{i=1}^{N_c} \log_2 \Big[1 + \tfrac{1}{\sqrt{\Gamma_B}} a_{ik}^{-1/2} \mathbf{h}_{ik} \mathbf{Q}_{ik} \mathbf{h}_{ik}^H a_{ik}^{-1/2} \tfrac{1}{\sqrt{\Gamma_B}} \Big] \tag{5}$$

with $(.)^{-1/2}$ the inverse square root. We can also rewrite the weighted rate sum for the MIMO MAC using $\det(\mathbf{I} + \mathbf{AB}) = \det(\mathbf{I} + \mathbf{BA})$:

$$R^{MAC} = \sum_{k=1}^{K} w_k \sum_{i=1}^{N_c} \log_2 \Big[\det \Big(\mathbf{I} + \tfrac{1}{\sqrt{\Gamma_M \Gamma_B}} a_{ik}^{-1/2} \mathbf{h}_{ik} \mathbf{B}_{ik}^{-1/2} \sqrt{\Gamma_B} a_{ik}^{1/2} \phi_{ik} a_{ik}^{1/2} \sqrt{\Gamma_B} \tfrac{1}{\sqrt{\Gamma_M \Gamma_B}} \mathbf{B}_{ik}^{-1/2} \mathbf{h}_{ik}^H a_{ik}^{-1/2} \Big) \Big] \tag{6}$$

By comparing R^{BC} and R^{MAC}, this leads to the following flipping formulas:

$$\mathbf{Q}_{ik} = \tfrac{\Gamma_B}{\Gamma_M} a_{ik} \phi_{ik} \mathbf{B}_{ik}^{-1} \tag{7}$$

These flipping formulas are much simpler than [1] owing to the DSL context. Therefore the equivalence between the MIMO MAC and the MIMO BC weighted rate sums as well as the equivalence between the total power constraint in the MIMO MAC and the MIMO BC still holds under a non-zero SNR gap if $\Gamma_M = \Gamma_B$. This duality result will be used in the following section to convert transmit covariance matrices in the MIMO MAC to their dual transmit covariance matrices in the MIMO BC[1].

[1] Note that from an implementation point of view, once the covariance matrices $(\mathbf{Q}_{ik})_{i=1\dots N_c}^{k=1\dots K}$ are determined, the transmitted data symbols $\mathbf{x}_i$ can be constructed as follows: 1) The $T \times 1$ vector of the M-QAM data symbols $\mathbf{s}_{ik}$ is precoded using the $T \times T$ $\mathbf{L}_{ik}$ matrix, i.e. $\mathbf{q}_{ik} = \mathbf{L}_{ik} \mathbf{s}_{ik}$, such that $\mathrm{E}[\mathbf{q}_{ik} \mathbf{q}_{ik}^H] = \mathbf{Q}_{ik} = \mathbf{L}_{ik} \mathbf{L}_{ik}^H$ with $\mathbf{L}_{ik}$ the Cholesky function (note that $\mathbf{Q}_{ik}$ is a positive semi-definite matrix). 2) Then $\mathbf{x}_i = \sum_{k=1}^{K} \mathbf{q}_{ik}$ will be sent on the T lines (the t^{th} element of $\mathbf{x}_i$ will be sent on the t^{th} line).

III. Transmit Covariance Matrices for Broadcast Channels under per-modem total power constraints and with non-zero SNR gap

In this section, we recall the power allocation algorithm for the MIMO BC under per-modem total power constraints [4] with the incorporation of the non-zero SNR gap [2]. The primal problem of finding transmit vector covariance matrices in the MIMO BC under a per-modem total power constraint P_t^{tot} for each modem is:

$$\begin{aligned} &\max_{(\mathbf{Q}_{ik})_{i=1\ldots N_c}^{k=1\ldots K}} R^{BC} \\ &\text{subject to} \sum_{k=1}^{K}\sum_{i=1}^{N_c}[\mathbf{Q}_{ik}]_{tt} \leq P_t^{tot}\ \forall t \\ &\mathbf{Q}_{ik} \succeq 0, i=1\ldots N_c, k=1\ldots K \end{aligned} \tag{8}$$

where $[\mathbf{Q}_{ik}]_{tt}$ is the t^{th} diagonal element of the transmit vector covariance matrix for user k over tone i, and R^{BC} is the weighted rate sum function for the MIMO BC given by equation (4). The Lagrangian decouples into a set of N_c smaller problem, thus reducing the complexity of equation (4). The BC dual objective function is:

$$F^{BC}(\mathbf{\Lambda}) = \max_{(\mathbf{Q}_{ik})_{i=1\ldots N_c}^{k=1\ldots K}} \mathcal{L}^{BC}(\mathbf{\Lambda}, (\mathbf{Q}_{ik})_{i=1\ldots N_c}^{k=1\ldots K}) \tag{9}$$

with $\mathbf{\Lambda} = \text{diag}(\lambda_1, \ldots, \lambda_T)$ the Lagrange multipliers associated with the MIMO BC per-modem total power constraints and

$$\begin{aligned} &\mathcal{L}^{BC}(\mathbf{\Lambda}, (\mathbf{Q}_{ik})_{i=1\ldots N_c}^{k=1\ldots K}) = \sum_{i=1}^{N_c}\Bigg(\sum_{k=1}^{K} w_k \log_2\Bigg[\\ &1 + \frac{\mathbf{H}_{ik}\mathbf{Q}_{ik}\mathbf{H}_{ik}^H}{\Gamma_B(\mathbf{I}+\mathbf{H}_{ik}(\sum_{j=1}^{k-1}\mathbf{Q}_{ij})\mathbf{H}_{ik}^H)}\Bigg] - \sum_{k=1}^{K}\text{Trace}(\mathbf{\Lambda}\mathbf{Q}_{ik})\Bigg) \\ &+\text{Trace}\left(\mathbf{\Lambda}\text{diag}(P_1^{tot}, \ldots, P_T^{tot})\right) \end{aligned} \tag{10}$$

The dual optimization problem is:

$$\begin{aligned} &\text{minimize} \quad F^{BC}(\mathbf{\Lambda}) \\ &\text{subject to} \quad \mathbf{\Lambda} \geq \mathbf{0} \end{aligned} \tag{11}$$

The duality between the MIMO MAC and the MIMO BC says that it is possible to achieve the same rates in both domains under the same total power constraint. We will use this argument to find the MIMO BC covariance matrices under per-modem total power constraints. The crucial step is then to rescale the channel matrix by the inverse square root of the Lagrange multiplier matrix as in [4] :

$$\mathbf{y}_i = \overbrace{\mathbf{H}_i\mathbf{\Lambda}^{-1/2}}^{\mathbf{H}'_i}\mathbf{x'}_i + \mathbf{n}_i \tag{12}$$

where $\mathbf{x'}_i = \mathbf{\Lambda}^{1/2}\mathbf{x}_i$. For this rescaled channel, the dual objective function in the BC becomes:

$$\begin{aligned} &\mathcal{L}^{BC}(\mathbf{\Lambda}, (\mathbf{Q'}_{ik})_{i=1\ldots N_c}^{k=1\ldots K}) = \sum_{i=1}^{N_c}\Bigg(\sum_{k=1}^{K} w_k \log_2\Bigg[\\ &1 + \frac{\mathbf{h'}_{ik}\mathbf{Q'}_{ij}\mathbf{h'}_{ik}^H}{\Gamma_B(\mathbf{I}+\mathbf{h'}_{ik}(\sum_{j=1}^{k-1}\mathbf{Q'}_{ij})\mathbf{h'}_{ik}^H)}\Bigg] - \sum_{k=1}^{K}\text{Trace}(\mathbf{Q'}_{ik})\Bigg) \\ &+\text{Trace}\left(\text{diag}(P'^{tot}_1, \ldots, P'^{tot}_T)\right) \end{aligned} \tag{13}$$

where $\mathbf{Q'}_{ik} = \mathbf{\Lambda}^{1/2}\mathbf{Q}_{ik}\mathbf{\Lambda}^{1/2}$ and $\text{diag}(P'^{tot}_1, \ldots, P'^{tot}_T) = \mathbf{\Lambda}^{1/2}\text{diag}(P_1^{tot}, \ldots, P_T^{tot})\mathbf{\Lambda}^{1/2}$. For a given set of Lagrange multipliers, the precoder matrix $\mathbf{\Lambda}^{1/2}$ then effectively transforms the per-modem total power constraints into a total power $\text{Trace}\left(\text{diag}(P'^{tot}_1, \ldots, P'^{tot}_T)\right)$, by hiding the Lagrange multipliers into the rescaled channels $\mathbf{H'}_{ik}$ and the new covariance matrices $\mathbf{Q'}_{ik}$. As the power allocation in the MIMO MAC is tractable [8], [9], one can calculate the transmit powers in the MIMO MAC and then transform these into transmit covariance matrices for the MIMO BC. The dual MIMO MAC, corresponding to the rescaled MIMO BC, can be written as:

$$\mathbf{v}_i = \mathbf{\Lambda}^{-1/2}\mathbf{H}_i^H\mathbf{u'}_i + \mathbf{w}_i \tag{14}$$

The dual objective function in the MIMO MAC then becomes:

$$F^{MAC}(\mathbf{\Lambda}) = \max_{(\mathbf{\Phi'}_{ik})_{i=1\ldots N_c}^{k=1\ldots K}} \mathcal{L}^{MAC}(\mathbf{\Lambda}, (\mathbf{\Phi'}_{ik})_{i=1\ldots N_c}^{k=1\ldots K}) \tag{15}$$

with

$$\begin{aligned} &\mathcal{L}^{MAC}(\mathbf{\Lambda}, (\mathbf{\Phi'}_{ik})_{i=1\ldots N_c}^{k=1\ldots K}) = \sum_{i=1}^{N_c}\Bigg(\sum_{k=1}^{K} w_k \log_2\Bigg[\\ &\det\Bigg(\mathbf{I} + \frac{\mathbf{h'}_{ik}^H\phi'_{ik}\mathbf{h'}_{ik}}{\Gamma_M(\mathbf{I}+\sum_{j=k+1}^{K}\mathbf{h'}_{ij}^H\phi'_{ij}\mathbf{h'}_{ij})}\Bigg)\Bigg] - \sum_{k=1}^{K}\phi'_{ik})\Bigg) \\ &+\text{Trace}\left(\text{diag}(P'^{tot}_1, \ldots, P'^{tot}_T)\right) \end{aligned} \tag{16}$$

The Lagrangian is maximized over the powers in the MIMO MAC by the MAC-iterative spectrum balancing (ISB) algorithm of [8]. When the SNR gap is zero, this algorithm is optimal owing to the convexity of the MIMO MAC problem [9]. However, when the SNR gap is non-zero, this algorithm becomes sub-optimal since for each tone the decoding orders give different rate sums. By setting $\Gamma_B = \Gamma_M$ and for a given $\mathbf{\Lambda}$, we can thus compute the power allocation in the MIMO MAC and use the duality formulas given in section II to obtain the transmit covariance matrices in the MIMO BC that achieve the same rates as in the MIMO MAC and under same total power constraint. The Lagrange multipliers are then adjusted so that the per-modem total power constraints are not violated in the MIMO BC. We define ϵ_t as the tolerance between the actual per-modem total power constraint $\sum_{i=1}^{N_c}[\mathbf{\Lambda}^{-1/2}(\sum_{k=1}^{K}\mathbf{Q'}_{ik})\mathbf{\Lambda}^{-1/2}]_{tt}$ and the target per-modem total power constraint P_t^{tot} for the t^{th} modem. The algorithm for adjusting the Lagrangian multiplier $\mathbf{\Lambda}$ is given in the Annex as Algorithm 1 and leads to the transmit vector covariance matrices in the MIMO BC under per-modem total power constraints.

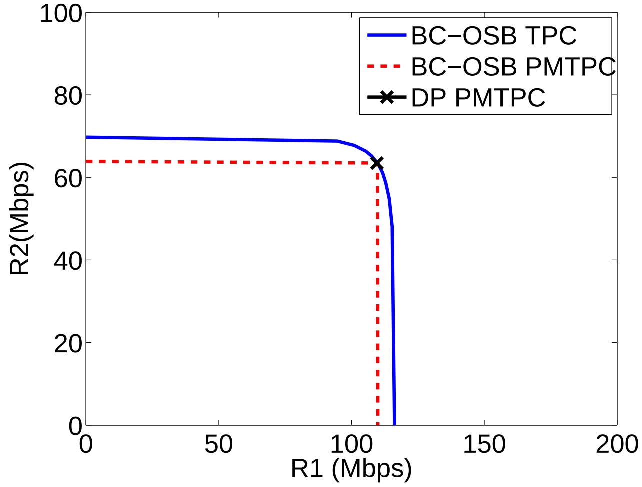

Fig. 1. Rate region of the MIMO BC algorithm in a VDSL2 system

The MIMO BC algorithm under a total power constraint and non-zero SNR gap is a particular case of the MIMO BC algorithm under per-modem total power constraints and non-zero SNR gap. In this case $\mathbf{\Lambda} = \lambda \mathbf{I}$. The same derivation as in the previous section can be given. The different formulas and the algorithm description can be easily modified accordingly.

IV. Results

The first set of simulation results are obtained for VDSL2 measured channels with 2 lines of 400 meters and 800 meters respectively. Spectral masks for VDSL2 fiber to the exchange (FTTex) are used [10], with SNR gap Γ=12.33 dB, an AWGN of -140 dBm/Hz and maximum transmit power P_j^{tot}=14.5 dBm per line. The frequency range is from 0 to 12 MHz with 4.3125 kHz spacing between subcarriers and 4 kHz symbol rate. The FDD band plan of VDSL2 corresponds to 2 frequency bands in the downlink scenario which are 138kHz-3.75MHz and 5.2MHz-8.5MHz.

Figure 1 shows the rates obtained by the MIMO BC algorithm under non-zero SNR gap and a total power constraint (TPC) P^{tot}=29 dBm or per-modem total power constraints (PMTPC) P_j^{tot}=14.5 dBm for the 2 possible detection orders. The difference between the possible detection orders is negligible owing to the diagonal dominance of the channel matrix. For PMTPC, we observe that the rates are independent of the weights given to the different users. This is due to the lower degree of freedom used in the optimization process compared to a total power constraint case. The MIMO BC algorithm under PMTPC and non-zero SNR gap is compared with the diagonalizing precoder (DP) combined with power optimization under PMTPC and non-zero SNR gap [11]. The DP of achieves 109.8 Mbps for the first user and 63.47 Mbps for the second user. Therefore the DP achieves most of the capacity under AWGN again owing to the diagonal dominance of the channel matrix.

The second set of simulation results involves a VDSL2 scenario with a 0 to 30 MHz bandwidth and with differential mode (DM) lines of 400 meters also exploiting phantom mode (PM) transmission [12] and a maximum transmit power P_j^{tot}=14.5 dBm per line. We set the number of transmitters $T = 3$, the number of users $K = 2$. In this case, 2 DM lines and 1 PM line are used for downlink with external noise coming from 2 VDSL2 DM lines of similar length whose PSD's are set at -60 dBm/Hz. We keep the FDD band plan of VDSL2 with similar per-modem total power constraints and we further transmit in the full 12-30 MHz bandwidth.

Rates (Mbps)	SVD TPC	SVD PMTPC	DP	MIMO BC
Total 2x3 Total	324.57	306.54	285.89	295.42

TABLE I
Comparison between different schemes

Table 1 shows a comparison between the MIMO BC algorithm under PMTPC with equal weight $w_1 = w_2 = 0.5$ and existing algorithms [11], [13]. The algorithms using singular value decomposition (SVD) under a TPC and PMTPC provide the optimal rate sum since we have coordination between the T modems at the transmit side and the K modems at the receive side [13]. There is a rate loss between SVD-based schemes (coordination at both sides) and the other schemes (coordination at the transmit side only) owing to the external (colored) noise that cannot be mitigated at the receive side. The MIMO BC algorithm under PMTPC and non-zero SNR gap shows a increased rate compared to the DP combined with power optimization under PMTPC and non-zero SNR gap. In fact, the DP cannot meet the per-modem total power constraint corresponding to the PM transmission due to its linear structure (matrix inversion of a non-square matrix).

V. Conclusion

In this paper, we have derived a unified framework for MIMO BC that can cope with per-modem total power constraints and non-zero SNR gap. Simulation results were given for VDSL2 channels as well as for a bonded VDSL2 scenario exploiting Phantom Mode (PM) transmission with external noise coming from VDSL2 disturbing lines. The MIMO BC algorithm did not show any significant improvement compared to existing algorithms like DP in the VDSL2 scenario with AWGN owing to the diagonal dominance of the channel matrix. However, when the VDSL2 channel matrix is non-square (i.e. the number of transmitters is larger than the number of users as for an extra differential mode line or phantom mode transmission), the MIMO BC algorithm shows better results than DP.

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ANNEX

Algorithm 1 MIMO BC algorithm under PMTPC and non-zero SNR gap

for $t = 1 \dots T$
    init $\lambda_t = 1$
    init $step_t = 2$
    init $b_t = 0$
    init $(\mathbf{Q'}_{ik})_{i=1\dots N_c}^{k=1\dots K} = \mathbf{0}$
end for
while $\exists t$ s.t. $|\sum_{i=1}^{N_c}[\mathbf{\Lambda}^{-1/2}(\sum_{k=1}^{K}\mathbf{Q'}_{ik})\mathbf{\Lambda}^{-1/2}]_{tt} - P_t^{tot}| > \epsilon_t$
    Iterative search $\max_{(\mathbf{\Phi'}_i)_{i=1\dots N_c}} \mathcal{L}^{MAC}(\mathbf{\Lambda}, (\mathbf{\Phi'}_i)_{i=1\dots N_c})$
    MAC-BC Duality
    for $k = 1$ to K $\forall i = 1 \dots N_c$
        $a'_{ik} = 1 + \mathbf{h'}_{ik}(\sum_{j=1}^{k-1}\mathbf{Q'}_{ij})\mathbf{h'}_{ik}^H$
        $\mathbf{B'}_{ik} = \mathbf{I} + \sum_{j=k+1}^{K}\mathbf{h'}_{ij}^H\phi'_{ij}\mathbf{h'}_{ij}$
        $\mathbf{Q'}_{ik} = a'_{ik}\phi'_{ik}\mathbf{B'}_{ik}^{-1}$
    end for
    if $\sum_{i=1}^{N_c}[\mathbf{\Lambda}^{-1/2}(\sum_{k=1}^{K}\mathbf{Q'}_{ik})\mathbf{\Lambda}^{-1/2}]_{tt} - P_t^{tot} < 0$
        $b_t = b_t + 1$
        $\lambda_t = \lambda_t / step_t$
        $step_t = step_t - 1/2^{b_t}$
    end if
    $\lambda_t = \lambda_t * step_t$
end while