SPACE-TIME BLOCK CODES WITH MAXIMAL DIVERSITY IN OFDM-BASED SYSTEMS

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Abstract

In this paper, Orthogonal Space Time Block Coding (OSTBC) is combined with linear precoding matrices to improve the exploitation of the different diversities for a multicarrier transmission. The linear precoding matrices based on Hadamard construction are proposed where a simple linear de-precoder can be chosen at the receiver part which performs closely to the Maximum Likelihood (ML) global precoded OSTBC decoder. The system is generalized to different numbers of transmit antennas and different sizes of precoding matrices. Performance results have been obtained in theoretical and real channel environments. It is shown that the combined scheme efficiently exploits the space, time and frequency diversities while using a linear decoder. This system leads to an OSTBC with maximal diversity for more than two antennas when a Zero Forcing (ZF) or a Minimum Mean Square Error (MMSE) equalizer is chosen.

1 Introduction

OSTBC was first proposed by Alamouti [1] and generalized by Tarokh [2]. In this paper, we put the scheme that we briefly presented in [3] that consists of combining linear precoding and OSTBC into general use. We showed in [3] that the system using Hadamard linear precoding matrices based on the Special Unitary SU(2) group and a simple linear de-precoder offers a good tradeoff between performance and complexity and performs very close to the global precoded OSTBC ML decoder. The interest of our system is that the precoding matrix size can be adjusted in order to exploit space, time and frequency diversities while the systems proposed in the literature are limited to small sizes for the precoding matrix due to the use of ML-like decoders.

The system keeps the initial rate of the OSTBC, i.e. 1 for the Alamouti code and 1/2 or 3/4 for Tarokh codes. In [3] and in this paper, the diversity of the system is demonstrated to be increased owing to linear precoding applied in the time, frequency, or space domains and using several groups of transmit antennas alternatively. It is shown that this specific combination of linear precoding and OSTBC results in an orthogonal system providing maximal diversity.

First the state of the art of STBC is given, focusing on systems with 4 transmit antennas. Then, the generalised scheme is presented, which can be adapted to various MIMO configurations with different numbers of transmit antennas. The linear precoding matrices are described, i.e. the complex Hadamard matrices based on SU(2) construction. It is shown that according to the chosen matrices used for the linear precoding, a non-centered gaussian diversity can be obtained for the detected symbols. Finally, simulation results are given for different precoding matrix sizes confirming the theoretical analysis in uncorrelated Rayleigh flat fading and real environments when combining with Orthogonal Frequency Division Multiplex (OFDM) modulation.

For this study, the uncorrelated frequency non-selective Rayleigh fading channel is time invariant during T OFDM symbol durations. Perfect channel estimation is assumed on the receiver side. The channel corresponds to a quasi-static flat fading well adapted to OFDM-like modulations. Hence, the theoretical channel response of the uncorrelated channels from transmitting antenna t to receiving antenna r can be represented by $h_{tr} = \rho_{tr} e^{i\theta_{tr}}$. For the real environment, simulations are carried out with independant BRAN A and BRAN E channels between each couple of transmit and receive antennas [6].

2 State of the art

As Tarokh demonstrated that no orthogonal design with full spatial diversity exists for more than two transmit antennas and rate 1 [1][2], some research is focused on Non Orthogonal STBC (NOSTBC). For the NOSTBC proposed in [4] and [5] by Jafarkhani and Tirkkonnen respectively, the codes are represented by a 4x4 matrix:

$$\mathbf{X}_{Jaf} = \begin{bmatrix} x_1 & -x_2^* & -x_3^* & x_4 \\ x_2 & x_1^* & -x_4^* & -x_3 \\ x_3 & -x_4^* & x_1^* & -x_2 \\ x_4 & x_3^* & x_2^* & x_1 \end{bmatrix} \tag{1}$$

$$\mathbf{X}_{Tir} = \begin{bmatrix} x_1 & -x_2^* & x_3 & -x_4^* \\ x_2 & x_1^* & x_4 & x_3^* \\ x_3 & -x_4^* & x_1 & -x_2^* \\ x_4 & x_3^* & x_2 & x_1^* \end{bmatrix} \tag{2}$$

with $\mathbf{x} = [x_1 \quad x_2 \quad x_3 \quad x_4]^T$ the transmitted vector whose components are Quadrature Amplitude Modulation

(QAM) symbols or Phase shift Keying (PSK) symbols, The total transmit power is P. Each antenna transmits a symbol over one symbol duration, therefore in a 4 transmit antenna system, each antenna transmits symbols at a power of $P/4$. Assuming the fading coefficients are constant over four consecutive symbol durations, the received vector is represented by:

$$\mathbf{r} = \mathbf{h}.\mathbf{X} + \mathbf{n} \tag{3}$$

where $\mathbf{r} = [r_1 \quad r_2 \quad r_3 \quad r_4]$ is the received signal and $\mathbf{h} = [h_1 \quad h_2 \quad h_3 \quad h_4]$ is the channel and $\mathbf{n} = [n_1 \quad n_2 \quad n_3 \quad n_4]$ is the Additive White Gaussian Noise (AWGN). This received vector must be rearranged for the decoding process leading to the vector $\mathbf{r}' = [r_1 \quad -r_2^* \quad -r_3^* \quad r_4]^T$ in the case of Jafarkhani's code or $\mathbf{r}' = [r_1 \quad -r_2^* \quad r_3 \quad -r_4^*]^T$ in the case of Tirkkonen's code leading to the following equation:

$$\mathbf{r}' = \mathbf{H}.\mathbf{x} + \mathbf{n}' \tag{4}$$

with $\mathbf{n}'$ the rearranged AWGN and

$$\mathbf{H}_{Jaf} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ -h_2^* & h_1^* & -h_4^* & h_3^* \\ -h_3^* & -h_4^* & h_1^* & h_2^* \\ h_4 & -h_3^* & -h_2 & h_1 \end{bmatrix} \tag{5}$$

$$\mathbf{H}_{Tir} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ -h_2^* & h_1^* & -h_4^* & h_3^* \\ h_3 & h_4 & h_1 & h_2 \\ -h_4^* & h_3^* & -h_2^* & h_1^* \end{bmatrix} \tag{6}$$

are the equivalent channel matrix representations. Applying the transconjugate of the equivalent channel matrix to the received vector leads to:

$$\mathbf{y} = \mathbf{H}^H.\mathbf{H}.\mathbf{x} + \mathbf{H}^H.\mathbf{n}' = \mathbf{\Lambda}.\mathbf{x} + \mathbf{H}^H.\mathbf{n}' \tag{7}$$

with

$$\mathbf{\Lambda}_{Jaf} = \begin{bmatrix} a & 0 & 0 & b \\ 0 & a & -b & 0 \\ 0 & -b & a & 0 \\ b & 0 & 0 & a \end{bmatrix} \tag{8}$$

$$\mathbf{\Lambda}_{Tir} = \begin{bmatrix} a & 0 & b & 0 \\ 0 & a & 0 & b \\ b & 0 & a & 0 \\ 0 & b & 0 & a \end{bmatrix} \tag{9}$$

where $a = \sum_{i=1}^{4} |h_i|^2$ and $b = 2Re(h_1 h_4^* - h_2 h_3^*)$ in the case of Jafarkani's code or $a = \sum_{i=1}^{4} |h_i|^2$ and $b = 2Re(h_1 h_3^* + h_2 h_4^*)$ in the case of Tirkkonen's code.

3 Linear Precoding with STBC

The Linear Precoding schemes with STBC transmission and reception are presented in Figure 1. First, the QAM or PSK symbols $\mathbf{x} = [x_1 \quad x_2 \quad \dots \quad x_L]^T$ are linear precoded by a complex Hadamard construction matrix based on SU(2) group of size $L \times L$ [3] such as:

$$\mathbf{\Theta}_L^{Had} = \sqrt{\frac{2}{L}} \begin{bmatrix} \mathbf{\Theta}_{L/2} & \mathbf{\Theta}_{L/2} \\ \mathbf{\Theta}_{L/2} & -\mathbf{\Theta}_{L/2} \end{bmatrix} \tag{10}$$

with $L = 2^n$, n $\in \mathbf{N}^*$, n ≥ 2 and:

$$\mathbf{\Theta}_2 = \begin{bmatrix} e^{j\theta_1}.\cos\eta & e^{j\theta_2}.\sin\eta \\ -e^{-j\theta_2}.\sin\eta & e^{-j\theta_1}.\cos\eta \end{bmatrix} \tag{11}$$

belonging to the Special Unitary group SU(2), therefore $det(\mathbf{\Theta_2}) = 1$ and $\Theta_2^{-1} = \Theta_2^H$.

The linear precoded symbols s are given by:

$$\mathbf{s} = \mathbf{\Theta}_L.\mathbf{x} \tag{12}$$

with $\mathbf{s} = [s_1 \quad s_2 \quad \dots \quad s_L]^T$ extended symbols from QAM or PSK modulations. The final step of the transmitter part is to map the symbol vector s into an OSTBC.

In the 4-transmit antenna system that we proposed in [3], the Alamouti OSTBC is chosen in order to keep the rate one for the new proposed scheme. The Alamouti OSTBC is applied alternatively to antennas 1 and 2 and then to antennas 3 and 4. Thus, the symbols are transmitted over the first group of transmit antennas 1 and 2 with a power $P/2$ over two symbol durations when the other antennas are switched off. Then, the other symbols are transmitted over the second group of transmit antennas 3 and 4. Therefore, as for the precedent NOSTBC codes for 4 antennas, the total transmit power per symbol duration is P. We obtain the following equivalent matrix:

$$\mathbf{S} = \begin{bmatrix} s_1 & -s_2^* & 0 & 0 \\ s_2 & s_1^* & 0 & 0 \\ 0 & 0 & s_3 & -s_4^* \\ 0 & 0 & s_4 & s_3^* \end{bmatrix} \tag{13}$$

The received vector must be rearranged leading to the vector $\mathbf{r}' = [r_1 \quad -r_2^* \quad r_3 \quad -r_4^*]^T$ and leads to the following channel matrix representation:

$$\mathbf{H} = \begin{bmatrix} h_1 & h_2 & 0 & 0 \\ -h_2^* & h_1^* & 0 & 0 \\ 0 & 0 & h_3 & h_4 \\ 0 & 0 & -h_4^* & h_3^* \end{bmatrix} \tag{14}$$

After the decoding process consisting of the transconjugate of the equivalent channel matrix representation, we get:

$$\mathbf{\Lambda} = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & b \end{bmatrix} \tag{15}$$

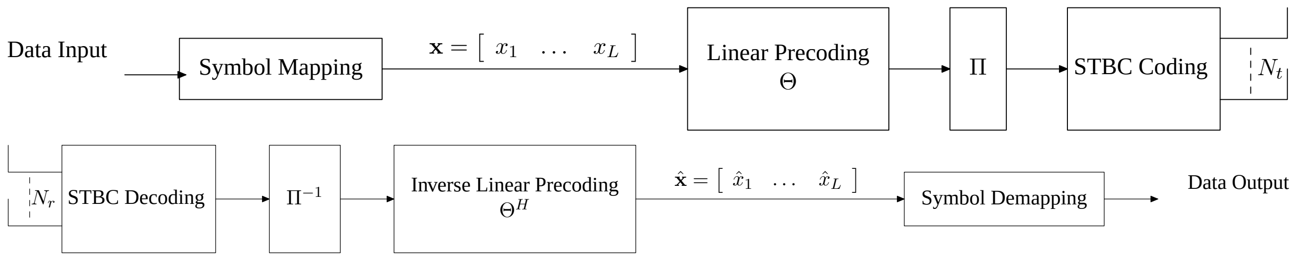

Figure 1: Linear Precoded STBC Transmitter and Receiver

where $a = \sum_{i=1}^{2} |h_i|^2$ and $b = \sum_{i=3}^{4} |h_i|^2$. So far, there is no gain compared to the classical Alamouti scheme described in [2]. Considering an interleaving process between Linear Precoding and STBC Coding for the transmitter part, after de-interleaving at the receiver side the diagonal elements are mixed up between blocks leading to:

$$\mathbf{\Lambda} = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix} \tag{16}$$

where $a = \sum_{i=1}^{2} |h_i|^2$, $b = \sum_{i=3}^{4} |h_i|^2$, $c = \sum_{i=5}^{6} |h_i|^2$, $d = \sum_{i=7}^{8} |h_i|^2$ provided by different Alamouti decoded blocks.

We have seen that before being space-time coded, the symbols are preliminarly linear precoded with a $L \times L$ unitary matrix based on Hadamard construction. In fact, this linear precoding has the effect of increasing the diversity order of the transmitted symbols. Using $\mathbf{\Theta}_L^{Had}$ for $L = 4$ and applying the coefficients $\eta = \frac{\pi}{4}$, $\theta_2 = \theta_1 - \frac{\pi}{2}$, $\theta_1 = \frac{5\pi}{4}$ leads to the global transmission and reception system described by:

$$\mathbf{A}_4 = \mathbf{\Theta}_4^H.\mathbf{\Lambda}.\mathbf{\Theta}_4 \tag{17}$$

$$\mathbf{A}_4 = \frac{1}{4}\begin{bmatrix} a+b+c+d & a-b+c-d & a+b-c-d & a-b-c+d \\ -a+b-c+d & a+b+c+d & -a+b+c-d & a+b-c-d \\ a+b-c-d & a-b-c+d & a+b+c+d & a-b+c-d \\ -a+b+c-d & a+b-c-d & -a+b-c+d & a+b+c+d \end{bmatrix} \tag{18}$$

The transconjugate of the channel $\mathbf{G} = \mathbf{H}^H$ is equivalent to an Maximum Ratio Combining (MRC) equalizer. The diagonal terms and off-diagonal terms have respectively diversity that tend towards a non-centered gaussian law and a centered gaussian law when L increases in an uncorrelated Rayleigh flat fading channel environment. Therefore, when using an MRC equalizer with perfect channel estimation, the global system is non-orthogonal. Here, the diagonal terms follow a χ^2_{16} law applying an Alamouti MRC equalizer in an uncorrelated Rayleigh flat fading channel environment. The interference terms follow a χ^2_8 law difference. With perfect channel estimation and a ZF equalizer, the interference terms are null as well as with an MMSE equalizer at high SNR.

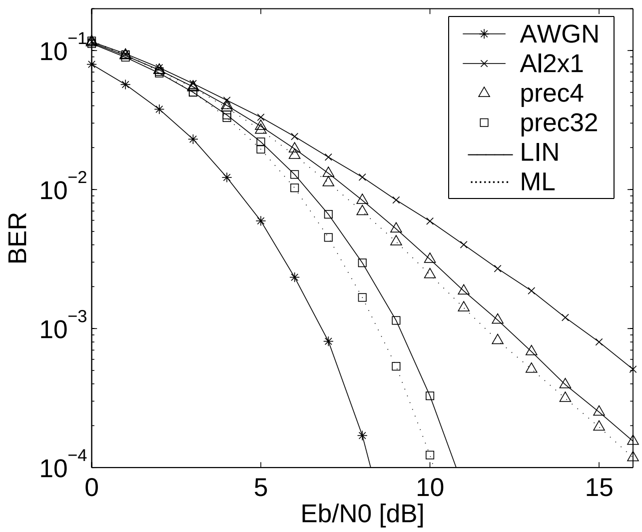

Figure 2: Performance of Alamouti with linear precoding for $L = 4$ and 32 and η=1 bps/Hz with linear or ML receiver over the theoretical channel

In the case of a ZF equalizer, the equalization matrix is :

$$\mathbf{G} = \frac{\mathbf{H}^H}{\mathbf{H}^H\mathbf{H}} \tag{19}$$

whereas in the case of an MMSE equalizer, the equalization matrix is :

$$\mathbf{G} = \frac{\mathbf{H}^H}{\mathbf{H}^H\mathbf{H} + \frac{1}{\gamma}\mathbf{I}} \tag{20}$$

where γ is the Signal to Noise Ratio at the receive antenna.

When using a ZF equalizer with perfect channel estimation, all the off-diagonal terms are null. In order to avoid enhancing of the noise, it is preferable to choose an MMSE equalizer, leading to very small interference terms and leading to a global orthogonal system at high SNR. We can generalize the proposed scheme to any size $L \times L$. For the Alamouti code, the generalization corresponds to the fol-

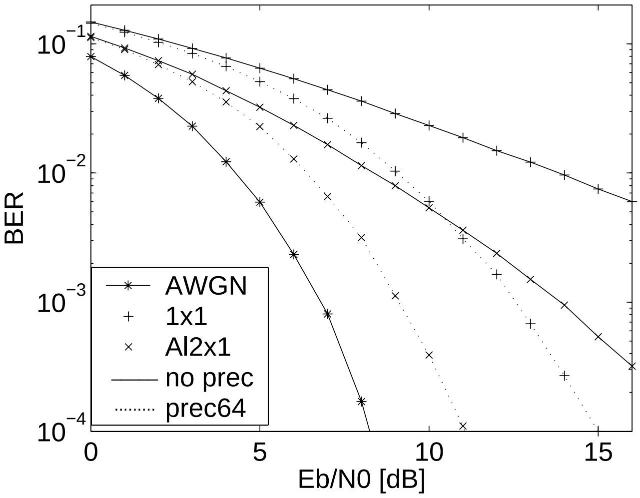

Figure 3: Performance of SISO and Alamouti OFDM-based systems without and with linear precoding for $L = 64$ and η=1bps/Hz over the theoretical channel

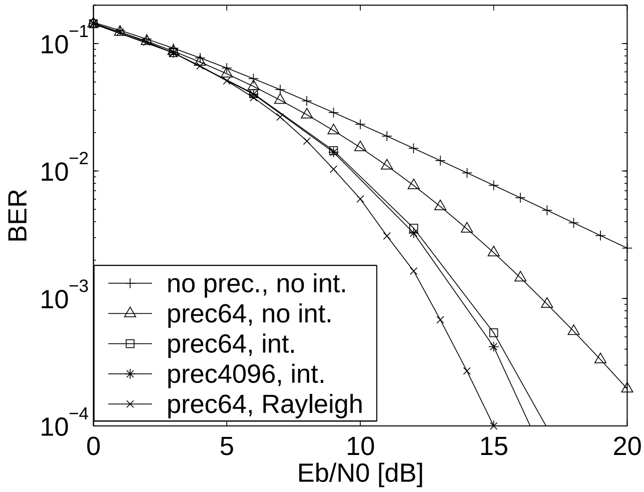

Figure 4: Performance of linear precoded SISO OFDM-based systems for BRAN A and 3km/h

lowing matrix:

$$\mathbf{S} = \begin{bmatrix} s_1 & -s_2^* & 0 & 0 & \dots & \dots & 0 & 0 \\ s_2 & s_1^* & 0 & 0 & \dots & \dots & 0 & 0 \\ 0 & 0 & s_3 & -s_4^* & \ddots & \ddots & 0 & 0 \\ 0 & 0 & s_4 & s_3^* & \ddots & \ddots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & \dots & s_{L-1} & -s_L^* \\ 0 & 0 & 0 & 0 & \dots & \dots & s_L & s_{L-1}^* \end{bmatrix} \tag{21}$$

with

$$\mathbf{s} = \mathbf{\Theta}_L.\mathbf{x} \tag{22}$$

Applying an Alamouti MRC equalizer in a flat Rayleigh fading channel environment with complex Hadamard linear precoding matrices of size $L \times L$, the diagonal elements follow a χ^2_{4L} law and the non diagonal elements follow a χ^2_{2L} law difference. This scheme can be generalized to any STBC and uses efficiently uses the space domain, keeping the rate of the initial code, i.e. 1 for the Alamouti code. The system leads to a global orthogonal transmission/reception scheme when a ZF or an MMSE equalizer is chosen for more than two antennas.

OFDM-based systems provide full frequency diversity owing to the orthogonality between sub carriers. Contrary to the precedent part where linear precoding was done in the time domain, it is possible to apply the linear precoding in the frequency and time domains. In order to apply linear precoding in the frequency domain, one may use a linear precoder of size $L \leq N_c$, where N_c is the number of sub-carriers. In order to apply linear precoding in the time and frequency domains, one may use a linear precoder of size $L \geq N_c$. Therefore precoded linear multi-carrier schemes with OSTBC have their temporal, spatial and frequency diversities exploited.

4 Simulation results

The simulations are carried out in an uncorrelated Rayleigh flat fading channel environment (theoretical channel) well adapted to OFDM-like modulations and two real channel environments which are BRAN A and BRAN E [6]. The chosen STBC is the Alamouti code. The MMSE equalization technique and different interleaved precoding matrix sizes have been implemented. In fact, we have checked that the MMSE equalizer provides the best results compared to the MRC or the ZF equalizer as it is demonstrated in section 3. Thus, only results with MMSE will be presented in this paper.

Figure 2 shows the performance of a linear precoded Alamouti system over the theoretical channel with complex Hadamard based SU(2) matrices for different sizes of precoding matrices ($L = 4$ and 32), with $N_t = 2^n, n \in \mathbf{N}^*$ corresponding to $N_t = 2, 4, 8, \dots$ antennas and $N_r = 1$. As expected, owing to a higher diversity order, the performance of the linear precoded OSTBC scheme performs better than the sole Alamouti scheme. The same performance are obtained whatever the value of n and $L = 32$. The performance of the linear receiver performs very close to the ML receiver with no match for the complexity implementation. Indeed, the complexity of an ML decoder increases according to the length of the precoding matrix size L. For $L = 32$ with a BPSK, 2^{32} codewords need to be compared. Therefore, there is a small loss of performance (0.5 dB at $BER = 10^{-4}$) compare to the price of complexity that should be used.

Figure 3 shows the performance of the linear precoded

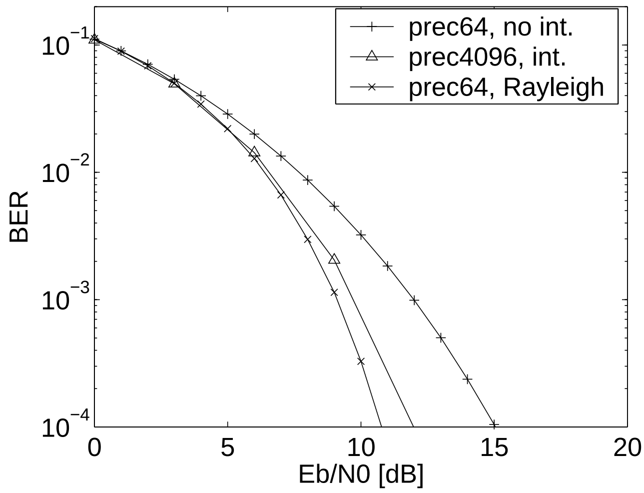

Figure 5: Performance of linear precoded Alamouti OFDM-based systems for BRAN A and 3km/h

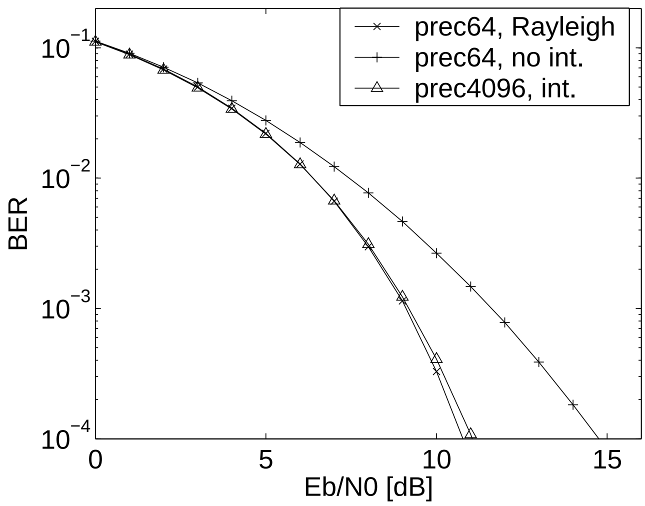

Figure 6: Performance of linear precoded Alamouti OFDM-based systems for BRAN E and 60km/h

SISO and Alamouti systems with a complex Hadamard based SU(2) matrix of size $L = 64$ over the theoretical channel with $N_t = 2^n$, $n \in \mathbf{N}^*$, $N_r = 1$ and $N_c = 64$ sub carriers. We can observe that there is a gain of 4 dB at $BER = 10^{-4}$ between the precoded SISO case and the precoded Alamouti case.

Figure 4 presents the performance of SISO systems in a BRAN A environment [6] with mobility 3 km/h and $N_c = 64$ sub carriers. Linear precoded SISO cases with and without interleaving are compared. For $L = 64$ without interleaving, we observe a loss in the diversity order from the theoretical case due to the coherence bandwidth of 3.18 MHz over the total bandwidth 20 MHz. An interleaver is added in order to exploit the temporal selectivity not fully exploited by the linear precoding. However this huge interleaver is not realistic for actual implementation but is used here for theoretical demonstration. Furthermore, an additional gain can be obtained using a larger size of linear precoding matrix, i.e. $L = 4096$ gives better performance than $L = 64$ with interleaving. Indeed, the curve $L = 4096$ with interleaver reaches almost the theoretical bound given by uncorrelated Rayleigh channels and linear precoding (prec64, Rayleigh).

Figure 5 and Figure 6 present the performance of Alamouti systems with linear precoding in a BRAN A (3km/h) and BRAN E (60 km/h) environments respectively [6]. The BRAN A or the BRAN E environments for the SISO case are adapted to the MIMO case by taking independant uncorrelated channels, each having 18 taps and needing a guard interval of 15 $\%$ over the OFDM symbol duration. A correlation could have been introduced by adding a correlation matrix between these different BRAN A or BRAN E channels. One can see that there is a reduction in performance with a non-interleaved $L = 64$ system compared to the theoretical case (prec64, Rayleigh). However, as previously demonstrated, by increasing the linear precoding size to $L = 4096$ and by applying an very large interleaver, the performance loss can be avoided. In this way, the performance obtained over the BRAN A or BRAN E environment would reach those of a linear precoded flat Rayleigh environment for Alamouti systems. Specifically, due to the temporal diversity provided by the BRAN E channel, the theoretical performance are reached with a smaller interleaver. Therefore linear precoding and interleaving are parameters that could be adjusted in real channel environments in order to reach the theoretical performance.

5 Conclusion

The proposed combination of linear precoding and OSTBC applied to several transmit antennas leads to an efficient exploitation of space, time and frequency diversities in OFDM systems. With a ZF equalizer or MMSE equalizer, the interference terms are null or quasi null when perfect channel estimation is performed, leading to a quasi-orthogonal system in the MMSE case. Thus, a simple linear decoder provides a very good performance versus complexity tradeoff compared to other systems proposed in the literature based on ML receivers. Simulation results with the specific linear precoders using OSTBC were adapted to multi-carrier modulations and real channel environements such as BRAN A or BRAN E. Therefore, these precoders can be applied to various MIMO transmissions in order to efficiently and simply exploit spatial, temporal and frequency diversities. As demonstrated in real channel environments, depending on the temporal channel characteristics, an interleaver may be added in order to reach optimal performance.

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