When does vectored Multiple Access Channels (MAC) optimal power allocation converge to an FDMA solution?

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***Abstract*— Vectored Multiple Access Channels (MAC) have attracted a lot of interest during the past few years. The optimal structure of vectored MAC in the uplink is based on the Successive Interference Canceller-Minimum Mean Square Error (SIC-MMSE). For equal weights and zero Signal to Noise Ratio (SNR) gap, the optimal transmit covariance matrices are found by iterative waterfilling. However, practical scenarios include non-zero SNR gap required to achieve the target probability of error at the desired data rate. With non-zero SNR gap, it is possible to achieve a weighted rate sum higher than iterative waterfilling by means of MAC-Optimal Spectrum Balancing (MAC-OSB). Moreover, it has been shown in the literature that the optimal power allocation for single-carrier flat scalar MAC with non-zero SNR gap is given by a Frequency Division Multiple Access (FDMA) type solution. In this paper, we investigate the problem of optimal power allocation for multi-carrier vectored MAC with non-zero SNR gap. Simulation results are given for VDSL2 channels and wireless channels. We confirm by simulations that FDMA is indeed the optimal power allocation in multi-carrier scalar MAC systems. When extending to multi-carrier vectored MAC systems, the optimal power allocation will tend towards a shared spectrum solution or a FDMA type solution depending on the level of crosstalk, the SNR, the SNR gap and the power constraints.**

I. INTRODUCTION

Vectored Multiple Access Channels (MAC) have attracted a lot of interest during the past few years [1], [2]. The optimal receiver structure of vectored MAC in the uplink is based on the the Successive Interference Canceller-Minimum Mean Square Error (SIC-MMSE) [1]. For equal weights and zero Signal to Noise Ratio (SNR) gap, the optimal transmit covariance matrices are found by iterative waterfilling [2]. However, practical scenarios include non-zero SNR gap required to achieve the target probability of error at the desired data rate. With non-zero SNR gap, it is possible to achieve a weighted rate sum higher than iterative waterfilling by means of MAC-Optimal Spectrum Balancing (MAC-OSB) [3]. In a recent paper, it has been shown that the optimal power allocation for single-carrier flat scalar Multiple Access Channels (MAC) with non-zero SNR gap is given by an Frequency Division Multiple Access (FDMA) type solution [4].

In this paper, we investigate the problem of optimal power allocation for multi-carrier vectored MAC with non-zero SNR gap. Therefore we extend the single-carrier flat scalar MAC with non-zero SNR gap to the multi-carrier vectored MAC with non-zero SNR gap. In the vectored MAC case, it is shown that the crosstalk plays an important role in the optimal power allocation. The optimal power allocation can tend towards a FDMA type solution or a shared solution of each tone between users, depending on the level of crosstalk scenario, the SNR, the SNR gap and the power constraints. Using the Lagrange multipliers in the vectored MAC objective function, we study the convergence of the multi-carrier vectored MAC with non-zero SNR gap toward these two solutions depending on these different parameters.

In section II, we first recall the comparison between the weighted rate sums for single-carrier systems FDMA, scalar MAC and vectored MAC with non-zero SNR gap. In section III, we extend the different weighted rate sums from the single-carrier case to the multi-carrier case. In fact, the primal vectored MAC capacity optimization problem subject to per-modem total power constraints and non-zero SNR gap is transformed into a collection of per-tone unconstrained vectored MAC capacity optimization problems using Lagrangian parameters. We derive optimal receiver structures in combination with optimal transmit covariance matrices which achieve vectored MAC channel capacity. Simulation results are given for VDSL2 channels and wireless channels. We confirm by simulations that FDMA is indeed the optimal power allocation in multi-carrier scalar MAC systems. When extending to multi-carrier MIMO MAC systems, the optimal power allocation will tend towards a shared spectrum solution or a FDMA type solution depending on the level of crosstalk, the SNR, the SNR gap and the power constraints.

We consider a vectored MAC with T receivers and K users, each user k having a single transmitter in an uplink scenario and using Discrete Multi-Tone (DMT) modulation with a cyclic prefix longer than the maximum delay spread of the channels. The transmission on tone i can then be modelled as:

$$\mathbf{v}_i = \mathbf{H}_i^H \mathbf{u}_i + \mathbf{w}_i \quad \text{where} \quad \mathbf{H}_i^H = \left[\begin{array}{ccc} \mathbf{h}_{i1}^H & \dots & \mathbf{h}_{iK}^H \end{array} \right] \tag{1}$$

where N_c is the number of subcarriers, $\mathbf{v}_i$ is the received

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vector of length T, $\mathbf{u}_i = [u_{i1} \dots u_{iK}]^T$ is the transmitted vector with $\phi_{ik} = E[u_{ik}u_{ik}^H]$ the transmitted power for user k and tone i, $\mathbf{H}_i$ the $K \times T$ Mutiple nput Multiple Output (MIMO) channel matrix and $\mathbf{w}_i$ the vector containing Additive White Gaussian Noise (AWGN). In this paper, we assume $E[\mathbf{w}_i\mathbf{w}_i^H] = E[\mathbf{n}_i\mathbf{n}_i^H] = \mathbf{I}$, which can always be obtained after a whitening step.

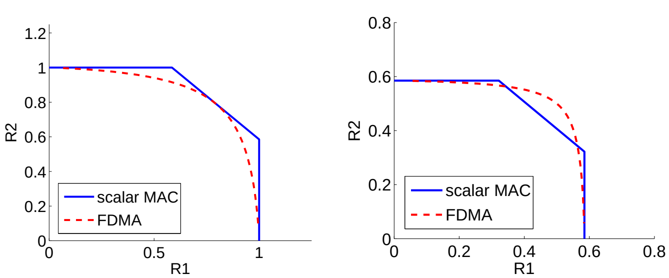
II. Comparison between weighted rate sums of vectored FDMA and vectored MAC in single-carrier systems with flat channels

In this part, we consider the weighted rate sum for single carrier-systems with flat channels and flat power allocation. The w_k's are the weights assigned to the different users. Assuming $w_1 < \dots < w_K$, we consider the weighted rate sum function of scalar MAC for a given decoding order 1,...,K-1,K (i.e. user K is decoded last) with non-zero SNR gap. We set the number of users $K = 2$ and the number of receivers $T = 1$. Therefore the 2×1 channel matrix is represented by $\mathbf{H} = [h_1 h_2]^T$. For single carrier-systems with flat channels and flat power allocation, the weighted rate sums in the 2-user case for the scalar MAC (shared solution with SIC-MMSE) are:

$$\begin{aligned} R_A^{MAC} &= w_1 log_2\left(1 + \tfrac{1}{\Gamma}\phi_1|h_1|^2\right) + \\ & w_2 log_2\left(1 + \tfrac{1}{\Gamma}\tfrac{\phi_2|h_2|^2}{1+\phi_1|h_1|^2}\right) \\ R_B^{MAC} &= w_2 log_2\left(1 + \tfrac{1}{\Gamma}\phi_2|h_2|^2\right) + \\ & w_1 log_2\left(1 + \tfrac{1}{\Gamma}\tfrac{\phi_1|h_1|^2}{1+\phi_2|h_2|^2}\right) \end{aligned} \tag{2}$$

The rate region for FDMA (or TDMA) is characterized by :

$$\begin{aligned} R^{FDMA} &= \alpha log_2\left(1 + \tfrac{1}{\alpha\Gamma}\phi_1|h_1|^2\right) + \\ & (1-\alpha) log_2\left(1 + \tfrac{1}{(1-\alpha)\Gamma}\phi_2|h_2|^2\right) \end{aligned} \tag{3}$$



(a) Γ=0 dB (b) Γ= 6 dB

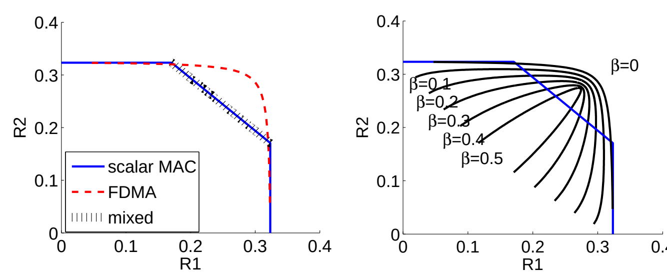
Fig. 1. Rate regions of scalar MAC and FDMA with zero and non-zero SNR gaps

The rate regions for FDMA and scalar MAC with zero SNR gap are shown on the left part of Figure 1. The corner points A and B of the scalar MAC rate region correspond to the different detection orders. The rate-sum for the two points is the same (angle of connecting line between A and B is 45 degrees). An in-between point on the line connecting A and B can be reached by time-sharing between two SIC's with different detection orders. The rate regions for FDMA and scalar MAC with non-zero SNR gap ($\Gamma = 6dB$) are shown on the right part of Figure 1. For the scalar MAC, inserting a non-zero SNR gap does not lead to a constant rate-sum (the line connecting the two points has an angle different from 45 degrees). Moreover, one can see that points outside the rate region of the scalar MAC can be reached by a FDMA strategy.

Now we try to give to the scalar MAC the required additional flexibility such that it can compute the FDMA points. For instance, in the 2 users case, in the α bandwidth (or time if TDMA is considered) the power loadings for the corner point A are given by (ϕ_{1A}, ϕ_{2A}) and in the $(1-\alpha)$ bandwidth the power loadings for the corner point B are given by (ϕ_{1B}, ϕ_{2B}). This mixed weighted rate sum is given by:

$$\begin{aligned} & R^{mixed} = \alpha\Big(w_1 log_2(1 + \tfrac{1}{\alpha\Gamma}\phi_{1A}|h_1|^2) + \\ & w_2 log_2(1 + \tfrac{1}{\alpha\Gamma}\tfrac{\phi_{2A}|h_2|^2}{1+\frac{1}{\alpha}\phi_{1A}|h_1|^2})\Big) + \\ & (1-\alpha)\Big(w_2 log_2(1 + \tfrac{1}{(1-\alpha)\Gamma}\phi_{2B}|h_2|^2) + \\ & w_1 log_2(1 + \tfrac{1}{(1-\alpha)\Gamma}\tfrac{\phi_{1B}|h_1|^2}{1+\frac{1}{(1-\alpha)}\phi_{2B}|h_2|^2})\Big) \end{aligned} \tag{4}$$

The left part of Figure 2 shows the rate region of the updated scalar MAC with non-zero SNR gap ($\Gamma = 12dB$) where the power loadings correspond to their respective bandwidth $\phi_{1A} = \alpha\phi_1$, $\phi_{1B} = (1-\alpha)\phi_1$, $\phi_{2A} = \alpha\phi_2$, and $\phi_{2B} = (1-\alpha)\phi_2$. One can see that the scalar MAC corner points A and B are found by setting α =0 or α =1. The right part of Figure 2 shows the rate region of the updated scalar MAC with non-zero SNR gap ($\Gamma = 12dB$) where the power loadings correspond to $\phi_{1A} = \beta\phi_1$, $\phi_{1B} = (1-\beta)\phi_1$), $\phi_{2A} = (1-\beta)\phi_2$, $\phi_{2B} = \beta\phi_2$). One can see that the FDMA points are found by setting β =0 or β =1 corresponding to $\phi_{1A} * \phi_{1B} = 0$, $\phi_{2A} * \phi_{2B} = 0$, $\phi_{1A} * \phi_{2A} = 0$, $\phi_{1B} * \phi_{2B} = 0$.



(a) Γ = 12 dB (b) Γ = 12 dB

Fig. 2. Updated scalar MAC rate region with two different power loadings strategies

Now we extend these results to the vectored MAC and vectored FDMA. The weighted rate sum of the vectored MAC is:

$$\begin{aligned} R_A^{MAC} &= w_1 log_2\left[det\left(\mathbf{I} + \tfrac{1}{\Gamma}\mathbf{h}_1^H\phi_1\mathbf{h}_1\right)\right] + \\ & w_2 log_2\left[det\left(\mathbf{I} + \tfrac{1}{\Gamma}\tfrac{\mathbf{h}_2^H\phi_2\mathbf{h}_2}{\mathbf{I}+\mathbf{h}_1^H\phi_1\mathbf{h}_1}\right)\right] \\ R_B^{MAC} &= w_2 log_2\left[det\left(\mathbf{I} + \tfrac{1}{\Gamma}\mathbf{h}_2^H\phi_2\mathbf{h}_2\right)\right] + \\ & w_1 log_2\left[det\left(\mathbf{I} + \tfrac{1}{\Gamma}\tfrac{\mathbf{h}_1^H\phi_1\mathbf{h}_1}{\mathbf{I}+\mathbf{h}_2^H\phi_2\mathbf{h}_2}\right)\right] \end{aligned} \tag{5}$$

The rate region for the vectored FDMA is characterized by :

$$R^{FDMA} = \alpha log_2\Big[det\left(\mathbf{I}+\tfrac{1}{\alpha\Gamma}\mathbf{h}_1^H\phi_1\mathbf{h}_1\right)\Big]+ (1-\alpha)log_2\Big[det\left(\mathbf{I}+\tfrac{1}{(1-\alpha)\Gamma}\mathbf{h}_2^H\phi_2\mathbf{h}_2\right)\Big] \quad (6)$$

The optimality of vectored FDMA or vectored MAC depends on the SNR gap, the SNR and the level of crosstalk. In the Figure 3 we set the number of users $K = 2$ and the number of receivers $T = 2$. Therefore the $2\times$ 2 channel matrix is represented by $\mathbf{H} = [\mathbf{h}_1\mathbf{h}_2]^T$, with $\mathbf{h}_1 = [h_{11}h_{12}]$ and $\mathbf{h}_2 = [h_{21}h_{22}]$. We can observe on this figure that while FDMA achieve higher rates compared to vectored MAC with strong crosstalk (crosstalk channels having the same power spectral density as direct channels $|h_{12}|^2 = |h_{11}|^2$ and $|h_{21}|^2 = |h_{22}|^2$), with weaker crosstalk channels vectored MAC offers a higher weighted rate sum ($|h_{12}|^2 = |h_{21}|^2 = 0$). Therefore there exists a point of crosstalk level where the vectored MAC becomes the optimal solution compared to vectored FDMA. It is possible to calculate the level of crosstalk between the optimal power allocation by vectored FDMA and vectored MAC by solving numerically the equivalence $R^{MAC} = R^{FDMA}$. The boundary between vectored MAC and vectored FDMA according to the crosstalk level $\rho = |h_{12}|/|h_{11}| = |h_{21}|/|h_{22}|$ is shown on Figure 4, where we vary the SNR gap and the SNR. The main conclusion of this figure is that the level of crosstalk should be quite high in order to have an OFDMA type solution as the optimal power allocation (with SNR=40dB and Γ=20 dB the level of crosstalk $\rho = 0.5$)

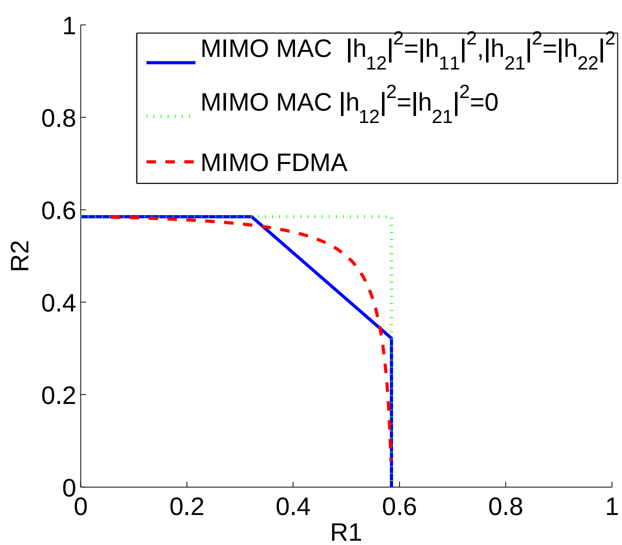


Fig. 3. $\Gamma > 0$ dB

As for the scalar MAC, we can add to the vectored MAC the required additional flexibility such that it can compute the FDMA points. As previously for the 2 users case, in the α bandwidth the power loadings for the corner point A are given by (ϕ_{1A}, ϕ_{2A}) and in the $(1-\alpha)$ bandwidth the power loadings for the corner point B are given by (ϕ_{1B}, ϕ_{2B}). The mixed

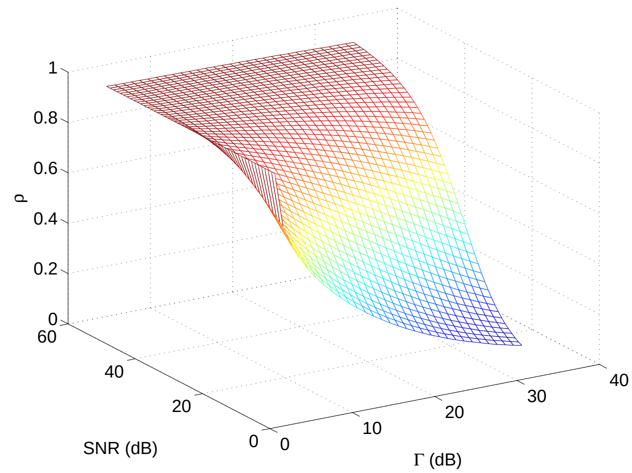


Fig. 4. $\Gamma > 0$ dB

weighted rate sum is given by:

$$R^{mixed} = \alpha\Big(w_1log_2\Big[det\left(\mathbf{I}+\tfrac{1}{\alpha\Gamma}\mathbf{h}_1^H\phi_{1A}\mathbf{h}_1\right)\Big]+ w_2log_2\Big[det\left(\mathbf{I}+\tfrac{1}{\alpha\Gamma}\tfrac{\mathbf{h}_2^H\phi_{2A}\mathbf{h}_2}{\mathbf{I}+\mathbf{h}_1^H\frac{1}{\alpha}\phi_{1A}\mathbf{h}_1}\right)\Big]\Big)+ (1-\alpha)\Big(w_2log_2\Big[det\left(\mathbf{I}+\tfrac{1}{(1-\alpha)\Gamma}\mathbf{h}_2^H\phi_{2B}\mathbf{h}_2\right)\Big]+ w_1log_2\Big[det\left(\mathbf{I}+\tfrac{1}{(1-\alpha)\Gamma}\tfrac{\mathbf{h}_1^H\phi_{1B}\mathbf{h}_1}{\mathbf{I}+\mathbf{h}_2^H\frac{1}{(1-\alpha)}\phi_{2B}\mathbf{h}_2}\right)\Big]\Big) \quad (7)$$

We can conclude the same conclustions for the updated vectored MAC with non-zero SNR gap as for the updated scalar MAC with non-zero SNR gap. Indeed, the vectored MAC corner points A and B are found by setting α =0 or α =1. In case of zero crosstalk channels ($|h_{12}|^2 = |h_{21}|^2 = 0$) the updated vectored MAC will provide a single optimal point (green curve on Figure 4). However, as the SNR gap is non-zero, the corner points A and B do not lead to the same rate sum. The FDMA points can be found by setting β =0 or β =1 to the power loadings $\phi_{1A} = \beta\phi_1$, $\phi_{1B} = (1-\beta)\phi_1$, $\phi_{2A} = (1-\beta)\phi_2$, $\phi_{2B} = \beta\phi_2$ whenever the crosstalk is large. This leads to non-overlapping spectra of the different users $\phi_{1A} * \phi_{1B} = 0$, $\phi_{2A} * \phi_{2B} = 0$, $\phi_{1A} * \phi_{2A} = 0$, $\phi_{1B} * \phi_{2B} = 0$.

III. Updated Vectored MAC-Optimal Spectrum Balancing (MAC-OSB) under per-modem total power constraints with non-zero SNR gap

In this part we investigate the problem of optimal power allocation for multi-carrier vectored MAC with non-zero SNR gap. We add to the vectored MAC-OSB [3] the required additional flexibility such that it can compute the FDMA points and compute the best corner point for each tone. The primal problem of finding optimal power allocations in the

MAC under a per-modem total power constraint P_k^{tot} is:

$$\begin{aligned} &\max_{\phi_{ik},\alpha} \sum_{i=1}^{N_c} R_i^{mixed} \\ &\text{subject to} \sum_{i=1}^{N_c} \phi_{ik} \leq P_k^{tot}\ \forall k \\ &\phi_{ik} \succeq 0, i = 1 \ldots N_c, k = 1 \ldots K \end{aligned} \tag{8}$$

with $\mathbf{\Phi}_{ik} = E[\mathbf{u}_{ik}\mathbf{u}_{ik}^H]$ the covariance matrix of transmitted symbols for user k over tone i. Assuming $w_1 < \cdots < w_K$, the weighted rate sum function considering MIMO MAC for a given decoding order 1,...,K-1,K (i.e. user K is decoded last) with non-zero SNR gap is given by:

$$C^{MAC} = \sum_{k=1}^{K} w_k \sum_{i=1}^{N_c} log_2 \left[det \left(\mathbf{I} + \frac{1}{\Gamma} \mathbf{B}_{ik}^{-1} \mathbf{H}_{ik}^H \mathbf{\Phi}_{ik} \mathbf{H}_{ik} \right) \right] \tag{9}$$

with Γ the SNR multiplier required to achieve the target probability of error at the desired data rate in the MIMO MAC, and $\mathbf{B}_{ik} = \mathbf{I} + \sum_{j=k+1}^{N} \mathbf{H}_{ij}^H \mathbf{\Phi}_{ij} \mathbf{H}_{ij}$. The above formula corresponds to the operation of SIC-MMSE which is an optimal receiver for given transmit powers. The idea of dual decomposition is to solve the primal problem by its Lagrangian (dual problem) and leads to the MAC-OSB solution algorithm [3]. The Lagrangian decouples into a set of N_c smaller problem, thus reducing the complexity of equation of the primal problem. By defining $\mathbf{\Lambda} = diag(\mathbf{\Lambda}_1, \ldots, \mathbf{\Lambda}_K)$ and $\mathbf{\Lambda}_k$ a diagonal matrix of Lagrange multipliers $diag(\lambda_{k1}, \ldots, \lambda_{kM_k})$, the MAC dual objective function is:

$$F^{MAC}(\mathbf{\Lambda}) = \max_{(\mathbf{\Phi}_{ik})_{i=1\ldots N_c}^{k=1\ldots K}} \mathcal{L}(\mathbf{\Lambda}, (\mathbf{\Phi}_{ik})_{i=1\ldots N_c}^{k=1\ldots K}) \tag{10}$$

with

$$\begin{aligned} &\mathcal{L}(\mathbf{\Lambda}, (\mathbf{\Phi}_{ik})_{i=1\ldots N_c}^{k=1\ldots K}) = \sum_{i=1}^{N_c} \left(\sum_{k=1}^{K} w_k log_2 \Bigg[\right. \\ &det \left(\mathbf{I} + \frac{\mathbf{H}_{ik}^H \mathbf{\Phi}_{ik} \mathbf{H}_{ik}}{\Gamma(\mathbf{I} + \sum_{j=k+1}^{K} \mathbf{H}_{ij}^H \mathbf{\Phi}_{ij} \mathbf{H}_{ij})} \right) \Bigg] - \left. \sum_{k=1}^{K} Trace(\mathbf{\Lambda}_k \mathbf{\Phi}_{ik}) \right) \\ &+ \sum_{k=1}^{K} Trace(\mathbf{\Lambda}_k diag(P_{k1}^{tot}, \ldots, P_{kM_k}^{tot})) \end{aligned} \tag{11}$$

The dual optimization problem is:

$$\begin{aligned} \text{minimize} \quad & F^{MAC}(\mathbf{\Lambda}) \\ \text{subject to} \quad & \mathbf{\Lambda} \geq \mathbf{0} \end{aligned} \tag{12}$$

By tuning the Lagrange multipliers, the per-modem total power constraints can be enforced. If the SNR gap is zero, for given Tx powers the detection order does not change the rate sum. Hence in a weighted rate sum, the weights define the detection order. Moreover, it has been shown in [5] that the dual optimization problem is a convex problem, therefore simple iterative algorithms can be used to find the optimal solution. With non-zero SNR-gap, however, for given Tx powers the detection order (in general) does change the rate sum. In this case, an exhaustive search on the different power loadings can be done [3]. The search for the optimal $\mathbf{\Lambda}$ involves evaluations of the dual objective function, i.e. maximizations of the Lagrangian, which is decoupled over the tones for a given $\mathbf{\Lambda}$. The optimization problem of MAC-OSB is solved by an exhaustive search on a per-tone basis.

For the scalar MAC-OSB algorithm, we set the number of users $K = 2$ with the number of transmitters for user 1 and 2 $M_1 = 1$ and $M_2 = 1$ respectively and the number of receivers $T = 1$. Therefore the 2×1 channel matrix is represented by $\mathbf{H}_i = [h_{i1} h_{i2}]^T$. The Lagrangian becomes:

$$\begin{aligned} &\mathcal{L}(\mathbf{\Lambda}, (\phi_{i1}, \phi_{i2})_{i=1\ldots N_c}) = \sum_{i=1}^{N_c} \left(w_1 log_2 \left(1 + \frac{\phi_{i1}|h_{i1}|^2}{\Gamma(1+\phi_{i2}|h_{i2}|^2)} \right) \right. \\ &\left. + w_2 log_2 \left(1 + \frac{\phi_{i2}|h_{i2}|^2}{\Gamma} \right) - \lambda_1 \phi_{i1} - \lambda_2 \phi_{i2} \right) \\ &+ \lambda_1 P_1^{tot} + \lambda_2 P_2^{tot} \end{aligned} \tag{13}$$

IV. Simulation Results

Simulations results are obtained on VDSL2 measured channels with 2 lines of 400 meters. An AWGN of -140 dBm/Hz and maximum transmit power P_j^{tot}=14.5 dBm per line are used. The frequency range is from 0 to 30 MHz with 4.3125 kHz spacing between subcarriers and 4 kHz symbol rate. The FDD band plan of VDSL2 corresponds to 3 frequency bands in the uplink scenario which are 25-138kHz, 3.75-5.2MHz and 8.5-30MHz. The optimal power allocation for the scalar MAC-OSB algorithm is given on Figure 5 for Γ=0 dB and on Figure 6 for Γ=10.8 dB. We can see on these figures that increasing the SNR gap indeed leads to an FDMA solution using the scalar MAC-OSB algorithm. The weighted rate sum ($w_1 = w_2$=0.5) for scalar MAC-OSB and Γ=0 dB is $w_1 R_1 + w_2 R_2$=138.39 Mbps with R_1=56.09 Mbps and R_2=220.69 Mbps. The weighted rate sum ($w_1 = w_2$=0.5) for scalar MAC-OSB and Γ=10.8 dB is $w_1 R_1 + w_2 R_2$=98.95 Mbps with R_1=94.66 Mbps and R_2=103.25 Mbps and .

For the MIMO MAC-OSB algorithm, we set the number of users $K = 2$ with the number of transmitters for user 1 and 2 $M_1 = 1$ and $M_2 = 1$ respectively and the number of receivers $T = 2$. Therefore the 2×2 channel matrix is represented by $\mathbf{H}_i = [\mathbf{h}_{i1} \mathbf{h}_{i2}]^T$, , with $\mathbf{h}_{i1} = [h_{i11} h_{i12}]$ and $\mathbf{h}_{i2} = [h_{i21} h_{i22}]$. The Lagrangian becomes:

$$\begin{aligned} &\mathcal{L}(\mathbf{\Lambda}, (\phi_{i1}, \phi_{i2})_{i=1\ldots N_c}) = \sum_{i=1}^{N_c} \left(w_1 log_2 \Bigg[\right. \\ &det \left(\mathbf{I} + \frac{\mathbf{h}_{i1}^H \phi_{i1} \mathbf{h}_{i1}}{\Gamma(\mathbf{I} + \mathbf{h}_{i2}^H \phi_{i2} \mathbf{h}_{ij})} \right) \Bigg] \\ &\left. + w_2 log_2 \left[det \left(\mathbf{I} + \frac{\mathbf{h}_{i2}^H \phi_{i2} \mathbf{h}_{i2}}{\Gamma} \right) \right] - \lambda_1 \phi_{i1} - \lambda_2 \phi_{i2} \right) \\ &+ \lambda_1 P_1^{tot} + \lambda_2 P_2^{tot} \end{aligned} \tag{14}$$

Simulations results are obtained on VDSL2 measured channels with 2 lines of 400 meters. The optimal power allocation for the MIMO MAC-OSB algorithm is given on Figure 7 for Γ=10.8 dB because we obtain the same set of PSD's over the tones with Γ=10.8 dB or Γ=0 dB. Therefore, with weak

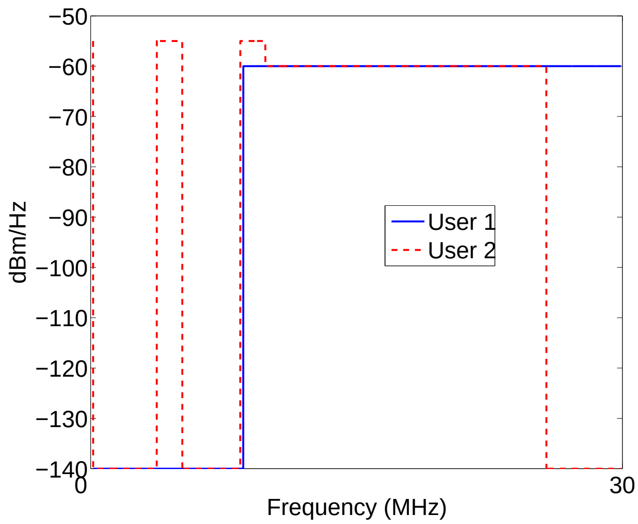


Fig. 5. Γ=0 dB

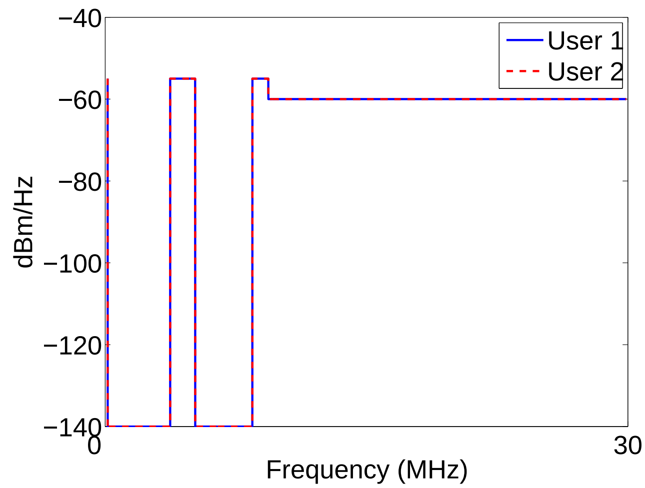


Fig. 7. Γ=10.8 dB

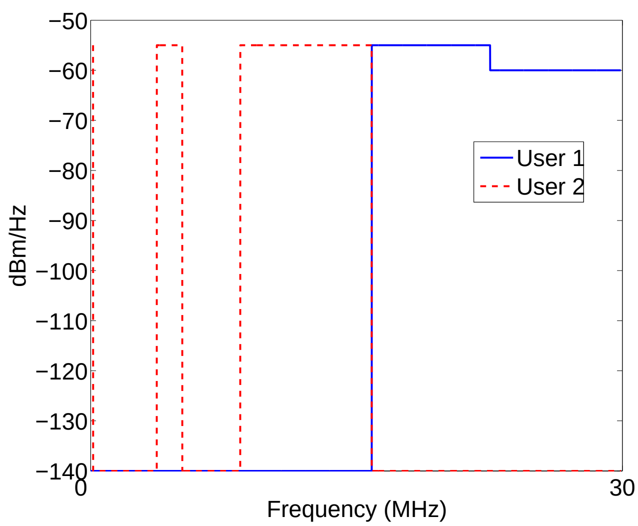


Fig. 6. Γ=10.8 dB

crosstalk channels as in VDSL2 and high SNR, the optimal solution is not an FDMA-type solution. The weighted rate sum ($w_1 = w_2$=0.5) for MIMO MAC-OSB and Γ=0 dB is $w_1R_1 + w_2R_2$=254.14 Mbps with R_1=254.95 Mbps and R_2=253.33 Mbps. The weighted rate sum ($w_1 = w_2$=0.5) for MIMO MAC-OSB and Γ=10.8 dB is $w_1R_1 + w_2R_2$=177.90 Mbps with R_1=177.11 Mbps and R_2=178.69 Mbps. If we consider the crosstalk channels having the same power spectral density as direct channels $|h_{12}|^2 = |h_{11}|^2$ and $|h_{21}|^2 = |h_{22}|^2$ for Γ=10.8 dB, we obtain the same FDMA-type solution as Figure 5.

The last simulation results are obtained on complex gaussian channels of mean 0 and variance 1 constant over 100 tones for direct and crosstalk channels, keeping the same FDD band plan for VDSL2 in the uplink. Again, we set the number of users $K = 2$ with the number of transmitters for user 1 and 2 $M_1 = 1$ and $M_2 = 1$ respectively and the number of receivers $T = 2$. The optimal power allocation for the MIMO MAC-OSB algorithm is given on Figure 8 for Γ=10.8 dB with an AWGN of -50 dBm/Hz giving a low SNR. Therefore, for complex gaussian channels with low SNR and high crosstalk, the optimal power allocation is given by an FDMA type solution. To summarize, FDMA is the optimal power allocation in multi-carrier scalar MAC systems. When extending to multi-carrier MIMO MAC systems, the optimal power allocation tends towards a shared spectrum solution for high SNR and low crosstalk. However, some simulations showed that the multi-carrier MIMO MAC-OSB algorithm can tend towards an FDMA type solution with low SNR and high crosstalk. Moreover, as multi-carrier scalar MAC systems, non-square systems with $T < K$ will likely lead to an FDMA type solution.

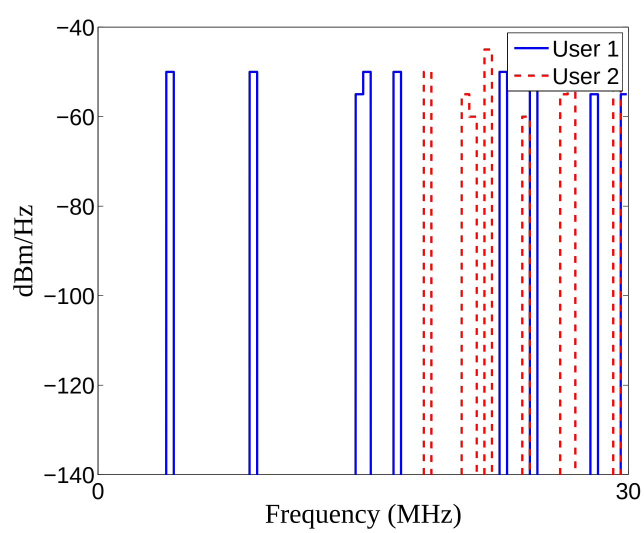


Fig. 8. Γ=10.8 dB

V. Conclusion

In this paper, we investigated the problem of optimal power allocation for multi-carrier MIMO MAC with non-zero SNR gap. Simulation results were given for VDSL2 channels and wireless channels. We saw that FDM is the optimal power allocation in multi-carrier scalar MAC systems. When extending to multi-carrier MIMO MAC systems, the optimal power allocation likely tends towards a shared spectrum solution for VDSL2 channels due to high SNR and low crosstalk. However, the multi-carrier MIMO MAC-OSB algorithm can tend towards an FDMA type solution for wireless channels with low SNR and high crosstalk. As multi-carrier scalar MAC systems, non-square systems with $T < K$ will likely lead to an FDMA type solution.

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