OPTIMAL POWER ALLOCATION UNDER PER-MODEM TOTAL POWER AND SPECTRAL MASK CONSTRAINTS IN XDSL VECTOR CHANNELS WITH ALIEN CROSSTALK

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ABSTRACT

In xDSL systems, in-domain crosstalk is easily dealt with based on zero-forcing receiver or precoding while out of domain or alien crosstalk requires a more advanced processing. In vector channels, Multiple Input Multiple Output (MIMO) signal processing mitigates both types of crosstalk, usually by means of a pre-whitening filter, Singular Value Decomposition (SVD) based transmission and waterfilling based power allocation. In this paper, we investigate the problem of power allocation in xDSL vector channels under in-domain and alien crosstalk. We propose a new power allocation algorithm to maximize the MIMO capacity under per-modem total power constraints and spectral mask constraints, leading to a generalized SVD-based transmission.

***Index Terms*—** xDSL, MIMO systems, crosstalk, power allocation, optimization methods

1. INTRODUCTION

The growing demand for high speed services on the last mile access calls for new paradigms offering an increased capacity and better performance. Thanks to the success of xDSL (ADSL in particular), new services that require higher data rates start to emerge and service providers begin to bond copper pairs to form a (more performant) broadband link. Multiple Input Multiple Output (MIMO) signal processing algorithms are then used to provide a suitable mitigation of the interference, in-domain crosstalk (i.e. self-crosstalk between bonded lines) as well as alien crosstalk.

In-domain crosstalk cancellation has been studied for 2-sided coordination vector channels [1, 2]. The optimal precoding and equalization as well as optimal Power Spectral Densities (PSD's) for vector channels are obtained through the SVD of the channel matrix combined with standard waterfilling. For 1-sided coordination Multiple Access Channels (MAC) or Broadcast Channels (BC), due to the diagonal dominance structure of the channel matrix, the optimal Generalized Decision Feedback Equalizer (GDFE) based solution can be reduced to a simple Zero Forcing (ZF) solution with transmit PSD's obtained by single-channel waterfilling [3]. Moreover, crosstalk avoidance has been studied for Interference Channels (IC) with no coordination between transmitters and receivers. The optimal transmit PSD's are found by means of Optimal Spectrum Balancing (OSB) [4].

Alien crosstalk cancellation uses the correlation of the noise to improve the performance of the transmission and hence to increase the capacity of the link. This correlation can appear in the spatial domain (between pairs), the frequency domain (between tones) or the mode domain (between common-mode and differential-mode). In a recent paper [5], it was shown that there is more benefit to exploit the noise correlation between pairs than the correlation between tones. With alien crosstalk, the diagonal dominance structure of the channel matrix of in-domain crosstalk is destroyed by the necessary pre-whitening, and so the above mentioned procedures are no longer applicable.

In this paper, we investigate the problem of power allocation for 2-sided coordination xDSL vector channels under in-domain crosstalk and alien crosstalk, exploiting the noise correlation between the bonded lines. Realistic per-modem total power constraints as well as spectral mask constraints are included in the optimization problem. The primal MIMO capacity optimization problem subject to power constraints coupled over the tones is transformed into a collection of per-tone unconstrained optimization problems using Lagrangian parameters. We derive optimal transmitter and receiver structures (precoders and equalizers) in combination with power allocation which achieve MIMO channel capacity. The per-modem constraints are found to lead to a generalized SVD-based transmission.

The derivation of the optimal power allocation under per-modem total power constraints and spectral mask constraints is given in section II. We then also derive the optimal Tx/Rx

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structure. Simulation results and conclusions are given in section III and IV.

2. POWER ALLOCATION UNDER PER-MODEM TOTAL POWER CONSTRAINTS AND SPECTRAL MASK CONSTRAINTS

In this section, the derivation of optimal PSD's in a xDSL vector channel with in-domain crosstalk and alien crosstalk and the corresponding optimal transmitter/receiver (Tx/Rx) structure are given under per-modem total power constraints as well as spectral mask constraints. We assume a vector channel with N transmit modems and N receive modems, where the transmitters use Discrete Multi-Tone (DMT) modulation with a cyclic prefix longer than the maximum delay spread of the channel. The transmission on one tone can then be modelled as:

$$\mathbf{y}_i = \mathbf{H}_i \mathbf{x}_i + \mathbf{n}_i \quad i = 1 \dots N_c \tag{1}$$

where N_c is the number of subcarriers, $\mathbf{x}_i$ is the vector of N transmitted signals on tone i, $\mathbf{y}_i$ the received signal vector, $\mathbf{H}_i$ the $N \times N$ MIMO channel matrix and $\mathbf{n}_i$ the vector of noise containing Additive White Gaussian Noise (AWGN) and alien crosstalk. Note that we do not assume that the alien crosstalk is synchronised with the MIMO binder nor that the cyclic prefix is longer than the maximum delay spread of the alien crosstalk in the binder. That means that there could be more correlation to exploit in the frequency domain since the noise is not decoupled over the tones, contrary to the transmitted symbols. In the xDSL vector channel power allocation context, it is relevant to consider a constraint on the power of each transmit modem separately instead of a constraint on the power for all modems together. Moreover, DSL standardisation often defines spectral masks that each transmitter has to satisfy.

2.1. Optimal power allocation

The primal problem of finding optimal PSD's for a MIMO binder under per-modem total power constraints P_j^{tot} and spectral mask constraints is:

$$\begin{array}{ll} max\; C(\mathbf{\Phi}_i)_{i=1\dots N_c} \text{ s.t.} & \sum_{i=1}^{N_c} (\mathbf{\Phi}_i)_{jj} \leq P_j^{tot}\ \forall \text{ j} \\ & (\mathbf{\Phi}_i)_{jj} \leq \phi_i^{mask,j}\ \forall \text{ j} \\ \mathbf{\Phi}_i \text{ positive semidefinite} & \end{array} \tag{2}$$

with $\mathbf{\Phi}_i$ the covariance matrix of transmitted symbols $\mathbf{\Phi}_i = E[\mathbf{x}_i \mathbf{x}_i^H]$ over tone i for the MIMO binder and with the objective function being the MIMO capacity summed over the N_c tones:

$$C(\mathbf{\Phi}_i)_{i=1\dots N_c} = \sum_{i=1}^{N_c} log_2 \left[det \left(\mathbf{I} + \frac{1}{\Gamma} \mathbf{H}_i \mathbf{\Phi}_i \mathbf{H}_i^H \mathbf{R}_i^{-1} \right) \right] \tag{3}$$

Here, $\mathbf{R}_i = E[\mathbf{n}_i \mathbf{n}_i^H]$ is the covariance matrix of the noise, and Γ is the SNR gap. The idea of dual decomposition is to solve (2) via its Lagrangian. The Lagrangian decouples into a set of N_c smaller problems, thus reducing the complexity of (2). Using Lagrange multipliers to transfer the per-modem total power constraints and the mask constraints into the objective function, the dual objective function becomes:

$$\begin{aligned} F(\mathbf{\Lambda}, \tilde{\mathbf{\Lambda}}_1, \dots, \tilde{\mathbf{\Lambda}}_{N_c}) = max \Bigg(& C(\mathbf{\Phi}_i)_{i=1\dots N_c} \\ & - \sum_{i=1}^{N_c} Trace((\mathbf{\Lambda} + \tilde{\mathbf{\Lambda}}_i)\mathbf{\Phi}_i) \Bigg) + Trace\left(\mathbf{\Lambda} diag(P_j^{tot})\right) \\ & + \sum_{i=1}^{N_c} Trace\left(\tilde{\mathbf{\Lambda}}_i diag(\phi_i^{mask,j})\right) \end{aligned} \tag{4}$$

with $\mathbf{\Lambda} = diag(\lambda_1, \dots, \lambda_N)$ a diagonal matrix of Lagrange multipliers corresponding to the per-modem total power constraints and $\tilde{\mathbf{\Lambda}}_i = diag(\tilde{\lambda}_{i1}, \dots, \tilde{\lambda}_{iN})$ a diagonal matrix of Lagrange multipliers corresponding to the spectral mask constraints for tone i. The dual optimization problem is:

$$\begin{array}{ll} \text{minimize} & F(\mathbf{\Lambda}, \tilde{\mathbf{\Lambda}}_1, \dots, \tilde{\mathbf{\Lambda}}_{N_c}) \\ \text{subject to} & \mathbf{\Lambda}^{ii}, \tilde{\mathbf{\Lambda}}_1^{ii}, \dots, \tilde{\mathbf{\Lambda}}_{N_c}^{ii} \geq 0 \quad \forall i \end{array} \tag{5}$$

Because the dual function is convex in $\mathbf{\Lambda}, \tilde{\mathbf{\Lambda}}_1, \dots, \tilde{\mathbf{\Lambda}}_{N_c}$, it has a unique minimum. As the duality gap is zero, this minimum corresponds to the global optimum of the primal problem [6]. The search for the optimal $\mathbf{\Lambda}, \tilde{\mathbf{\Lambda}}_1, \dots, \tilde{\mathbf{\Lambda}}_{N_c}$ in (5) involves evaluations of the dual objective function (4), i.e. maximizations of the Lagrangian, which is decoupled over the tones for a given $\mathbf{\Lambda}, \tilde{\mathbf{\Lambda}}_1, \dots, \tilde{\mathbf{\Lambda}}_{N_c}$.

$$\begin{aligned} & F(\mathbf{\Lambda}, \tilde{\mathbf{\Lambda}}_1, \dots, \tilde{\mathbf{\Lambda}}_{N_c}) = \\ & \sum_{i=1}^{N_c} max \Bigg(log_2 \left[det \left(\mathbf{I} + \tfrac{1}{\Gamma} \mathbf{H}_i \mathbf{\Phi}_i \mathbf{H}_i^H \mathbf{R}_i^{-1} \right) \right] \\ & - Trace((\mathbf{\Lambda} + \tilde{\mathbf{\Lambda}}_i)\mathbf{\Phi}_i) \Bigg) + Trace\left(\mathbf{\Lambda} diag(P_j^{tot})\right) \\ & + \sum_{i=1}^{N_c} Trace\left(\tilde{\mathbf{\Lambda}}_i diag(\phi_i^{mask,j})\right) \end{aligned} \tag{6}$$

By exploiting the Cholesky decomposition $\mathbf{R}_i = \mathbf{L}_i \mathbf{L}_i^H$ where $\mathbf{L}_i$ is a lower triangular matrix (whose inverse will be used to whiten the noise at the receive side), using the property $det(\mathbf{I} + \mathbf{AB}) = det(\mathbf{I} + \mathbf{BA})$) and by defining the SVD $\frac{1}{\sqrt{\Gamma}} \mathbf{L}_i^{-1} \mathbf{H}_i (\mathbf{\Lambda} + \tilde{\mathbf{\Lambda}}_i)^{-1/2} = \mathbf{U}_i \mathbf{D}_i \mathbf{V}_i^H$, we can rewrite the optimization problem as:

$$F(\mathbf{\Lambda}, \tilde{\mathbf{\Lambda}}_1, \dots, \tilde{\mathbf{\Lambda}}_{N_c}) = \sum_{i=1}^{N_c} max\Bigg(log_2[det(\mathbf{I} + \mathbf{U}_i\mathbf{D}_i\mathbf{V}_i^H(\mathbf{\Lambda} + \tilde{\mathbf{\Lambda}}_i)^{1/2}\mathbf{\Phi}_i (\mathbf{\Lambda} + \tilde{\mathbf{\Lambda}}_i)^{1/2}\mathbf{V}_i\mathbf{D}_i\mathbf{U}_i^H)] - Trace(\mathbf{V}_i^H(\mathbf{\Lambda} + \tilde{\mathbf{\Lambda}}_i)^{1/2}\mathbf{\Phi}_i (\mathbf{\Lambda} + \tilde{\mathbf{\Lambda}}_i)^{1/2}\mathbf{V}_i)\Bigg) + Trace\left(\mathbf{\Lambda} diag(P_j^{tot})\right) + \sum_{i=1}^{N_c} Trace\left(\tilde{\mathbf{\Lambda}}_i diag(\phi_i^{mask,j})\right) \tag{7}$$

By setting $\tilde{\mathbf{\Phi}}_i = \mathbf{V}_i^H(\mathbf{\Lambda} + \tilde{\mathbf{\Lambda}}_i)^{1/2}\mathbf{\Phi}_i(\mathbf{\Lambda} + \tilde{\mathbf{\Lambda}}_i)^{1/2}\mathbf{V}_i$:

$$F(\mathbf{\Lambda}, \tilde{\mathbf{\Lambda}}_1, \dots, \tilde{\mathbf{\Lambda}}_{N_c}) = \sum_{i=1}^{N_c} max\left(log_2\left[det\left(\mathbf{I} + \mathbf{D}_i^2\tilde{\mathbf{\Phi}}_i\right)\right] - Trace(\tilde{\mathbf{\Phi}}_i)\right) + Trace\left(\mathbf{\Lambda} diag(P_j^{tot})\right) + \sum_{i=1}^{N_c} Trace\left(\tilde{\mathbf{\Lambda}}_i diag(\phi_i^{mask,j})\right) \tag{8}$$

Here it is observed that off-diagonal elements in $\tilde{\mathbf{\Phi}}_i$ merely reduce the determinant, hence that the optimal $\tilde{\mathbf{\Phi}}_i$ is diagonal. In order to find the maximum, we compute the derivative of the function:

$$\frac{dF(\mathbf{\Lambda}, \tilde{\mathbf{\Lambda}}_1, \dots, \tilde{\mathbf{\Lambda}}_{N_c})}{d\tilde{\mathbf{\Phi}}_i} = diag\left[\left(\mathbf{D}_i^{-2} + \tilde{\mathbf{\Phi}}_i\right)^{-1}\right] - \mathbf{I} = 0 \tag{9}$$

Therefore the optimal power allocation under per-modem total power constraints and spectral mask constraints is given by:

$$\mathbf{\Phi}_i = (\mathbf{\Lambda} + \tilde{\mathbf{\Lambda}}_\mathbf{i})^{-1/2}\mathbf{V}_i\left[\mathbf{I} - \mathbf{D}_i^{-2}\right]^+\mathbf{V}_i^H(\mathbf{\Lambda} + \tilde{\mathbf{\Lambda}}_\mathbf{i})^{-1/2} \tag{10}$$

where the $[.]^+$ operation is inserted in order to obtain positive semi-definite $\mathbf{\Phi}_i$'s. One can note that the precoder formulas are now a function of the Lagrange multipliers. Formula (10) provides a closed form waterfilling solution for MIMO systems under per-modem total power constraints and spectral mask constraints once the Lagrange multipliers are set.

2.2. Optimal Tx/Rx structure

In this section we specify the optimal Tx/Rx structure for the optimal power allocation. First, a pre-whitening operation is performed on the received vector $\mathbf{y}_i$, based on the Cholesky factor of the noise covariance matrix:

$$\mathbf{L}_i^{-1}\mathbf{y}_i = \mathbf{L}_i^{-1}\mathbf{H}_i\mathbf{x}_i + \mathbf{L}_i^{-1}\mathbf{n}_i \tag{11}$$

Then, we calculate the SVD based on the optimal setting of the Lagrange multipliers $\frac{1}{\sqrt{\Gamma}}\mathbf{L}_i^{-1}\mathbf{H}_i(\mathbf{\Lambda}_{opt} + \tilde{\mathbf{\Lambda}}_{i,opt})^{-1/2} = \mathbf{U}_i\mathbf{D}_i\mathbf{V}_i^H$ and multiply the transmitted symbols by $(\mathbf{\Lambda}_{opt} + \tilde{\mathbf{\Lambda}}_{i,opt})^{-1/2}\mathbf{V}_i$ and the received symbols by $\mathbf{U}_i^H$ leading to:

$$\tilde{\mathbf{y}}_i = \mathbf{U}_i^H\mathbf{L}_i^{-1}\mathbf{H}_i\mathbf{x}_i + \tilde{\mathbf{n}}_i \tag{12}$$

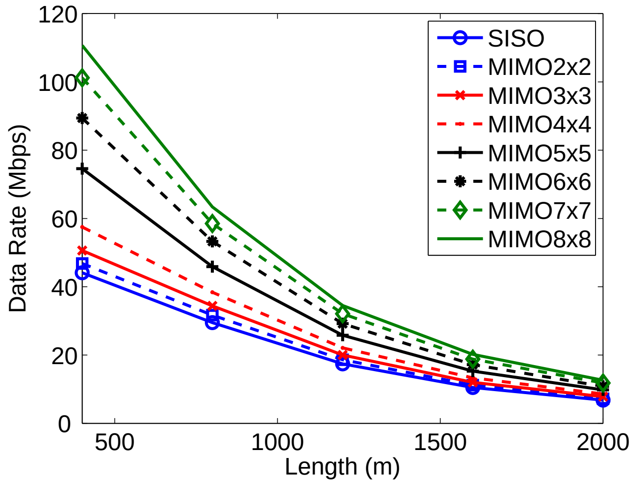

Fig. 1. Capacity per line with 2-sided coordination in a downlink scenario under per-modem total power constraints with spectral mask constraints

This corresponds to N equivalent SISO systems are given by:

$$\tilde{\mathbf{y}}_i = \mathbf{D}_i\tilde{\mathbf{x}}_i + \tilde{\mathbf{n}}_i \tag{13}$$

where $E[\tilde{\mathbf{n}}_i\tilde{\mathbf{n}}_i^H] = \mathbf{I}$. The final step is to waterfill over the different singular values subject to the total power constraint P_j^{tot} $\forall j$ and the spectral mask constraints (which will be satisfied automatically). As previously mentioned, owing to the dual objective function $F(\mathbf{\Lambda}, \tilde{\mathbf{\Lambda}}_1, \dots, \tilde{\mathbf{\Lambda}}_{N_c})$ being continuous differentiable, the search algorithm can use a gradient-descent like method to find the optimal Lagrange multipliers and is guaranteed to converge. The algorithm tries to converge under the per-modem total power constraints over the tones and inside this optimisation tries to converge on a per-tone basis to also satisfy the spectral mask constraints. The complete algorithm description of power allocation under per-modem total power constraints and spectral mask constraints can be found in [7].

3. RESULTS

The simulations results were obtained for different line lengths by concatenation of a 400 meters France Telecom binder with 8 lines. Spectral masks from VDSL2 were used [8], with Γ=10.8 dB (with Shannon gap 9.8 dB, margin 6 dB and coding gain 5 dB), and Additive White Gaussian Noise (AWGN) of -140 dBm/Hz and a maximum transmit power per line P_j^{tot}=14.5 dBm. The frequency range is from 0 to 12 MHz with 4.3125 kHz spacing between subcarriers and 4 kHz symbol rate. **Fig.1** shows the capacity per line for 2-sided coordination of a downlink MIMO binder for a varying number of coordinated pairs and a varying number of alien crosstalk lines under per-modem total power constraints and spectral mask constraints. A binder of 8 lines is used, with the num-

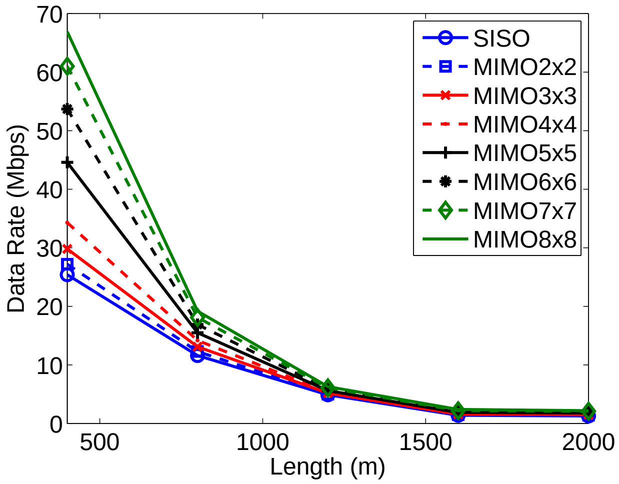

Fig. 2. Capacity per line with 2-sided coordination in an uplink scenario under per-modem total power constraints with spectral mask

ber of coordinated pairs N going from 1 to 8 and the number of alien crosstalk lines from 7 to 0 respectively. In each case, all $\binom{8}{N}$ combinations are used providing an average capacity. One can see that there is a benefit from bonding under alien crosstalk since the capacity per line increases from 44e6 in the SISO case to 110e6 in the MIMO8x8 case, where there is no alien crosstalk and thus maximum performance is achieved. When the number of alien crosstalkers is smaller than the number of coordinated pairs, there is a larger improvement in terms of capacity. This is due to the fact that the covariance matrix of the alien crosstalk becomes rank deficient and thus provides more cancellation performance on the first singular values of the prewhitened channel matrix. **Fig.2** shows the results for an equivalent uplink scenario. For short lines, one can see that the capacity increases from 25.4e6 in the SISO case to 66.9e6 in the MIMO8x8 case. However, there is only a small difference between these two cases for long lines in the uplink scenario compared to the downlink scenario due to the VDSL2 spectral masks. **Table 1** shows the relative gain between the SISO case and the different MIMO cases. It is clear from this table that the gain is large for short lines for the downlink or the uplink scenario. The gain is reduced for longer lines, especially so for the uplink scenario.

4. CONCLUSION

In this paper we investigated the problem of optimal power allocation in xDSL vector channels with alien crosstalk. We have described the power allocation algorithm and optimal Tx/Rx structures under per-modem total power constraints and spectral mask constraints. Several properties characterize this power allocation algorithm, one being the dependence of the precoder matrix on the optimal Lagrange multipliers. Secondly, the optimal PSD's are found in closed form, leading to a continuous power loading. Simulation results were given for a MIMO binder of 8 lines with different lengths in a VDSL2 context.

$\times$ e6	400m d/u	800m d/u	1200m d/u	1600m d/u
2x2	2.7/1.8	2.2/0.7	1.2/0.1	0.7/0.1
3x3	6.5/4.4	4.9/1.5	2.6/0.3	1.5/0.2
4x4	13.4/8.9	8.9/2.6	4.7/0.4	2.8/0.3
5x5	30.5/19.2	16.4/3.9	8.4/0.7	4.8/0.5
6x6	45.3/28.3	23.3/5.3	11.8/1	6.7/0.7
7x7	57.1/35.6	29.0/6.5	14.7/1.2	8.3/0.8
8x8	66.6/41.5	33.9/7.6	17.1/1.4	9.7/1

Table 1. Capacity gain between SISO capacity and bonding in downlink and uplink scenarios under per-modem total power constraints with spectral mask constraints

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