Near-optimal performance iterative MMSE receiver for spatial multiplexing MC-CDMA

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I. INTRODUCTION

The Multi-Carrier Code Division Multiple Access (MC-CDMA) technique associating the Orthogonal Frequency Division Multiplex (OFDM) modulation and the access technique CDMA has been presented for the first time in 1993 [1]. The MC-CDMA technique spreads the data symbols in the frequency domain and has very good performance in a downlink transmission.

On the other hand, the work of Foshini has introduced a new way to exploit multi-antenna systems by sending different data symbols at the transmitter side and by inverting the Multiple Input Multiple Output (MIMO) channel matrix at the receiver side [2]. The capacity of this MIMO scheme called spatial multiplexing increases linearly with the minimum number between transmit and receive antennas [3].

Moreover, the combination between spatial multiplexing and MC-CDMA was studied in [4], [5]. In this paper, a near-optimal performance iterative Minimum Mean Square Error (MMSE) receiver is proposed, where equalization and despreading can be performed jointly. The performance are given for full and half loads in an uncorrelated MIMO context.

II. PRESENTATION OF THE TRANSMITTER

The MIMO MC-CDMA transmitter and receiver are presented in Figure 1.

First, N_t vectors of data symbols belonging to different users are spreaded separtly owing to a Walsh-Hadamard matrix. Then, the N_t spreaded vectors are send to the N_t transmit antennas. An OFDM modulation is applied on each transmit antenna. This scheme is well adapted in the case of a downlink transmission because the system is synchronous. At the receiver side, an OFDM demodulation is appled, then an equalization matrix corresponding to a spatial decoding and finally the despreading of the different users. The spatial decoding and the despreading can be performed jointly depending on the complexity afforded. The received vector $\mathbf{r}$ can be expressed by:

$$\mathbf{r} = \mathcal{H}\mathcal{C}\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{x}$ is a vector of length N_tN_u which is the concatenation of N_t vectors of length N_u corresponding to the different users,

$$\mathcal{H} = \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} & \dots & \mathbf{H}_{1t} & \dots & \mathbf{H}_{1N_t} \\ \mathbf{H}_{21} & \mathbf{H}_{22} & \dots & \mathbf{H}_{2t} & \dots & \mathbf{H}_{2N_t} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \mathbf{H}_{r1} & \mathbf{H}_{r2} & \dots & \mathbf{H}_{rt} & \dots & \mathbf{H}_{rN_t} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \mathbf{H}_{N_r1} & \mathbf{H}_{N_r2} & \dots & \mathbf{H}_{N_rt} & \dots & \mathbf{H}_{N_rN_t} \end{bmatrix} \tag{2}$$

is the channel matrix of size $N_rL_c \times N_tL_c$ where $\mathbf{H}_{rt}$ is a diagonal matrix of size $L_c \times L_c$, each component of the diagonal matrix corresponding to the channel impuse response of a subcarrier in the frequency domain k $h_{rt,k} = \rho_{rt,k}e^{i\theta_{rt,k}}$, $\mathcal{C} = \mathbf{I}_{N_t} \otimes \mathbf{C}$ is the spreading matrix $N_tL_c \times N_tN_u$ where $\mathbf{C}$ is the multi-user sequence matrix of size $L_c \times N_u$ and $\mathbf{n}$ the noise vector of length N_rL_c.

III. DISJOINT EQUALIZATION-DESPREADING RECEIVER

In order to retrieve transmitted symbols which have been corrupted by the multi-antenna channel and the spreading between users, a Single User (SU) detection technique consists of a disjoint equalization MIMO matrix $\mathcal{G}$ and despreading. These SU detection techniques have been demonstrated to offer the best tradeoff between performance and complexity in a Single Input Single Output (SISO) or Space Time Block Coding (STBC) context [6], [7]. Using a MMSE equalizer in a spatial multiplexing context, the matrix $\mathcal{G}$ is equal to:

$$\mathcal{G} = (\mathcal{H}^H\mathcal{H} + \frac{L_c}{N_u\gamma}\mathcal{I})^{-1}\mathcal{H}^H \tag{3}$$

with γ the Signal to Noise Ratio (SNR) at the receive antenna. After the equalization step, this leads to a vector $\hat{\mathbf{s}}$ corresponding to the estimated spreaded symbols:

$$\hat{\mathbf{s}} = \mathcal{G}\mathbf{r} = \mathcal{G}\mathcal{H}\mathcal{C}\mathbf{x} + \mathcal{G}\mathbf{n} \tag{4}$$

After despreading, this leads to the vector $\hat{\mathbf{x}}$ corresponding to the symbols of the N_u users:

$$\hat{\mathbf{x}} = \mathcal{C}^T\hat{\mathbf{s}} = \mathcal{C}^T\mathcal{G}\mathcal{H}\mathcal{C}\mathbf{x} + \mathcal{C}^T\mathcal{G}\mathbf{n} \tag{5}$$

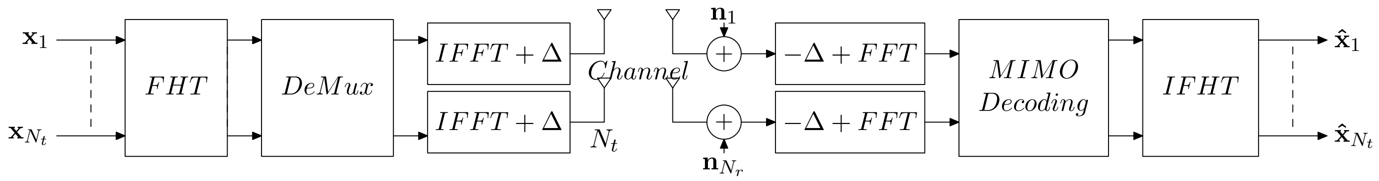

Fig. 1. Disjoint equalization-despreading MIMO MC-CDMA receiver

IV. JOINT EQUALIZATION-DESPREADING RECEIVER

It is also possible to treat jointly the multi-antenna interference and the spreading. The transmitter and the receiver in the case of joint reception is depicted in Figure 2.

The joint equalization matrix and despreading MMSE is equal to:

$$\mathcal{F} = (\mathcal{C}^T\mathcal{H}^H\mathcal{H}\mathcal{C} + \frac{1}{\gamma}\mathcal{I})^{-1}\mathcal{C}^T\mathcal{H}^H \tag{6}$$

with γ the SNR at the receive antenna. The matrix size to invert increases linearly with the length of the spreading sequences and the nunber of transmit and receive antennas. However with a system at full load, there is an equivalence between disjoint MMSE receiver and joint MMSE receiver.

$$\begin{aligned}\mathcal{F} =& (\mathcal{H}^H\mathcal{H}\mathcal{C} + \frac{1}{\gamma}\mathcal{C})^{-1}(\mathcal{C}^T)^{-1}\mathcal{C}^T\mathcal{H}^H \\ =& \, (\mathcal{H}^H\mathcal{H}\mathcal{C} + \frac{1}{\gamma}\mathcal{C})^{-1}\mathcal{H}^H \\ =& \, \mathcal{C}^{-1}(\mathcal{H}^H\mathcal{H} + \frac{1}{\gamma}\mathcal{I})^{-1}\mathcal{H}^H \\ =& \, \mathcal{C}^T(\mathcal{H}^H\mathcal{H} + \frac{1}{\gamma}\mathcal{I})^{-1}\mathcal{H}^H\end{aligned} \tag{7}$$

V. ITERATIVE RECEIVERS

The insertion of channel coding for MIMO MC-CDMA systems allows the use of an iterative receiver at the receiver side. The iterative receiver for MIMO MC-CDMA systems is depicted on Figure 3.

First, the interference canceller needs a first estimation of transmitted symbols. This first estimation is obtained using a SU-MMSE MIMO MC-CDMA technique. Then, it is necessary to calculate the extrinsic information on the coded bits after soft demodulation, deinterleaving and channel decoding. After a soft modulation and spreading of the new data symbols, the formula of the interference canceller is equal to:

$$\hat{\mathbf{s}}^p = (diag\Gamma + \frac{L_c}{N_u\gamma}\mathcal{I})^{-1}(\mathcal{H}^H\mathbf{r} - (\Gamma - diag\Gamma)\hat{\mathbf{s}}^{p-1}) \tag{8}$$

with

$$\Gamma = \mathcal{H}^H\mathcal{H} \tag{9}$$

For a MIMO MC-CDMA system using an interference canceller performing jointly the cancellation of multi-antenna interference and spreading interference, this leads to the following formula:

$$\hat{\mathbf{x}}^p = (diag\Gamma + \frac{1}{\gamma}\mathcal{I})^{-1}(\mathcal{C}^T\mathcal{H}^H\mathbf{r} - (\Gamma - diag\Gamma)\hat{\mathbf{x}}^{p-1}) \tag{10}$$

with

$$\Gamma = \mathcal{C}^T\mathcal{H}^H\mathcal{H}\mathcal{C} \tag{11}$$

The complexity of the joint iterative receiver has a complexity linearly superior to the disjoint one owing to the inversion of a diagonal matrix.

VI. SIMULATION RESULTS

In this section the performance of the non-iteraitve and iterative MIMO MC-CDMA receivers presented are compared for different loads. A system with 4 transmit and 4 receive antennas with QPSK modulation is chosen and multiuser sequences of length $L_c = 64$. The number of subcarrier is equal to $N_c = 64$. A convolutive code $(23, 35)_o$ with rate $R = 1/2$ is chosen leading to a spectral efficiency 4 bps/Hz. 4 iterations are performed at the receiver side. The channels used for the simulations are decorrelated Rayleigh flat fading channels per subcarrier corresponding to any OFDM Rayleigh frequency and temporal selective channel with perfect interleaving. The Additive White Gaussian Noise (AWGN) QPSK curve corresponds to the optimal performance that can be reached in a SISO or a MIMO context.

Figure 4 shows the performance of a MIMO 4x4 MC-CDMA system at full load $N_u = 64$. The gain between an iterative receiver and a non iterative receiver is 6.5 dB at $BER = 10^{-4}$. A SU detection technique with MMSE equalizer is chosen for the first iteration and a joint iterative receiver for the following iterations. This leads to the optimal performance, that is the Matched Filter Bound (MFB) close to the performance of a gaussian channel (SISO AWGN). This performance can

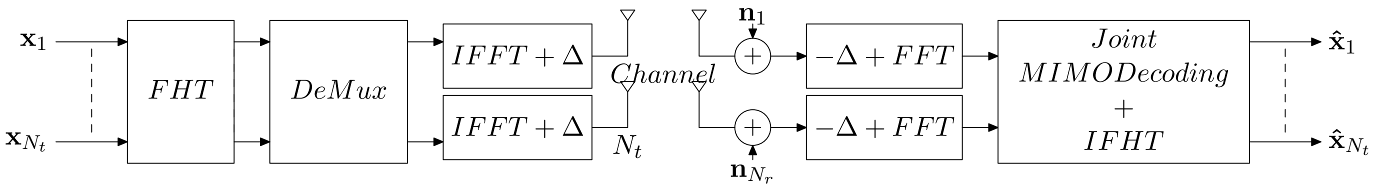

Fig. 2. Joint equalization-despreading MIMO MC-CDMA receiver

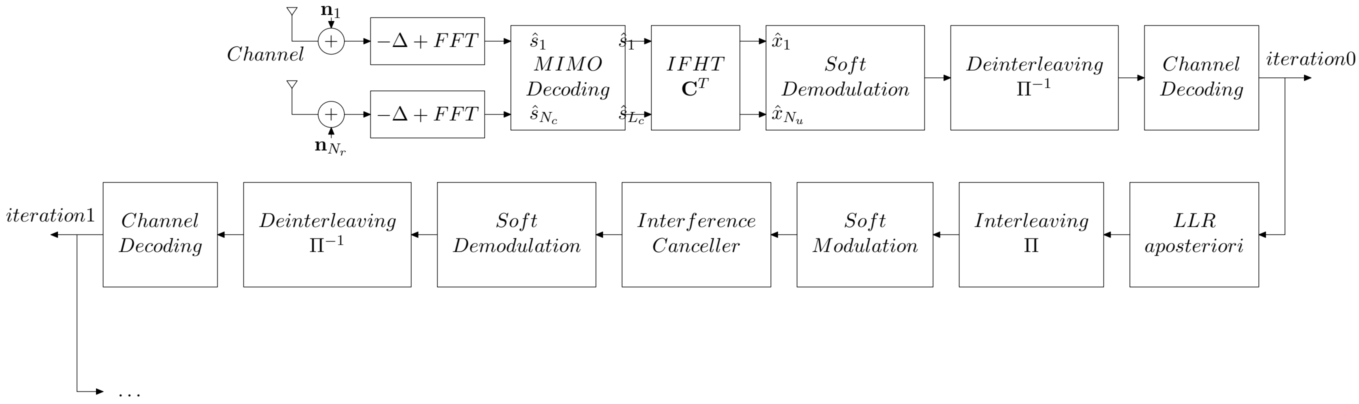

Fig. 3. Iterative MC-CDMA receiver

be reached owing to the optimal exploitation of spatial, frequency and temporal diversities. The complexity of the iterative receiver is still affordable since a SU detection technique with MMSE equalizer needs the inversion of the MIMO channel matrix and the joint iterative MMSE receiver needs the inversion of a diagonal matrix.

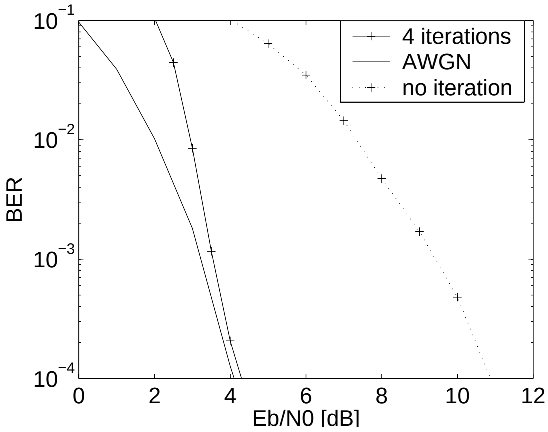

Fig. 4. Performance between iterative and non iterative MIMO 4x4 MC-CDMA receivers at full load with $N_c = N_u = L_c = 64$ at 4 bps/Hz

Figure 5 shows the performance of the same MIMO 4x4 MC-CDMA system with an iterative receiver at full and half load. The performance of the MFB is reached whatever the load when an iterative receiver is applied at the receiver side. However, the system with a lower load obtains better performance at low SNR.

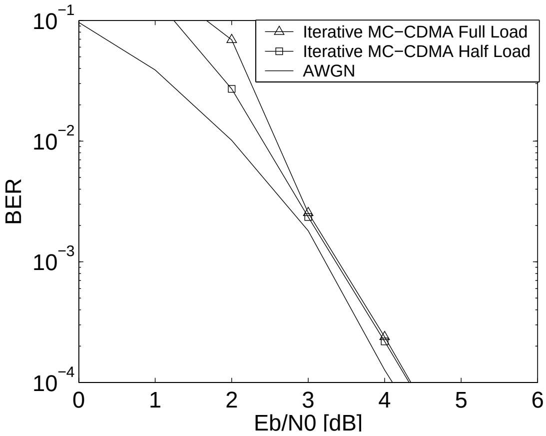

Fig. 5. Performance between full and half load MIMO 4x4 MC-CDMA systems with iterative receivers at 4 bps/Hz

VII. CONCLUSION

In this article, a simple iterative receiver for spatial multiplexed MC-CDMA systems has been presented. SU detection technique provides a very good tradeoff between performance and complexity. Non-iterative SU detection technique with MMSE equalizer and iterative receivers have been compared in terms of performance.

The simulations uses a MIMO 4x4 system with 64 unorrelated channels and a low complexity receiver including a SU detection technique with MMSE equalizer at the first iteration and a joint MMSE interference canceller for the following iterations at full load. For decorrelated flat Rayleigh channels per subcarrier representing any frequency and selective Rayleigh channel with OFDM modulation and perfect interleaving, the gain between a non iterative and iterative receiver is 6.5 dB. Moreover, the performance of the iterative receiver reaches the performance of the MFB close to the gaussian performance owing to high diversity order exploited. For lower loads, the iterative receiver has better performance at low SNR.

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