Performance Analysis of Linear Precoded Space-Time Block Codes with Channel Coding

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***Abstract*— In this paper, we combine Convolutional Codes (CC) and Turbo Codes (TC) as channel coding with Space-Time Block Coding (STBC) in order to exploit spatio-temporal diversities. Moreover, linear precoding matrices are combined to efficiently improve the space-time diversity order of the multiple antenna system. The proposed system implements a simple Minimum Mean Square Error (MMSE) linear decoder at the reception side, which has very low complexity compared to the Maximum Likelihood (ML) decoders currently used in such systems. Performance results obtained by simulations are given for a 2 transmit antennas system with different precoding matrix sizes over flat Rayleigh fading channels for a global system including channel coding. The described scheme can easily and efficiently be adapted to a different system with more transmit antennas.**

I. Introduction

In 1996, Foschini demonstrated that the spectral efficiency increases linearly with the minimum number of transmit or receive antennas [1] when using multiple antennas systems. Another way to exploit multiple antennas is to use STBC. The initial two-transmit antennas Orthogonal STBC (OSTBC) proposed by Alamouti [2] is merely decoded with a linear operation. Another advantage of this code is its unitary rate. Then, Tarokh [3] extended OSTBC to 3 or 4 transmit antennas with linear decoding as well, but resulting in lower 1/2 and 3/4 rate codes. Since, many studies have been carried out to find out space-time codes with more than two antennas and rate one, resulting in Quasi Orthogonal STBC (QOSTBC) [4, 5], but requiring more complex than linear decoders such as Maximum Likelihood (ML) decoder.

In parallel, linear precoding also called constellation rotation was demonstrated to efficiently exploit time diversity for Single Input Single Output (SISO) systems [6]. In [7], a system combining the STBC proposed in [4] with linear precoding is presented, but the complexity of its ML detector varies exponentially with the length of the precoding matrix.

Since 1993, Turbo codes are demonstrated to be very efficient to exploit the coding diversity, performing close to the Shannon limit [8]. In this paper, we tested the influence of the channel coding for the system presented in [9], where linear precoding and OSTBC are efficiently combined. As channel coding scheme, we have implemented the robust duo binary turbo channel encoder that is proposed in DVB-RCT standard [10], as well as CC. At the receiver, before channel decoding a simple linear MMSE decoder offers a good trade-off between performance and complexity thanks to low interference terms.

II. Linear Precoding

In this section, we present the unitary linear precoding matrices which are the Fourier, the Vandermonde and the complex Hadamard matrices for a size $L \times L$.

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The linear precoding matrix obtained with Fourier Transform matrix construction is:

$$\mathbf{\Theta}_L^{FFT} = \frac{1}{\sqrt{L}} \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & w^1 & w^2 & \dots & w^{L-1} \\ 1 & w^2 & w^4 & \dots & w^{2(L-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & w^{L-1} & w^{2(L-1)} & \dots & w^{(L-1)(L-1)} \end{bmatrix} \quad (1)$$

with $w = e^{\frac{2j\pi}{L}}$.

The linear precoding matrix obtained with Vandermonde matrix construction is:

$$\mathbf{\Theta}_L^{Van} = \mathrm{diag}[1, \alpha, \alpha^2, \dots, \alpha^{L-1}].\mathbf{\Theta}_L^{FFT} \quad (2)$$

$$\mathbf{\Theta}_L^{Van} = \frac{1}{\sqrt{L}} \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ \theta_1 & \theta_2 & \theta_3 & \dots & \theta_{L-1} \\ \theta_1^2 & \theta_2^2 & \theta_3^2 & \dots & \theta_{L-1}^2 \\ \vdots & \vdots & \vdots & & \vdots \\ \theta_1^{L-1} & \theta_2^{L-1} & \theta_3^{L-1} & \dots & \theta_{L-1}^{L-1} \end{bmatrix} \quad (3)$$

with $\theta_i = \alpha w^i = \alpha e^{\frac{2ij\pi}{L}}$.

The linear precoding based on the complex Hadamard construction base on SU(2) group matrix is :

$$\mathbf{\Theta}_L^{Had} = \sqrt{\frac{2}{L}} \begin{bmatrix} \mathbf{\Theta}_{L/2} & \mathbf{\Theta}_{L/2} \\ \mathbf{\Theta}_{L/2} & -\mathbf{\Theta}_{L/2} \end{bmatrix} \quad (4)$$

with $L = 2^n$, n $\in \mathbf{N}^*$, n ≥ 2 and:

$$\mathbf{\Theta}_2 = \begin{bmatrix} e^{j\theta_1}.\cos\eta & e^{j\theta_2}.\sin\eta \\ -e^{-j\theta_2}.\sin\eta & e^{-j\theta_1}.\cos\eta \end{bmatrix} \quad (5)$$

belonging to the Special Unitary group SU(2), therefore $det(\mathbf{\Theta_2}) = 1$ and $\Theta_2^{-1} = \Theta_2^H$.

Figure 1 represents the 3-D Gaussian density of linearly precoded Quadrature Phase Shift Keying (QPSK) symbols by one of the three linear precoding matrices of size $L = 256$. In fact, the projection of a uniform probability distribution over an L-sphere onto one or two dimensions is a non-uniform probability distribution that approaches a Gaussian density as L increases. When using Quadrature Amplitude Modulation (QAM) symbols equiprobably distributed, the result of the projection over a sphere of dimension L is a 3-D Gaussian density when L is large, as for $L = 256$.

III. Theoretical performance with diversity

In this section we provide the theoretical performance of a system with L independent branches in a flat Rayleigh fading. Assuming that a code word $\tilde{\mathbf{x}}$ is detected whereas the code word $\mathbf{x}$ is sent, for a fixed channel and with an Additive White Gaussian Noise (AWGN) whose variance is $\sigma^2 = \frac{N_0}{2}$ per dimension, the Pairwise Error Probability (PEP) can be expressed with the following formula :

$$P(\mathbf{x} \rightarrow \tilde{\mathbf{x}}|\mathbf{H}) = Q\left(\frac{d(\mathbf{x}, \tilde{\mathbf{x}})}{2\sigma}\right) \quad (6)$$

where $d(\mathbf{x}, \tilde{\mathbf{x}})$ denotes the Euclidean distance between code word $\mathbf{x}$ and $\tilde{\mathbf{x}}$ and $Q(y)$ is the Gaussian tail function.

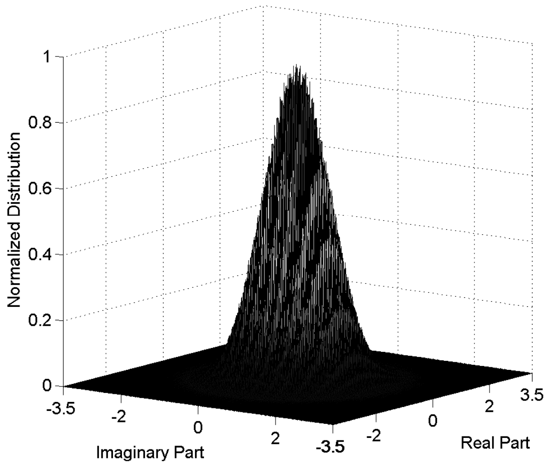

Fig. 1. linear precoded QPSK matrices for L=256

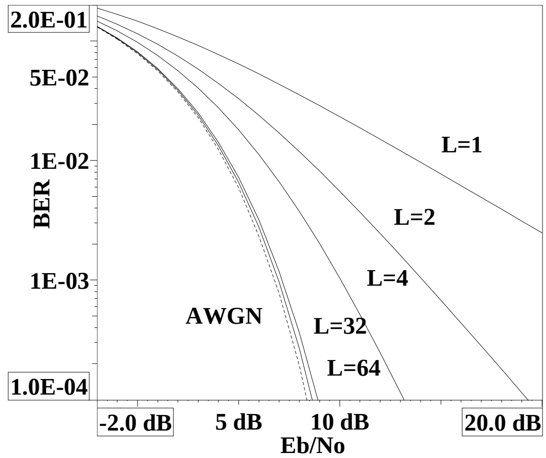

Fig. 2. Bit error rate for system with L-branch diversity

If we call $\mathbf{H}$ the matrix representation of the channel, the formula becomes :

$$P(\mathbf{x} \rightarrow \tilde{\mathbf{x}}|\mathbf{H}) = Q\left(\sqrt{\frac{||\mathbf{Hx} - \mathbf{H}\tilde{\mathbf{x}}||^2}{2N_0}}\right) \quad (7)$$

Let N the length of the code word, the probability that $\mathbf{x}$ and $\tilde{\mathbf{x}}$ differ at $L < N$ positions, we obtain :

$$P_L(\mathbf{x} \rightarrow \tilde{\mathbf{x}}) = Q\left(\sqrt{\sum_{l=1}^{L} |h_l|^2 \frac{2E_s}{N_0}}\right) \quad (8)$$

If all the fading amplitudes h_l are independent and identically Rayleigh distributed, it can be shown that $\sum_{l=1}^{L-1} |h_l|^2$ follows a chi-square probability law with $2L$ degrees of freedom. By evaluating this integral for a BPSK modulation, we obtain [11]:

$$P_L(\mathbf{x} \rightarrow \tilde{\mathbf{x}}) = \frac{1}{2}\left[1 - \mu \sum_{k=0}^{L} \binom{2k}{k} \left(\frac{1-\mu^2}{4}\right)^k\right] \quad (9)$$

with

$$\mu = \sqrt{\frac{E_s/N_0}{1 + E_s/N_0}} \quad (10)$$

Fig. 2 shows the bit error probability obtained with (9) for a L-branch diversity. We can denotes that when diversity L increases, the bit error probability tends towards AWGN curve.

IV. Association of linear precoding with OSTBC

In this section, the association of linear precoding and OSTBC is presented. Then the effect of linear precoding on the symbols and the noise is investigated.

A. Description of the proposed scheme

Let N the number of symbols to be transmitted and T the number of symbol durations over which the channel is constant. The rate R of OSTBC is the ratio N/T. Let $\mathbf{x} = [x_1 \dots x_L]^T$ complex symbols. We consider a $L \times L/R$ matrix such that the entries $\mathbf{S}(x_1, \dots, x_L)$ are complex linear combinations of x_i, $i = 1, \dots, L$ and their conjugates according to part I and $\mathbf{S}^H(x_1, \dots, x_L)\mathbf{S}(x_1, \dots, x_L) = \mathcal{D}$, where $\mathcal{D}$ is a diagonal matrix whose components are linear combinations of $|s_i|^2$, $i = 1, \dots, L$ [3]. For instance, with the Alamouti code [2], this corresponds to the following matrix:

$$\mathbf{S} = diag(\mathbf{S}_1, \dots, \mathbf{S}_j, \dots, \mathbf{S}_{L/2}) \quad (11)$$

with

$$\mathbf{S}_j = \begin{bmatrix} s_{2j-1} & -s_{2j}^* \\ s_{2j} & s_{2j-1}^* \end{bmatrix} \quad (12)$$

and $\mathbf{s} = [s_1 \ \dots \ s_L]^T$ being linear precoded symbols such as $\mathbf{s} = \mathbf{\Theta}_L.\mathbf{x}$. The received vector $\mathbf{r} = [r_1 \ \dots \ r_L]$ is represented by $\mathbf{r} = \mathbf{h}.\mathbf{S} + \mathbf{n}$ where $\mathbf{h} = [h_1 \ \dots \ h_L]$ is the channel and $\mathbf{n} = [n_1 \ \dots \ n_L]$ is the AWGN. The received vector must be rearranged leading to the vector $\mathbf{r}' = [r_1 \ -r_2^* \ \dots \ r_{L-1} \ -r_L^*]^T$ and leading to the equation $\mathbf{r}' = \mathbf{H}.\mathbf{s} + \mathbf{n}'$ with $\mathbf{n}'$ the modified AWGN $\mathbf{n}' = [n_1 \ -n_2^* \ \dots \ n_{L-1} \ -n_L^*]^T$ and

$$\mathbf{H} = diag(\mathbf{H}_1, \dots, \mathbf{H}_j, \dots, \mathbf{H}_{L/2}) \quad (13)$$

with

$$\mathbf{H}_j = \begin{bmatrix} h_{2j-1} & h_{2j} \\ -h_{2j}^* & h_{2j-1}^* \end{bmatrix} \quad (14)$$

the equivalent channel matrix representation. The decoding process consists of applying a Maximum Ratio Combiner (MRC) equalizer to the equivalent received vector which is the transconjugate of the equivalent channel matrix :

$$\mathbf{y} = \mathbf{H}^H\mathbf{H}.\mathbf{s} + \mathbf{H}^H.\mathbf{n}' = \mathbf{\Lambda}.\mathbf{s} + \mathbf{H}^H.\mathbf{n}' \quad (15)$$

with

$$\mathbf{\Lambda} = diag(\mathbf{\Lambda}_1, \dots, \mathbf{\Lambda}_j, \dots, \mathbf{\Lambda}_{L/2}) \quad (16)$$

and

$$\mathbf{\Lambda}_j = \lambda_j \mathbf{I}_2 = (|h_{2j-1}|^2 + |h_{2j}|^2)\mathbf{I}_2 \quad (17)$$

Considering an interleaving process long enough, the diagonal elements are mixed up between leading to:

$$\mathbf{\Lambda} = diag(\lambda_1, \ldots, \lambda_L) \quad (18)$$

We have seen that before being space-time coded, the symbols are preliminarily linear precoded with a $L \times L$ unitary matrix as described in part II. The final step consists of applying the inverse linear precoding to the vector $\mathbf{y}$:

$$\mathbf{z} = \mathbf{\Theta}_L^H.\mathbf{y} = \mathbf{\Theta}_L^H.\mathbf{\Lambda}.\mathbf{\Theta}_L\mathbf{x} + \mathbf{\Theta}_L^H.\mathbf{H}^H.\mathbf{n}' \quad (19)$$

B. Effect of linear precoding on the symbols

This linear precoding will have the effect of increasing the diversity order of the transmitted symbols. For instance, using $\mathbf{\Theta}_L^{Van}$ for $L = 4$ and applying the coefficients $\alpha = \frac{\pi}{4}$, this leads to the global hermitian circulant matrix:

$$\mathbf{A}_4 = \frac{1}{4}\begin{bmatrix} a & b & c & b^* \\ b^* & a & b & c \\ c & b^* & a & b \\ b & c & b^* & a \end{bmatrix} \quad (20)$$

with

$$\begin{aligned} a &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \\ b &= \lambda_1 - \lambda_3 - j(\lambda_2 - \lambda_4) \\ c &= \lambda_1 - \lambda_2 + \lambda_3 - \lambda_4 \end{aligned} \quad (21)$$

The transconjugate of the channel $\mathbf{G} = \mathbf{H}^H$ is equivalent to an Maximum Ratio Combining (MRC) equalizer. When L increases in a flat Rayleigh fading channel environment, the diagonal terms have a diversity that tend towards a non-centered Gaussian law and interferences that tend towards a centered Gaussian law on the non-diagonal terms. Therefore, when using an MRC equalizer with perfect channel estimation, the global system is non-orthogonal. For Vandermonde matrices, the diagonal terms follow a χ^2_{16} law when an Alamouti MRC equalizer is chosen in a flat Rayleigh fading channel environment. The interference term noted c follows a χ^2_8 law difference and b follows a χ^2_4 law difference per dimension. In the case of a ZF equalizer, the equalization matrix is :

$$\mathbf{G} = \frac{\mathbf{H}^H}{\mathbf{H}^H\mathbf{H}} \quad (22)$$

In the case of an MMSE equalizer, the equalization matrix is :

$$\mathbf{G} = \frac{\mathbf{H}^H}{\mathbf{H}^H\mathbf{H} + \frac{1}{\gamma}\mathbf{I}} \quad (23)$$

where γ is the Signal to Noise Ratio at the receive antenna. When using a ZF equalizer with perfect channel estimation, all the non diagonal terms are null. In order to avoid enhancing the noise, it is preferable to choose an MMSE equalizer, leading to very small interference terms and leading to a global orthogonal system at high SNR.

C. Effect of linear precoding on the noise

However, this linear precoding will not improve the performance at low SNR. For instance, with the Alamouti code the noise terms become:

$$\mathbf{K} = \mathbf{H}^H.\mathbf{n}' = [K_1 \ \ldots \ K_j \ \ldots \ K_L] \quad (24)$$

with

$$K_j = h^*_{2j-1}n_{2j-1} + h_{2j}n^*_{2j} \quad \text{or} \quad K_j = h^*_{2j}n_{2j-1} - h_{2j-1}n^*_{2j} \quad (25)$$

With an MRC equalizer, the variance of this noise has the following form:

$$E[K_j^2] = 2N_0(|h_{2j-1}|^2 + |h_{2j}|^2) \quad (26)$$

per real dimension. With a ZF equalizer, the variance of noise becomes

$$E[K_j^2] = \frac{2N_0}{|h_{2j-1}|^2 + |h_{2j}|^2} \quad (27)$$

per real dimension. Applying the linear deprecoder will have the effect of averaging the variance of K_j, the noise becomes:

$$\mathbf{M} = \mathbf{\Theta_L^H}\mathbf{K} = \mathbf{\Theta_L^H}\mathbf{H}^H.\mathbf{n}' = [M_1 \ \ldots \ M_j \ \ldots \ M_L] \quad (28)$$

With a ZF equalizer, the variance of the noise becomes:

$$E[M_j^2] = \frac{2N_0}{L}\sum_{j=1}^{L}(|h_{2j-1}|^2 + |h_{2j}|^2)^{-1} \quad (29)$$

per real dimension. Owing to the last equation, the linear precoding has little impact on the noise.

V. Simulation results

The simulations are carried out in a Rayleigh flat fading channel environment well adapted to OFDM-like modulation with Alamouti and Tarokh codes, MMSE equalizer and interleaving for different sizes of precoding matrices. In fact, the MMSE equalizer provides the best results compared to the MRC or the ZF equalizer.

Figure 3 shows the performance of the linear precoded Alamouti system with complex Hadamard based SU(2) matrices for different sizes of precoding matrices ($L = 4$ and 64) and $N_r = 1$ without channel coding. We observe that the specified precoded system with $L = 4$ outperforms of 2 dB the sole Alamouti scheme without precoding at $BER = 10^{-3}$. One can see that the slopes of the simulated curves are parallel to the slopes of the theoretical curves. However, a shift can be seen between the simulated and the theoretical curves because of a loss in performance due to the variance of the noise described in part IV-C.

Figure 4 provides BER performance of the linear precoded system for one (SISO) and two antennas (Alamouti) with non systematic convolutionnal channel coding of half rate with constraint length $K = 7$ and $(133, 171)_o$ polynomial generators. In spite of the diversity gain brought by the channel coding, linear precoding exploits an aditionnal diversity leading to an higher slope for both SISO and Alamouti schemes.

Finally, the performance of both schemes (SISO and Alamouti) are carried out in Figure 5 with duo-binary turbo channel coding of half rate proposed in DVB-RCT standard and packet

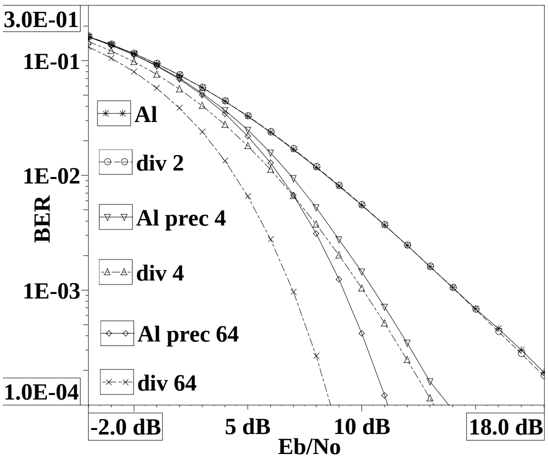

Fig. 3. Linear precoded Alamouti scheme

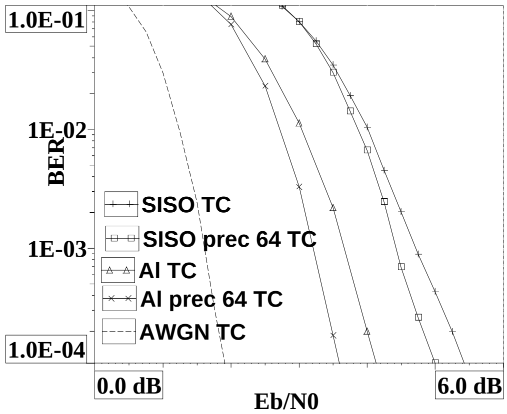

Fig. 5. Linear precoded SISO and Alamouti schemes with duo-binary turbo-coding

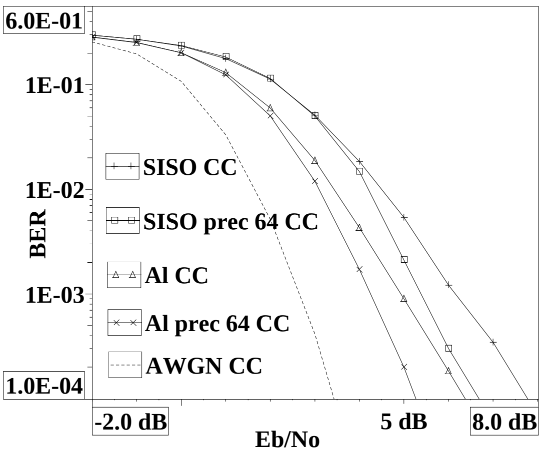

Fig. 4. Linear precoded SISO and Alamouti schemes with convolutionnal coding

size 432 [10]. The proposed schemes with linear precoding have a $0.5dB$ performance gain compared to their respective systems without precoding. Moreover, the use of a precoding matrix allows to achieve better performance even with a very low value of precoding matrix owing to the additional diversity brought to the turbo-decoding.

VI. CONCLUSION

In this paper, we have proposed a new scheme relying on the combination of OSTBC, issued on the Alamouti scheme, and a linear precoding. This new scheme leads to an efficient exploitation of space-time diversity. In addition, as a linear decoder is used at the reception side, the complexity linearly increases with the precoding matrix size but not exponentially compared to ML detectors often used in such precoding systems. Like this, our system described may be applied to other OSTBC codes and several antenna configurations. The performance results show that a simple linear decoder is sufficient when using an OSTBC combined with linear precoding. This conclusion remains valuable when a channel coding is used. To conclude, we can say that the use of our proposed schemes for two or more transmit antennas are good choices of transmission chain for future wireless communication systems.

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