Design of algorithms for Dynamic Spectrum Management (DSM) level-3 in xDSL: Comparison and application to phantom mode signals

Vincent Le Nir, Marc Moonen, Mamoun Guenach

Abstract— **In xDSL, the information is carried differentially through two copper wires. This transmission mode is called Differential-Mode (DM). While Common-Mode reception can mitigate part of the colored noise, the major drawback of CM transmission is the egress caused to other lines. The use of Multiple Input Multiple Output (MIMO) binder allows to further use a differential signal between two common modes referred to as Phantom Mode (PM) tranmission causing no significant egress to other lines. PM transmission increases the capacity by a factor (2N-1)/N (N the number of differential lines). In this paper we study three types of algorithms for optimal power allocation in MIMO systems with colored noise using PM transmission under practical implementation (per-modem total power constraints and non-zero SNR gap). First, the two-sided coordination case is studied leading to a SVD-based algorithm. Then, one-sided algorithms for Multiple Access Channels (MAC) and Broadcast Channels (BC) are investigated. Finally, we compare these Dynamic Spectrum Management (DSM) level-3 algorithm with a DSM level-2 algorithm for Interference Channels (IC). The three types of algorithms are compared in the simulation results applying a VDSL2 scenario.**

Index Terms— **MIMO systems, Optimization methods**

I. Introduction

Triple-play services fo xDSL calls for new techniques increasing capacity and performance in the physical layer. In xDSL, the information is carried differentially through two copper wires. This tranmission mode is called Differential-Mode (DM). Moreover, the Common-Mode (CM) can be further used at the receive side in order to mitigate colored noise coming from various sources (Radio Frequency Interference (RFI), alien crosstalk etc.) [1], [2]. However, CM transmission would result into a non-compliance with electromagnetic compatibility owing to the egress. Recently, Phantom Mode (PM) tranmission has been investigated for Multiple Input Multiple Output (MIMO) binders [3], [4]. PM transmission consists of using a differential signal between two common modes (see Fig. 1) causing no significant egress to other lines. Considering a MIMO binder with N differential lines, PM transmission increases the capacity by a factor (2N-1)/N.

In this paper we study three types of algorithms for optimal power allocation in MIMO systems with colored noise using PM reception and/or tranmission under practical implementation (per-modem total power constraints and non-zero SNR gap). First, the two-sided coordination structure is studied where the Channel State Information (CSI) is available at both the transmitter and the receiver. We extend the convential SVD algorithm with standard waterfilling under a total power constraint [5] to a SVD algorithm under more realistic per-modem total power constraints as first presented in [6]. Then, one-sided coordination structures for Multiple Access Channels (MAC) and Broadcast Channels (BC) are investigated. In this case, CSI is available only at the transmitter for BC or only at the receiver for MAC. We implement the MAC-Optimal Spectrum Balancing (MAC-OSB) algorithm [7] and the BC-Optimal Spectrum Balancing (BC-OSB) algorithm under per-modem total power constraints and non-zero SNR gap as first introduced in [8]. We compare these algorithms with traditional approches for non-colored noise such as Zero Forcing (ZF) for the MAC [9] and the Diagonalizing Precoder for the BC [10]. Finally, we compare these Dynamic Spectrum Management (DSM) level-3 algorithms to a DSM level-2 algorithm called Optimal Spectrum Balancing (OSB) for Interference Channels (IC) where there is no coordination between the transmitters nor the receivers [11].

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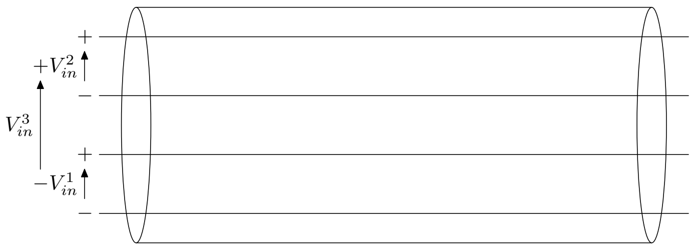

Fig. 1. Phantom mode Tranmission

In section II, we first recall the optimal power allocation for two-sided coordination vector channels with colored noise under per-modem total power constraints [6]. The conventional waterfilling algorithm under a total power constraint can easily be derived from the equations. In section III, we recall the MAC-OSB algorithm [7] and the BC-OSB algorithm [8] under per-modem total power constraints. Simulation results are given for bonded VDSL2 systems. We compare the different algorithm with non-zero SNR gap and colored noise coming from VDSL2 disturbing lines.

II. Two-sided coordination: SVD based algorithms

In this paragraph concerning two-sided coordination structures, we recall the SVD-based algorithm with colored noise under per-modem total power constraints. We assume that transmitters use Discrete Multi-Tone (DMT) modulation with a cyclic prefix longer than the maximum delay spread of the channel. The transmission over one tone can then be modelled as:

$$\mathbf{y}_i = \mathbf{H}_i \mathbf{x}_i + \mathbf{n}_i \quad i = 1 \dots N_c \tag{1}$$

where N_c is the number of subcarriers, $\mathbf{x}_i$ is the vector of N transmitted signals on tone i, $\mathbf{y}_i$ the received signal vector, $\mathbf{H}_i$ the $N \times N$ MIMO channel matrix and $\mathbf{n}_i$ the vector of colored noise. The primal problem of finding optimal PSD's for a MIMO binder under per-modem total power constraints P_j^{tot} is:

$$\begin{aligned} &\max_{(\mathbf{\Phi}_i)_{i=1\dots N_c}} \quad C(\mathbf{\Phi}_i)_{i=1\dots N_c} \\ &\text{subject to} \sum_{i=1}^{N_c} [\mathbf{\Phi}_i]_{jj} \leq P_j^{tot} \ \forall j \\ &\mathbf{\Phi}_i \succeq 0, i = 1 \dots N_c \end{aligned} \tag{2}$$

with $\mathbf{\Phi}_i$ the covariance matrix of transmitted symbols $\mathbf{\Phi}_i = E[\mathbf{x}_i \mathbf{x}_i^H]$ over tone i for the MIMO binder and with the objective function being the MIMO capacity summed over the N_c tones [12]:

$$C(\mathbf{\Phi}_i)_{i=1\dots N_c} = \sum_{i=1}^{N_c} log_2 \left[det \left(\mathbf{I} + \mathbf{H}_i \mathbf{\Phi}_i \mathbf{H}_i^H \mathbf{R}_i^{-1} \right) \right] \tag{3}$$

Here, $\mathbf{R}_i$ is the covariance matrix of the noise $\mathbf{R}_i = E[\mathbf{n}_i \mathbf{n}_i^H]$. The idea of dual decomposition is to solve (2)-(3) via its Lagrangian

[13]. The Lagrangian decouples into a set of N_c smaller problem, thus reducing the complexity of equation (2)-(3). The dual objective function is:

$$F(\mathbf{\Lambda}) = \max_{(\mathbf{\Phi}_i)_{i=1\dots N_c}} \mathcal{L}(\mathbf{\Lambda}, (\mathbf{\Phi}_i)_{i=1\dots N_c}) \tag{4}$$

with $\mathbf{\Lambda}$ a diagonal matrix of Lagrange multipliers $diag(\lambda_1, \dots, \lambda_N)$ and

$$\begin{aligned} \mathcal{L}(\mathbf{\Lambda}, (\mathbf{\Phi}_i)_{i=1\dots N_c}) = \sum_{i=1}^{N_c} \Bigg(& log_2 \left[det\left(\mathbf{I} + \mathbf{H}_i \mathbf{\Phi}_i \mathbf{H}_i^H \mathbf{R}_i^{-1}\right)\right] \\ & - Trace(\mathbf{\Lambda}\mathbf{\Phi}_i)\Bigg) + Trace\left(\mathbf{\Lambda} diag(P_j^{tot})\right) \end{aligned} \tag{5}$$

The dual optimization problem is:

$$\begin{aligned} \underset{\mathbf{\Lambda}}{\text{minimize}} \quad & F(\mathbf{\Lambda}) \\ \text{subject to} \quad & \lambda_j \geq 0 \quad \forall j \end{aligned} \tag{6}$$

The objective and constraint functions are differentiable and the Slater's conditions are satisfied, therefore the duality gap is zero and the minimum of the dual function corresponds to the global optimum of the primal problem [13]. Therefore, the search for the optimal $\mathbf{\Lambda}$ involves evaluations of the dual objective function, i.e. maximizations of the Lagrangian, which is decoupled over the tones for a given set λ_j's. By exploiting the Cholesky decomposition $\mathbf{R}_i = \mathbf{L}_i \mathbf{L}_i^H$, by defining the ($\mathbf{\Lambda}$-dependent) SVD $\mathbf{L}_i^{-1}\mathbf{H}_i \mathbf{\Lambda}^{-1/2} = \mathbf{U}_i \mathbf{D}_i \mathbf{V}_i^H$ and by setting $\tilde{\mathbf{\Phi}}_i = \mathbf{V}_i^H \mathbf{\Lambda}^{1/2} \mathbf{\Phi}_i \mathbf{\Lambda}^{1/2} \mathbf{V}_i$, we can reformulate the optimization problem as:

$$\begin{aligned} \mathcal{L}(\mathbf{\Lambda}, (\tilde{\mathbf{\Phi}}_i)_{i=1\dots N_c}) = \sum_{i=1}^{N_c} \Bigg(& log_2 \left[det\left(\mathbf{I} + \mathbf{D}_i^2 \tilde{\mathbf{\Phi}}_i\right)\right] \\ & - Trace(\tilde{\mathbf{\Phi}}_i)\Bigg) + Trace\left(\mathbf{\Lambda} diag(P_j^{tot})\right) \end{aligned} \tag{7}$$

We compute the derivative of the function in order to find the maximum, therefore the optimal power allocation is given by:

$$\mathbf{\Phi}_i = \mathbf{\Lambda}^{-1/2} \mathbf{V}_i \left[\frac{\mathbf{I}}{ln(2)} - \mathbf{D}_i^{-2} \right]^+ \mathbf{V}_i^H \mathbf{\Lambda}^{-1/2} \tag{8}$$

The complete algorithm of power allocation under per-modem total power constraints is given in [6]. After calculating the optimal Lagrange multipliers, we can calculate for each tone the SVD of the whitened channel scaled by the Lagrange multipliers $\mathbf{L}_i^{-1}\mathbf{H}_i \mathbf{\Lambda}_{opt}^{-1/2} = \mathbf{U}_i \mathbf{D}_i \mathbf{V}_i^H$, where $\mathbf{\Lambda}_{opt}$ is the optimal setting for the Lagrange multipliers, we multiply the transmitted symbols by $\mathbf{\Lambda}_{opt}^{-1/2}\mathbf{V}_i$ and the received symbols by $\mathbf{U}_i^H$ leading to:

$$\mathbf{U}_i^H \mathbf{L}_i^{-1} \mathbf{y}_i = \mathbf{U}_i^H \mathbf{L}_i^{-1} \mathbf{H}_i \mathbf{\Lambda}_{opt}^{-1/2} \mathbf{V}_i \tilde{\mathbf{x}}_i + \mathbf{U}_i^H \mathbf{L}_i^{-1} \mathbf{n}_i \tag{9}$$

This leads to parallel SISO systems as defined by:

$$\mathbf{U}_i^H \mathbf{L}_i^{-1} \mathbf{y}_i = \mathbf{D}_i \tilde{\mathbf{x}}_i + \mathbf{U}_i^H \mathbf{L}_i^{-1} \mathbf{n}_i \tag{10}$$

For practical implementations, we introduce the SNR gap Γ referred as the code gap in [14] which is the SNR multiplier required to achieve the target probability of error at the desired data rate. Considering the same Γ for the different virtual channels, the optimal $\mathbf{\Phi}_i$ is given by:

$$\mathbf{\Phi}_i = \mathbf{\Lambda}^{-1/2} \mathbf{V}_i \left[\frac{\mathbf{I}}{ln(2)} - \Gamma \mathbf{D}_i^{-2} \right]^+ \mathbf{V}_i^H \mathbf{\Lambda}^{-1/2} \tag{11}$$

A. Remark

The power allocation problem under a total power constraint can be derived directly from the problem of power allocation under per-modem total power constraints. In this case $\mathbf{\Lambda} = \lambda \mathbf{I}$. The optimal power allocation under total power constraint is then given by:

$$\mathbf{\Phi}_i = \lambda^{-1/2} \mathbf{V}_i \left[\frac{\mathbf{I}}{ln(2)} - \Gamma \mathbf{D}_i^{-2} \right]^+ \mathbf{V}_i^H \lambda^{-1/2} \tag{12}$$

III. One-sided coordination: MAC-OSB and BC-OSB algorithms

We consider a MIMO Broadcast Channel (BC) serving N users in a xDSL downstream scenario. The transmission on one tone can then be modelled as:

$$\mathbf{y}_i = \mathbf{H}_i \mathbf{x}_i + \mathbf{n}_i \quad \mathbf{H}_i = \begin{bmatrix} \mathbf{h}_{i1} \\ \vdots \\ \mathbf{h}_{iN} \end{bmatrix} \quad i = 1 \dots N_c \tag{13}$$

where N_c is the number of subcarriers, $\mathbf{x}_i$ and $\mathbf{y}_i$ are respectively the transmitted vector and the received vector of size N, $\mathbf{H}_i$ the $N \times N$ MIMO channel matrix and $\mathbf{n}_i$ the vector containing colored noise. $\mathbf{x}_i = \sum_{j=1}^{N} \mathbf{q}_{ij}$ is the sum of N user vectors $\mathbf{q}_{ij}$ of size N intended to the concerned user for the BC. The dual MIMO Multiple Access Channel (MAC) for the dual uplink scenario with N users can be written as:

$$\mathbf{v}_i = \mathbf{H}_i^H \mathbf{u}_i + \mathbf{w}_i \quad \text{where} \quad \mathbf{H}_i^H = \begin{bmatrix} \mathbf{h}_{i1}^H & \dots & \mathbf{h}_{iN}^H \end{bmatrix} \tag{14}$$

where $\mathbf{u}_i = [u_{i1} \dots u_{iN}]^T$ is the transmitted vector on tone i, $\mathbf{v}_i$ is the received signal vector of length N, and $\mathbf{w}_i$ is the vector containing colored noise. In this paper, we assume $E[\mathbf{n}_i \mathbf{n}_i^H] = \mathbf{I}$ (this is without loss of generality, as correlation in $\mathbf{n}_i$ cannot be exploited anyway), so that in the dual MAC channel $E[\mathbf{w}_i \mathbf{w}_i^H] = \mathbf{I}$ (in the MAC, a whitening operation can always be applied to the received vector such that the noise is white). The goal of this section is to find optimal transmit vector covariance matrices in the MAC and in the BC under per-modem total power constraints.

A. Multiple Access Channel - Optimal Spectrum Balancing (MAC-OSB)

The primal problem of finding optimal power allocations in the MAC under a per-modem total power constraint P_j^{tot} is:

$$\begin{aligned} & \max_{(\mathbf{\Phi}_i)_{i=1\dots N_c}} C^{MAC}(\mathbf{\Phi}_i)_{i=1\dots N_c} \\ & \text{subject to} \sum_{i=1}^{N_c} \phi_{ij} \leq P_j^{tot} \ \forall j \\ & \mathbf{\Phi}_i \succeq 0, i = 1 \dots N_c \end{aligned} \tag{15}$$

with $\mathbf{\Phi}_i = E[\mathbf{u}_i \mathbf{u}_i^H] = diag(\phi_{i1}, \dots, \phi_{iN})$ the transmit covariance matrix for tone i and $C^{MAC}(\mathbf{\Phi}_i)_{i=1\dots N_c}$ the weighted rate sum defined by:

$$\begin{aligned} & C^{MAC}(\mathbf{\Phi}_i)_{i=1\dots N_c} = \\ & \sum_{j=1}^{N} w_j \sum_{i=1}^{N_c} log_2 \left[det\left(\mathbf{I} + \frac{\mathbf{h}_{ij}^H \phi_{ij} \mathbf{h}_{ij}}{\Gamma(\mathbf{I} + \sum_{k=1}^{j-1} \mathbf{h}_{ik}^H \phi_{ik} \mathbf{h}_{ik})} \right) \right] \end{aligned} \tag{16}$$

The w_j's are the weights assigned to the different users. In the MAC, the optimal detection order is actually defined by the weights and the user with the largest weight is decoded last [7]. We assume a decreasing order of weights $w_1 > \dots > w_K$. The SNR gap Γ

referred as the code gap in [14] is the SNR multiplier required to achieve the target probability of error at the desired data rate. The dual objective function in the MAC is given as:

$$F^{MAC}(\mathbf{\Lambda}) = \max_{(\mathbf{\Phi}_i)_{i=1\dots N_c}} \mathcal{L}^{MAC}(\mathbf{\Lambda}, (\mathbf{\Phi}_i)_{i=1\dots N_c}) \quad (17)$$

with

$$\begin{aligned} &\mathcal{L}^{MAC}(\mathbf{\Lambda}, (\mathbf{\Phi}_i)_{i=1\dots N_c}) = \sum_{i=1}^{N_c} \Bigg(\sum_{j=1}^{N} w_j log_2 \Big[\\ &det \left(\mathbf{I} + \frac{\mathbf{h}_{ij}^H \phi_{ij} \mathbf{h}_{ij}}{\Gamma(\mathbf{I} + \sum_{k=1}^{j-1} \mathbf{h}_{ik}^H \phi_{ik} \mathbf{h}_{ik})} \right) \Big] - Trace(\mathbf{\Lambda \Phi}_i) \Bigg) \\ &+ Trace\left(\mathbf{\Lambda} diag(P_1^{tot}, \dots, P_N^{tot})\right) \end{aligned} \quad (18)$$

with $\mathbf{\Lambda}$ a diagonal matrix of Lagrange multipliers $diag(\lambda_1, \dots, \lambda_N)$. By tuning the Lagrange multipliers, the per-modem total power constraints can be enforced. Because the dual objective function is convex in $\mathbf{\Lambda}$, it has a unique minimum. As the duality gap is zero [15], this minimum corresponds to the global optimum of the primal problem. The search for the optimal $\mathbf{\Lambda}$ involves evaluations of the dual objective function, i.e. maximizations of the Lagrangian which, however, is decoupled over the tones for a given $\mathbf{\Lambda}$. A complete algorithm description is given as [7].

B. Broadcast Channels - Optimal Spectrum Balancing (BC-OSB)

The BC primal problem is defined as:

$$\begin{aligned} &\max_{(\mathbf{Q}_{ij})_{i=1\dots N_c, j=1\dots N}} C^{BC}(\mathbf{Q}_{ij})_{i=1\dots N_c, j=1\dots N} \\ &\text{subject to} \sum_{j=1}^{N} \sum_{i=1}^{N_c} \mathbf{Q}_{ij,ll} \leq P_l^{tot} \ \forall l \\ &\mathbf{Q}_{ij} \succeq 0, i = 1 \dots N_c, j = 1 \dots N \end{aligned} \quad (19)$$

where $E[\mathbf{x}_i \mathbf{x}_i^H] = E[\sum_{j=1}^{N} \mathbf{q}_{ij} \mathbf{q}_{ij}^H] = \sum_{j=1}^{N} \mathbf{Q}_{ij}$ corresponds to the sum of N transmit vector covariance matrices for the different users over tone i, P_l^{tot} the power budget for modem l and $C^{BC}(\mathbf{Q}_{ij})_{i=1\dots N_c, j=1\dots N}$ the weighted rate sum function for a given encoding order 1,...,N-1,N (i.e. user 1 is encoded first) with Dirty Paper Coding (DPC) is [16]:

$$\begin{aligned} &C^{BC}(\mathbf{Q}_{ij})_{i=1\dots N_c, j=1\dots N} = \\ &\sum_{j=1}^{N} w_j \sum_{i=1}^{N_c} log_2 \left[det \left(1 + \frac{\mathbf{h}_{ij} \mathbf{Q}_{ij} \mathbf{h}_{ij}^H}{\Gamma(1 + \mathbf{h}_{ij} (\sum_{k=j+1}^{N} \mathbf{Q}_{ik}) \mathbf{h}_{ij}^H)} \right) \right] \end{aligned} \quad (20)$$

Assuming a decreasing order of weights $w_1 > \dots > w_K$, as the MAC-BC duality dictates a reverse of the decoding/encoding order, in the BC the user with the largest weight has indeed to be encoded first. Thus, the first term of the sum represents the rate of user 1, which is encoded under the crosstalk of the other users. The last term of the sum represents the rate of user N after having removed the crosstalk from the other users. The weighted rate sum function is neither convex nor concave [16], therefore finding the optimal transmit vector covariance matrices in the BC is a difficult task. Fortunately, the duality between the MAC and the BC states that it is possible to achieve the same rates in both domains under the same total power constraint. As the optimal power allocation in the MAC is tractable, one can calculate optimal transmit vector covariance matrices in the MAC and transform these into optimal transmit vector covariance matrices in the BC. We aim to exploit MAC-BC duality theory under per-modem total power constraints using a similar approach. However, the MAC optimal power allocation cannot be converted directly into BC optimal transmit vector covariance matrices because the MAC-BC duality does not preserve per-modem total power constraints. To bypass this problem we apply a transformation to the dual objective function leading to an equivalent objective function with a total power constraint, and then we exploit MAC-BC duality. This transformation consists of a rescaling of the channel matrices by a precoding matrix operation. The BC dual objective function corresponding is:

$$F^{BC}(\mathbf{\Lambda}) = \max_{(\mathbf{Q}_{ij})_{i=1\dots N_c, j=1\dots N}} \mathcal{L}^{BC}(\mathbf{\Lambda}, (\mathbf{Q}_{ij})_{i=1\dots N_c, j=1\dots N}) \quad (21)$$

with

$$\begin{aligned} &\mathcal{L}^{BC}(\mathbf{\Lambda}, (\mathbf{Q}_{ij})_{i=1\dots N_c, j=1\dots N}) = \sum_{i=1}^{N_c} \Bigg(\sum_{j=1}^{N} w_j log_2 \Big[\\ &1 + \frac{\mathbf{h}_{ij} \mathbf{Q}_{ij} \mathbf{h}_{ij}^H}{\Gamma(1 + \mathbf{h}_{ij} (\sum_{k=j+1}^{N} \mathbf{Q}_{ik}) \mathbf{h}_{ij}^H)} \Big] - \sum_{j=1}^{N} Trace(\mathbf{\Lambda Q}_{ij}) \Bigg) \\ &+ Trace\left(\mathbf{\Lambda} diag(P_1^{tot}, \dots, P_N^{tot})\right) \end{aligned} \quad (22)$$

with $\mathbf{\Lambda}$ a diagonal matrix of Lagrange multipliers $diag(\lambda_1, \dots, \lambda_N)$. The dual optimization problem is:

$$\begin{aligned} &\text{minimize} \quad F^{BC}(\mathbf{\Lambda}) \\ &\text{subject to} \quad \lambda_i \geq 0 \quad \forall i \end{aligned} \quad (23)$$

Rescaling the channel matrices by the inverse square root of the Lagrange multiplier matrix leads to:

$$\mathbf{y}_i = \overbrace{\mathbf{H}_i \mathbf{\Lambda}^{-1/2}}^{\mathbf{H}'_i} \mathbf{x'}_i + \mathbf{n}_i \quad (24)$$

where $\mathbf{x'}_i = \mathbf{\Lambda}^{1/2} \mathbf{x}_i$. For this equivalent channel, the dual objective function in the BC becomes:

$$F^{BC}(\mathbf{\Lambda}) = \max_{(\mathbf{Q'}_{ij})_{i=1\dots N_c, j=1\dots N}} \mathcal{L}^{BC}(\mathbf{\Lambda}, (\mathbf{Q'}_{ij})_{i=1\dots N_c, j=1\dots N}) \quad (25)$$

with

$$\begin{aligned} &\mathcal{L}^{BC}(\mathbf{\Lambda}, (\mathbf{Q'}_{ij})_{i=1\dots N_c, j=1\dots N}) = \sum_{i=1}^{N_c} \Bigg(\sum_{j=1}^{N} w_j log_2 \Big[\\ &det \left(1 + \frac{\mathbf{h'}_{ij} \mathbf{Q'}_{ij} \mathbf{h'}_{ij}^H}{\Gamma(1 + \mathbf{h'}_{ij} (\sum_{k=j+1}^{N} \mathbf{Q'}_{ik}) \mathbf{h'}_{ij}^H)} \right) \Big] - \sum_{j=1}^{N} Trace(\mathbf{Q'}_{ij}) \Bigg) \\ &+ Trace\left(diag(P'^{tot}_1, \dots, P'^{tot}_N) \right) \end{aligned} \quad (26)$$

where $\mathbf{h'}_{ij} = \mathbf{h}_{ij} \mathbf{\Lambda}^{-1/2}$, $\mathbf{Q'}_{ij} = \mathbf{\Lambda}^{1/2} \mathbf{Q}_{ij} \mathbf{\Lambda}^{1/2}$ and $diag(P'^{tot}_1, \dots, P'^{tot}_N) = \mathbf{\Lambda}^{1/2} diag(P_1^{tot}, \dots, P_N^{tot}) \mathbf{\Lambda}^{1/2}$ (there we also used the property $Trace(\mathbf{AB}) = Trace(\mathbf{BA})$). One can see that (26) corresponds to (18) with $\mathbf{\Lambda} = \mathbf{I}$, and so that the precoder matrix $\mathbf{\Lambda}^{-1/2}$ transforms the per-modem total power constraints into a single total power constraint by hiding the Lagrange multipliers into the equivalent channels $\mathbf{h'}_{ij}$ and the new covariance matrices $\mathbf{Q'}_{ij}$. We can now invoke MAC-BC duality theory to transform (26) into an equivalent MAC dual objective function. The dual objective function in the MAC is given as:

$$F^{MAC}(\mathbf{\Lambda}) = \max_{(\mathbf{\Phi'}_i)_{i=1\dots N_c}} \mathcal{L}^{MAC}(\mathbf{\Lambda}, (\mathbf{\Phi'}_i)_{i=1\dots N_c}) \quad (27)$$

with

$$\mathcal{L}^{MAC}(\mathbf{\Lambda}, (\mathbf{\Phi'}_i)_{i=1\ldots N_c}) = \sum_{i=1}^{N_c} \left(\sum_{j=1}^{N} w_j log_2 \left[\right.\right.$$
$$det \left(\mathbf{I} + \frac{\mathbf{h'}_{ij}^H \phi'_{ij} \mathbf{h'}_{ij}}{\Gamma(\mathbf{I} + \sum_{k=1}^{j-1} \mathbf{h'}_{ik}^H \phi'_{ik} \mathbf{h'}_{ik})} \right) \left.\right] - Trace(\mathbf{\Phi'}_i) \left.\right) \tag{28}$$
$$+ Trace \left(diag(P'^{tot}_1, \ldots, P'^{tot}_N) \right)$$

For a given $\mathbf{\Lambda}$, we can then compute the optimal power allocation in the MAC and use a modified version including the SNR gap of the duality formulas described in [16] to obtain the optimal transmit vector covariance matrices in the BC that achieve the same rate as in the MAC (first term in $\mathcal{L}^{MAC}$ and $\mathcal{L}^{BC}$) with the same total power (second term in $\mathcal{L}^{MAC}$ and $\mathcal{L}^{BC}$). The Lagrange multipliers are then adjusted so that the per-modem total power constraints are enforced. A complete algorithm description is given as [8].

IV. Results

In the simulation results, we compare the SVD-based algorithm and the MAC-OSB, BC-OSB under per-modem total power constraints but also the OSB algorithm for Interference Channels (IC) [11]. We also use simplified algorithms performing Iterative Spectrum Balacing (ISB) which calculate the psd's of the users iteratively by fixing the psd's of the other users (the OSB approach uses an exhaustive search).

The simulations are carried out over measured channels from a a binder of four lines, therefore giving a 7×7 channels matrix (4 DM and 3 PM) over 30 MHz bandwidth. We use this channel in a VDSL2 context with the FDD ITU 30a band plan which provides 3 frequency bands in the downlink scenario, namely 138kHz-3.75MHz, 5.2MHz-8.5MHz and 12MHz-18.1MHz. For the uplink, the 4 frequency bands are 25kHz-138kHz, 3.75MHz-5.2MHz, 8.5MHz-12MHz and 18.1MHz-30MHz. The maximum transmit power per line is P_j^{tot}=14.5 dBm, with 8.625 kHz spacing between subcarriers and 8 kHz symbol rate. Colored noise consists of 2 other DM lines at fixed mask -60 dBm/Hz and Additive White Gaussian Noise (AWGN) at -140 dBm/Hz, with SNR gap Γ=10.8 dB.

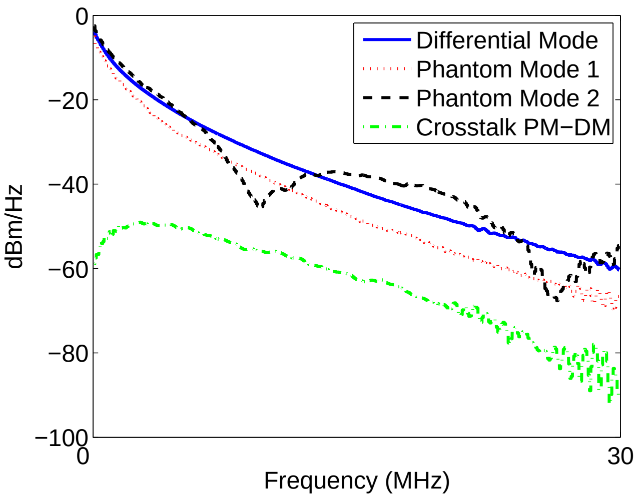

Fig. 2. Examples of direct and crosstalk channels for DM and PM

In Fig.2 we can see the different channels for DM and PM. The "Phantom Mode 1" corresponds to the first level of PM, that is the PM frequency response of the differential between 2 common-modes (between line 1 and line 2 or between line 3 and line 4). The "Phantom Mode 2" corresponds to the second level of PL, that is the differential between the previous 2 PM. One can see that there is more selectivity in frequency in the latter.

In Table I, 2 users at 400 meters from the CO/RT are served in a downstream scenario with their Differential-Mode (DM) (coordination at the transmit side with 2×2 channel matrix). This system is compared with the exploitation of their Phantom-mode (PM) at the transmit side (giving a 2×3 channel matrix) and the exploitation of their PM at the transnmit and the receive side (coordination at the transmit and the receiving side with 3×3 channel matrix). The power carried on the PM line is also 14.5 dBm thus 50% more power is added to the system. We also compare the full binder 4×4 system with the 4×7 system exploiting the 3 PM at the transmit side and the 7×7 exploiting the 3 PM at the transmit and the receive side. The downstream frequency bands are used in this simulation. What can be seen from this table is that:

- SVD gives the best performance (upper bound)
- BC-OSB and BC-ISB rates are equal and smaller than SVD (whitening not possible)
- BC-ISB and DP give equal rates when the number of transmitters equals the number of receivers
- BC-ISB gives higher rates than DP when the number of transmitters is larger than the number of receivers
- Only small increase in rates with only PM exploitation at the transmit side owing to small crosstalk channels compared to direct channels although 50% power increase for the 2×3 case.
- Benefit from DSM level-3 (SVD, BC-OSB, BC-ISB, DP) compared to DSM level-2 (OSB)
- Huge benefit from PM exploitation at the transmit and the receive side (50% power increase for the 3×3 case)

Sum rate (Mbps)	2x2	2x3	3x3
SVD PMTPC	300.8	306.9	483.0
BC-OSB PMTPC	291.2	294.8	-
BC-ISB PMTPC	291.2	294.8	-
DP PMTPC	291.2	291.4	-
OSB	221.4	-	-

Sum rate (Mbps)	4x4	4x7	7x7
SVD PMTPC	459.8	487.3	1031.6
BC-ISB PMTPC	289.0	289.7	-

TABLE I
Downstream scenario

In Table II for the upstream scenario, the PM line is used at the CO/RT giving a 3×2 channel matrix. We therefore compare the 2×2 channel matrix with the 3×2 exploiting the PM at the receive side and the 3×3 exploiting the PM at the transmit and the receive side. We also compare the full binder 4×4 system with the 7×4 system exploiting the 3 PM at the receive side and the 7×7 exploiting the 3 PM at the transmit and the receive side. The upstream frequency bands are used in this simulation. What can be seen from this table is that:

- SVD gives the best performance (upper bound)
- MAC-OSB and MAC-ISB rates are equal and close to SVD
- MAC-ISB gives higher rates than ZF because ZF does not exploit whitening
- Only small increase in rates with only PM exploitation at the receive side owing to small crosstalk channels compared to direct channels.
- Benefit from DSM level-3 (SVD, MAC-OSB, MAC-ISB, ZF) compared to DSM level-2 (OSB)

- Huge benefit from PM exploitation at the transmit and the receive side

Sum rate (Mbps)	2x2	3x2	3x3
SVD PMTPC	169.4	175.8	258.7
MAC-OSB PMTPC	169.3	175.8	-
MAC-ISB PMTPC	169.3	175.8	-
ZF PMTPC	160.1	160.2	-
OSB	110.5	-	-

Sum rate (Mbps)	4x4	7x4	7x7
SVD PMTPC	238.3	238.9	523.6
MAC-ISB PMTPC	232.1	232.8	-
ZF PMTPC	166.5	167.2	-

TABLE II
UPSTREAM SCENARIO

V. CONCLUSION

In this paper we have studied three types of algorithms for optimal power allocation in MIMO systems with colored noise using PM transmission under practical implementation (per-modem total power constraints and SNR gap). First, the two-sided coordination case has been studied leading to a SVD-based algorithm under per-modem total power constraints and SNR gap. This algorithm gives the best possible achievable rates, but at the price of perfect knowledge of CSI at both the transmit and the receive side. Then, one-sided algorithms for Multiple Access Channels (MAC) and Broadcast Channels (BC) were investigated. An algorithm called MAC-ISB is shown to perform closely to the SVD-based algorithm owing to the whitening operation at the receive side. An algorithm called BC-ISB performs better than DP when the number of transmitters is larger than the number of receivers. Finally, these algorithms referred to as DSM level-3 algorithms performs better than DSM level-2 algorithm with no coordination at the receiver nor at the transmitter (OSB). In VDSL2 downstream or upstream scenario with PM exploitation at the transmit or the receive side only, there is no significant gains as they are driven by the crosstalk channels (More gain can be seen in wireless channels where crosstalk channels are as powerful as the direct channels). In order to achieve large gains, PM exploitation must be done at the receive and the transmit side simultaneously (more than 100% gain).

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