Space-Time Block Codes with Full Diversity using Linear Precoding and Linear Receivers

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***Abstract*— In order to increase the diversity order of a multi-antenna system at a moderate cost of complexity, one may choose Orthogonal Space-Time Block Coding (OSTBC). In this paper, OSTBC is combined with linear precoding matrices to improve the space-time-frequency diversity. Different linear precoding matrices based on either Fourier, Vandermonde or Hadamard constructions are proposed where a simple linear de-precoder can be chosen at the receiver part which performs very close to the Maximum Likelihood (ML) decoder. The system is generalized to different numbers of transmit antennas and different sizes of precoding matrices. It is shown that the combined scheme efficiently exploits the space-time-frequency diversity using a linear decoder leading to an OSTBC with maximal diversity for more than two antennas when a Zero Forcing (ZF) or when an Minimum Mean Square Error (MMSE) equalizer is chosen.**

I. Introduction

Since the work of Foschini [1], there has been a huge interest concerning Multiple Input Multiple Output (MIMO) systems. The main result of MIMO systems is that the capacity varies linearly with the minimum of N_t transmit and N_r receive antennas. Parallel to his work, Orthogonal Space-Time Block Coding (OSTBC) was demonstrated to be a good trade-off between performance and complexity to exploit spatial diversity in multi-antenna systems. The initial OSTBC proposed by Alamouti [2] for 2 transmit antennas is merely decoded with a linear operation. Another advantage of the Alamouti code is its rate equal to 1. Then, Tarokh [3] extended OSTBC to 3 or 4 transmit antennas with linear decoding as well, but resulting in lower 1/2 and 3/4-rate codes for any complex constellation. Since, many studies were carried out to find space-time codes with more than two antennas and rate one, all resulting in Non-Orthogonal Space-Time Block Codes (NOSTBC) [4, 5, 6, 7], thus requiring more complexity at the receiver than linear decoders.

In parallel, linear precoding was demonstrated to efficiently exploit time diversity for Single Input Single Output (SISO) systems [8, 9] as well as MIMO systems including OSTBC [10]. In [11], a system combining the NOSTBC proposed in [4] with linear precoding is presented, but the complexity of its Maximum Likelihood (ML) decoder varies exponentially with the length of the precoding matrix. Linear precoding adapted to space-time transmission in order to find new NOSTBC was carried out in [12, 13, 14] but also using ML-like decoders.

In this paper, we generalize the scheme that we briefly presented in [15] that consists of combining linear precoding and OSTBC. We showed in [15] that the system using Hadamard linear precoding matrices based on the Special Unitary SU(2) group and a simple linear de-precoder offers a good performance/complexity tradeoff and performs very close to the ML decoder. The system conserves the initial rate of the OSTBC, i.e. 1 for the Alamouti code and 1/2 or 3/4 for Tarokh codes. In [15] and in this paper, we demonstrate that the diversity of the system increases owing to linear precoding applied in the time, frequency, or space domains using several groups of transmit antennas alternatively. A comparison between several linear precoding matrices based on either Fourier, Vandermonde or Hadamard constructions is provided. It is shown that this specific combination of linear precoding and OSTBC results in an orthogonal system providing full diversity.

First, the linear precoding matrices are described, i.e. the Fourier, Vandermonde and complex Hadamard matrices, the last one based on SU(2) construction. We demonstrate why our system allows a simple linear de-precoder while other NOSTBC based systems require ML-like decoders. Then, the generalised scheme is presented, which can be adapted to various MIMO configurations with different numbers of transmit antennas. It is shown that because of the chosen matrices used for the linear precoding, a non-centered gaussian diversity can be obtained for the detected symbols. This paper focuses on the maximization of diversity and the minimization of interference using linear precoding with a linear de-precoding at the receiver part while the systems proposed in the literature focus on the optimization of the minimal product distance of the linear precoding at high Signal to Noise Ratio (SNR) using an ML-like decoder. Finally, simulation results are given for different sizes of precoding matrices confirming the theoretical analysis.

For this study, uncorrelated frequency non-selective Rayleigh fading and time invariance during T symbol durations are assumed as well as perfect channel estimation at the receiver side. The channel corresponds to a quasi-static flat fading well adapted to Orthogonal Frequency Division Multiplex OFDM-like modulations. Hence, the theoretical channel response of the uncorrelated channels from transmitting antenna t to receiving antenna r can be represented by $h_{tr} = \rho_{tr} e^{i\theta_{tr}}$.

II. Linear Precoding

In this section, the unitary linear precoding matrices described are the Fourier, the Vandermonde and the complex Hadamard matrices for a size $L \times L$. Then, the state of the art for linear precoding without OSTBC is given, focusing on the constellation transmitted for different sizes of precoding matrices. Finally, it is shown that most of the schemes proposed in the literature are limited to small size precoding matrices because of the use of an ML-like decoder.

A. Linear Precoding matrices

1) Fourier matrices: The linear precoding matrix obtained with Fourier Transform matrix construction is:

$$\mathbf{\Theta}_L^{FFT} = \frac{1}{\sqrt{L}} \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & w^1 & w^2 & \dots & w^{L-1} \\ 1 & w^2 & w^4 & \dots & w^{2(L-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & w^{L-1} & w^{2(L-1)} & \dots & w^{(L-1)(L-1)} \end{bmatrix} \tag{1}$$

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with $w = e^{\frac{2j\pi}{L}}$.

2) Vandermonde matrices: The linear precoding matrix obtained with Vandermonde matrix construction is:

$$\mathbf{\Theta}_L^{Van} = \text{diag}[1, \alpha, \alpha^2, \ldots, \alpha^{L-1}].\mathbf{\Theta}_L^{FFT} \tag{2}$$

$$\mathbf{\Theta}_L^{Van} = \frac{1}{\sqrt{L}} \begin{bmatrix} 1 & 1 & 1 & \ldots & 1 \\ \theta_1 & \theta_2 & \theta_3 & \ldots & \theta_{L-1} \\ \theta_1^2 & \theta_2^2 & \theta_3^2 & \ldots & \theta_{L-1}^2 \\ \vdots & \vdots & \vdots & & \vdots \\ \theta_1^{L-1} & \theta_2^{L-1} & \theta_3^{L-1} & \ldots & \theta_{L-1}^{L-1} \end{bmatrix} \tag{3}$$

with $\theta_i = \alpha w^i = \alpha e^{\frac{2ij\pi}{L}}$.

3) Complex Hadamard matrices based on SU(2) group: The linear precoding based on the complex Hadamard construction matrix is [15]:

$$\mathbf{\Theta}_L^{Had} = \sqrt{\frac{2}{L}} \begin{bmatrix} \mathbf{\Theta}_{L/2} & \mathbf{\Theta}_{L/2} \\ \mathbf{\Theta}_{L/2} & -\mathbf{\Theta}_{L/2} \end{bmatrix} \tag{4}$$

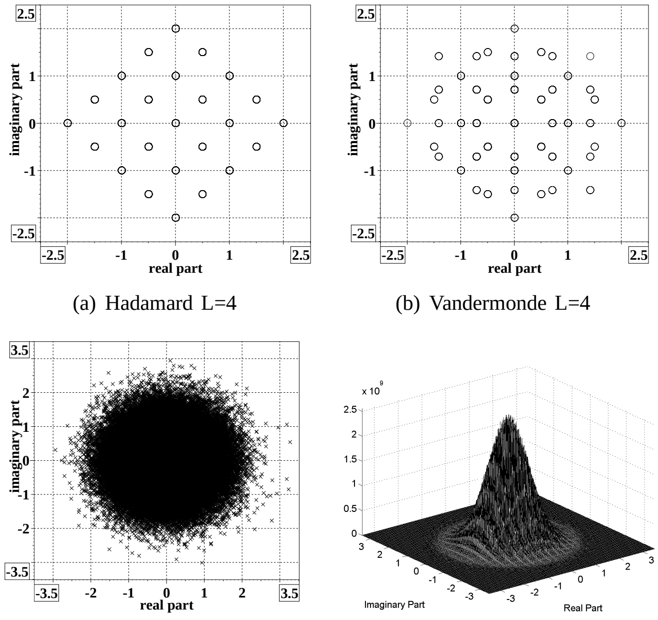
with $L = 2^n$, n $\in \mathbf{N}^*$, n ≥ 2 and:

$$\mathbf{\Theta}_2 = \begin{bmatrix} e^{j\theta_1}.\cos\eta & e^{j\theta_2}.\sin\eta \\ -e^{-j\theta_2}.\sin\eta & e^{-j\theta_1}.\cos\eta \end{bmatrix} \tag{5}$$

belonging to the Special Unitary group SU(2), therefore $det(\mathbf{\Theta_2}) = 1$ and $\Theta_2^{-1} = \Theta_2^H$.

B. State of the art

In [8, 9], the authors first used Vandermonde matrices as linear precoding matrices for a SISO system in a Rayleigh environment in order to increase the diversity of the global system. In [16], this operation is seen as constellation expansion. They demonstrated that in order to exploit the ultimate shaping gain (1.53 dB), the constellation must be uniformly distributed in a hypersphere. Indeed, the projection of a uniform probability distribution over an L-sphere onto one or two dimensions is a non-uniform probability distribution that approaches a Gaussian density as L increases. When using Quadrature Amplitude Modulation (QAM) symbols equiprobably distributed, the result of the projection over a sphere of dimension L is a 2-D gaussian density when L is large. This effect can be seen on Figures 1 (d) representing the 3-D gaussian density of linearly precoded Quadrature Phase Shift Keying (QPSK) symbols by a complex Hadamard matrix based on SU(2) group of size $L = 256$, which has the same shape as Vandermonde matrix or Fourier matrix for $L = 256$. Figure 1 (a) respresents linear precoded QPSK symbols with a complex Hadamard matrix based on SU(2) group of size $L = 4$. It can be demonstrated that Fourier matrices obtain the same number of constellation points as complex Hadamard matrices based on SU(2) group. On Figures 1 (b)(c), two Vandermonde matrices of size $L = 4$ and 256 are used to linearly precode QPSK symbols.



(a) Hadamard L=4 (b) Vandermonde L=4 (c) Vandermonde L=256 (d) 3-D Hadamard L=256 distribution

Fig. 1. Transmitted constellations, linear precoded QPSK for L=4 and L=256 with Hadamard and Vandermonde matrices

C. Decoding

In [10, 12, 13, 14], an ML or spheric decoder is used in order to retrieve the linear precoded information in several space-time architectures because of the interference terms. Thus, with a ML-like decoder, the maximum sizes are up to $L = 32$ in a SISO environment [8] and up to $L = 8$ in a MIMO environment [11]. However, in [9] the authors implemented linear decoding based on the Minimum Square Error (MSE) criterion with size $L = 256$ in a SISO environment. Similarly in [15], we demonstrated that linear decoding provides good performance in a MIMO environment using a specific linear precoding and OSTBC. As shown in the sequel, the receiver may merely consist of the transconjugate of the initial precoding matrix $\mathbf{\Theta}^H$ and a linear demapping according to the chosen constellation.

III. Linear precoding associated with OSTBC

The space-time architecture applied to the precoded symbols is an OSTBC scheme. Therefore, the transmission chain is linear because the receiver has a linear decoding complexity. In the following sections, an OSTBC description based on channel representation and the STBC state of the art is given. Then, the linear precoding with OSTBC is explained.

A. OSTBC representation

The rate R of OSTBC is defined as the ratio between the number of transmit symbols N and the number of symbol durations T necessary for their transmission.

With the 1-rate Alamouti code [2] for $N_t = 2$ transmit and $N_r = 1$ receive antennas can be represented as follows:

$$\mathcal{G}_2 = \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix} \tag{6}$$

Under the assumption that the fading coefficients are constant over two consecutive symbol durations, the code is represented by:

$$\mathbf{r} = \mathbf{H}.\mathbf{s} + \mathbf{n} \tag{7}$$

where $\mathbf{r} = [r_1\, r_2]^T$ is the received signal over two consecutive symbol durations, $\mathbf{s} = [s_1\, s_2]^T$ is the transmitted signal, $\mathbf{n} = [n_1\, n_2]^T$ is the Additive White Gaussian Noise (AWGN),

$$\mathbf{H} = \begin{bmatrix} h_1 & h_2 \\ -h_2^* & h_1^* \end{bmatrix} \tag{8}$$

is the equivalent channel matrix for the 2 successive symbol durations over 2 antennas, and h_i is the channel response of the transmit antenna i. The length of the channel representation given by the number of rows of $\mathbf{H}$ is $L_c = 2$. The decoding step consists of applying the transpose conjugate of the channel matrix to the equivalent received vector. The decoding leads to:

$$\hat{\mathbf{s}} = \mathbf{\Lambda}.\mathbf{s} + \mathbf{n}' \tag{9}$$

where $\hat{\mathbf{s}} = [\hat{s}_1\ \hat{s}_2]^T$ is the estimated symbol vector after decoding, $\mathbf{\Lambda} = \mathbf{H}^H.\mathbf{H} = \lambda.\mathbf{I}_2$, where $(.)^H$ stands for the transconjugate, $\mathbf{I}_2$ the identity 2x2 matrix, $\lambda = |h_1|^2 + |h_2|^2$ and $\mathbf{n}' = \mathbf{H}^H.\mathbf{n}$.

This matrix representation can be extended to other OSTBC such as the Tarokh codes [3]. For $\mathcal{G}_3$, $\mathcal{G}_4$, $\mathcal{H}_3$ and $\mathcal{H}_4$ the channel representations $\mathbf{H}$ respectively are:

$$\mathbf{H}_{\mathcal{G}_3} = \begin{bmatrix} h_1 & -h_2 & -h_3 & 0 & h_1^* & -h_2^* & -h_3^* & 0 \\ h_2 & h_1 & 0 & -h_3 & h_2^* & h_1^* & 0 & -h_3^* \\ h_3 & 0 & h_1 & h_2 & h_3^* & 0 & h_1^* & h_2^* \\ 0 & h_3 & -h_2 & h_1 & 0 & h_3^* & -h_2^* & h_1^* \end{bmatrix}^T \tag{10}$$

$$\mathbf{H}_{\mathcal{G}_4} = \begin{bmatrix} h_1 & -h_2 & -h_3 & -h_4 & h_1^* & -h_2^* & -h_3^* & -h_4^* \\ h_2 & h_1 & h_4 & -h_3 & h_2^* & h_1^* & h_4^* & -h_3^* \\ h_3 & -h_4 & h_1 & h_2 & h_3^* & -h_4^* & h_1^* & h_2^* \\ h_4 & h_3 & -h_2 & h_1 & h_4^* & h_3^* & -h_2^* & h_1^* \end{bmatrix}^T \tag{11}$$

$$\mathbf{H}_{\mathcal{H}_3} = \begin{bmatrix} h_1 & 0 & h_2^* & -\frac{h_3}{2} & -\frac{h_3^*}{2} & \frac{h_3}{2} & -\frac{h_3^*}{2} \\ h_2 & 0 & -h_1^* & \frac{h_3}{2} & -\frac{h_3^*}{2} & \frac{h_3}{2} & \frac{h_3^*}{2} \\ \frac{h_3}{\sqrt{2}} & \frac{h_3}{\sqrt{2}} & 0 & 0 & \frac{h_1^*+h_2^*}{\sqrt{2}} & 0 & \frac{h_1^*-h_2^*}{\sqrt{2}} \end{bmatrix}^T \tag{12}$$

$$\mathbf{H}_{\mathcal{H}_4} = \begin{bmatrix} h_1 & 0 & h_2^* & \frac{h_4-h_3}{2} & -\frac{h_3^*+h_4^*}{2} & \frac{h_3-h_4}{2} & -\frac{h_3^*+h_4^*}{2} \\ h_2 & 0 & -h_1^* & \frac{h_3-h_4}{2} & -\frac{h_3^*+h_4^*}{2} & \frac{h_3-h_4}{2} & \frac{h_3^*+h_4^*}{2} \\ \frac{h_3+h_4}{\sqrt{2}} & \frac{h_3-h_4}{\sqrt{2}} & 0 & 0 & \frac{h_1^*+h_2^*}{\sqrt{2}} & 0 & \frac{h_1^*-h_2^*}{\sqrt{2}} \end{bmatrix}^T \tag{13}$$

The length of these channel representations L_c is equal to 8 for $\mathcal{G}_3$ and $\mathcal{G}_4$ and 7 for $\mathcal{H}_3$ and $\mathcal{H}_4$. The OSTBC decoding gives the following result:

$$\lambda = \sum_{i=1}^{N_t} |h_i|^2 \tag{14}$$

This λ value applies to every OSTBC code.

B. State of the art

As Tarokh demonstrated that no orthogonal design exists for 4 transmit antennas, 4 diversity order and rate 1 [3], the research focused on NOSTBC. These codes can also be expressed by (7) where $\mathbf{r} = [r_1\ r_2\ r_3\ r_4]^T$ is the received signal and $\mathbf{s} = [s_1\ s_2\ s_3\ s_4]^T$ the transmitted one. The total transmit power is P. Each antenna transmits a symbol over one symbol duration, therefore each antenna transmits symbols at a power of $P/4$. The matrix representation described in III-A can be extended to these NOSTBC schemes [4, 5, 6, 7]. For the NOSTBC proposed in [4] and [5] by Jafarkhani and Tirkkonnen respectively, $\mathbf{H}$ is therefore a 4x4 matrix equal to:

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_2 \\ -\mathbf{H}_2^* & \mathbf{H}_1^* \end{bmatrix} \text{ and } \mathbf{H} = \begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_2 \\ \mathbf{H}_2 & \mathbf{H}_1 \end{bmatrix} \tag{15}$$

with $\mathbf{H}_i$ the equivalent channel matrix for 2 successive symbol durations over 2 antennas corresponding to (8). In the case of the NOSTBC proposed in [4], $\mathbf{\Lambda}$ of formula (9) becomes a non-orthogonal matrix of the form:

$$\mathbf{\Lambda} = \mathbf{H}^H.\mathbf{H} = \begin{bmatrix} a & 0 & 0 & b \\ 0 & a & -b & 0 \\ 0 & -b & a & 0 \\ b & 0 & 0 & a \end{bmatrix} \tag{16}$$

where $a = \sum_{i=1}^{4} |h_i|^2$ and $b = 2Re(h_1 h_4^* - h_2 h_3^*)$. This scheme needs an ML-like detector because of the intrinsic interference. In [10], the authors propose to apply a linear precoding to a non-orthogonal matrix [4] and to use a ML decoder at the receiver part. The implementation complexity of these systems increases exponentially with the size of the precoding matrix, because they all require ML or similar decoders due to high interference terms.

C. The new proposed scheme

In the 4-transmit antenna system that we proposed in [15], the Alamouti OSTBC is chosen in order to keep the rate one for the new proposed scheme. The Alamouti OSTBC is applied alternatively to antennas 1 and 2 and then to antennas 3 and 4. Thus, the symbols are transmitted over the first group of transmit antennas 1 and 2 with a power $P/2$ over two symbol durations when the other antennas are switched off. Then, the other symbols are transmitted over the second group of transmit antennas 3 and 4. Therefore, as for the precedent NOSTBC codes for 4 antennas, the total transmit power per symbol duration is P. We obtain the following equivalent matrix:

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1 & 0 \\ 0 & \mathbf{H}_2 \end{bmatrix} \tag{17}$$

of size 4x4. At the reception, in (9) we get $\mathbf{\Lambda} = \mathbf{\Lambda}_4 = \mathbf{H}^H.\mathbf{H} = diag(\lambda_1, \lambda_1, \lambda_2, \lambda_2)$ and $\lambda_i = |h_{2i-1}|^2 + |h_{2i}|^2$ with $i \in \mathbf{N}^*$ leading to a 2 channel diversity order. So far, there is no gain compared to the classical Alamouti scheme described in [2].

Before being space-time coded, the symbols are preliminarly linear precoded with a $L \times L$ unitary matrix based on Hadamard construction. This linear precoding will have the effect of increasing the diversity order of the transmitted symbols. Using $\mathbf{\Theta}_L^{Had}$ for $L=4$ and applying the coefficients $\eta=\frac{\pi}{4}$, $\theta_2=\theta_1-\frac{\pi}{2}$, $\theta_1=\frac{5\pi}{4}$, this leads to the SU(2) matrix:

$$\mathbf{\Theta}_2=\frac{1}{2}\begin{bmatrix} -1-j & -1+j \\ 1+j & -1+j \end{bmatrix} \tag{18}$$

The global transmission and reception system is described by:

$$\mathbf{A}_4=\mathbf{\Theta}_4^H.\mathbf{\Lambda}_4.\mathbf{\Theta}_4 \tag{19}$$

$$\mathbf{A}_4=\frac{1}{2}\begin{bmatrix} \lambda_1+\lambda_2 & 0 & \lambda_1-\lambda_2 & 0 \\ 0 & \lambda_1+\lambda_2 & 0 & \lambda_1-\lambda_2 \\ \lambda_1-\lambda_2 & 0 & \lambda_1+\lambda_2 & 0 \\ 0 & \lambda_1-\lambda_2 & 0 & \lambda_1+\lambda_2 \end{bmatrix} \tag{20}$$

The diagonal elements of $\mathbf{A}_4$ are all equal to:

$$a_{ii}=\frac{1}{2}\sum_{l=1}^{L=4}|h_l|^2 \quad \forall i\in[1\ldots4] \tag{21}$$

Thus, owing to linear precoding, the exploited channel diversity order increases from 2 to 4. Moreover, the interference terms are either null or equal to:

$$a_{ik}=\frac{1}{2}(\sum_{l=1}^{2}|h_l|^2-\sum_{l=3}^{4}|h_l|^2) \quad i,k\in[1\ldots4]_{i\neq k} \tag{22}$$

As shown in [15] and detailed in the part III-D, a simple linear decoding can be applied at the receiver part when linear precoding is associated with an OSTBC scheme like the Alamouti code for several antenna configurations. In the sequel, the generalisation of the described scheme is given for several OSTBC and linear precoding matrices.

D. Generalisation of the proposed scheme

In the proposed system OSTBC is applied by blocks of M symbols multiple of N according to the following equivalent channel representation $\mathbf{H}$ of size $L_cM/N\times M$:

$$\mathbf{H}=\begin{bmatrix} \mathbf{H}_1 & 0 & \ldots & 0 & \ldots & 0 \\ 0 & \mathbf{H}_2 & \ddots & 0 & \ldots & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{H}_j & \ddots & 0 \\ 0 & \ldots & 0 & \ddots & \ddots & 0 \\ 0 & \ldots & 0 & \ldots & 0 & \mathbf{H}_{M/N} \end{bmatrix} \tag{23}$$

with $\mathbf{H}_j$ the j^{th} equivalent channel representation of the OSTBC and L_c the length of the channel representation of the OSTBC with N_t antennas as defined in the part III-A. This means that $\mathbf{H}_j$ of size $L_c\times N$ is independant from the previous or future blocks. These blocks can be applied either in the time or in the frequency domain, requiring a channel constant over T adjacent time symbols or T adjacent subcarriers. However, these blocks can be applied in the space-time domain or space-frequency domain, that means that a multiple of the N_t transmit antennas (the total transmit antennas mN_t with $1\leq m\leq M/N$ antennas) can be used to send symbols over a first group of N_t antennas while the other antennas are switched off. If $m\geq 2$, the following time or frequency block will be sent on a different group of antennas, keeping the orthogonality of the resulting code. For instance, with the Alamouti code, the channel matrix can represent the channel matrix in either time or frequency domains using 2 transmit antennas. This channel matrix can also represent the channel matrix in the space-time or space-frequency domain using 4, 8 or more transmit antennas if only two transmit antennas are sending symbols while the other transmit antennas remain idle. However, the total transmit power should remain a constant P. Therefore, each antenna of the first group of N_t antennas will transmit at a power of P/N_t while the antennas of the other groups are switched off. For the following block, the first group will be switched off and each antenna of the following group will transmit at a power of P/N_t. Therefore, this system can be adapted in order to exploit the space, time or frequency domains depending on the channel characteristics. At the reception, $\mathbf{\Lambda}$ in (9) becomes:

$$\begin{array}{l}\mathbf{\Lambda}=\mathbf{H}^H.\mathbf{H}=diag(\mathbf{\Lambda}_1,\ldots,\mathbf{\Lambda}_j,\ldots,\mathbf{\Lambda}_{M/N}) \\ \text{with } \mathbf{\Lambda}_j=\lambda_j\mathbf{I}_N\end{array} \tag{24}$$

According to the OSTBC chosen, the diagonal elements of the equivalent transmission/reception are:

$$\lambda_j=\sum_{i=1}^{N_t}|h_i^j|^2 \tag{25}$$

with j the time or spatial index of the decoded OSTBC block. If an interleaving and de-interleaving operation is included, the λ_j's are mixed up and the initial matrix becomes an equivalent matrix $\mathbf{\Lambda}$ (9) with:

$$\mathbf{\Lambda}=diag(\lambda_1,\lambda_2,\ldots,\lambda_{j-1},\lambda_j,\ldots,\lambda_{M-1},\lambda_M) \tag{26}$$

Before being space-time coded, the symbols are preliminarly linear precoded with a unitary matrix $\mathbf{\Theta}_L$ of size $L\times L$. Whatever the unitary matrices based on Fourier, Vandermonde or complex Hadamard precoding matrices, the different eigenvalues λ_j provided by the OSTBC process are added, providing full diversity on the diagonal terms. On the other hand, these matrices provide differences between eigenvalues on the non diagonal terms.

The transconjugate of the channel $\mathbf{H}^H$ is equivalent to an Maximum Ratio Combining (MRC) equalizer. The diagonal terms have a diversity that tend towards a non-centered gaussian law and interferences that tend towards a centered gaussian law on the non-diagonal terms when L increases in a flat Rayleigh fading channel environment. Therefore, when using an MRC equalizer with perfect channel estimation, the global system is non-orthogonal.

With a ZF equalizer and an MMSE, the λ values become:

$$\lambda=\frac{\sum_{i=1}^{N_t}|h_i|^2}{\sum_{i=1}^{N_t}|h_i|^2}=1 \quad \lambda=\frac{\sum_{i=1}^{N_t}|h_i|^2}{\sum_{i=1}^{N_t}|h_i|^2+\frac{1}{\gamma}} \tag{27}$$

where γ is the Signal to Noise Ratio at the receive antenna.

When using a ZF equalizer with perfect channel estimation, all the non diagonal terms are null. In order to avoid enhancing the noise, it is preferable to choose an MMSE equalizer, leading to very small interference terms and leading to a global orthogonal system at high SNR. If we apply these unitary matrices to the diagonal matrix $\mathbf{\Lambda}_L$ corresponding to the OSTBC coding and decoding process, the global linear transmission/reception scheme is given by:

$$\mathbf{A}_L = \mathbf{\Theta}_L^{-1}.\mathbf{\Lambda}_L.\mathbf{\Theta}_L = \mathbf{\Theta}_L^H.\mathbf{\Lambda}_L.\mathbf{\Theta}_L \tag{28}$$

The linear de-precoding consists of applying $\mathbf{\Theta}_L^H$, keeping the global transmission chain linear. The rate of the initial OSTBC is conserved, i.e. one for the Alamouti code.

IV. Formulation with different linear precoding matrices

A general formulation of OSTBC combined with the specific linear precoding matrices such as the Vandermonde, Fourier and complex Hadamard matrices is described.

A. Vandermonde Matrices and Fourier Transform Matrices

The global transmission/reception scheme with the Vandermonde matrices is:

$$\mathbf{A}_L = \mathbf{\Theta}_L^{Van^{-1}}.\mathbf{\Lambda}_L.\mathbf{\Theta}_L^{Van} = \mathbf{\Theta}_L^{FFT^H}.\mathbf{\Lambda}_L.\mathbf{\Theta}_L^{FFT} \tag{29}$$

Owing to equation (2), the global transmission/reception formula of the Vandermonde and Fourier matrices is similar but the transmitted constellation is different as shown in the part II-A. The global formula can be rewritten as follows:

$$\mathbf{A}_L = \frac{1}{L}\sum_{i=1}^{L}\lambda_i.\mathbf{I}_L + \mathbf{J} \tag{30}$$

with $\mathbf{I}_L$ the identity matrix of size $L \times L$ and $\mathbf{J}$ the matrix of interference terms being differences between these λ_l's. As $\mathbf{\Theta}_L^{FFT}$ is a unitary matrix, all the diagonal elements of $\mathbf{A}_L$ a_{ii} are equal to:

$$a_{ii} = \frac{1}{L}\sum_{l=1}^{L}\lambda_l \ \ \forall i \in [1 \dots L] \tag{31}$$

while the interference terms are all sum of differences between these λ_l's. One of these interference terms is:

$$a_{ik} = \frac{1}{L}\sum_{l=0}^{L/2-1}(-1)^l(\lambda_{2l+1} + j\lambda_{2l+2}) \quad i,k \in [1 \dots L]_{i \neq k} \tag{32}$$

The other terms of interference are also sum of differences between eigenvalues and may be complex interferences. With a linear precoding matrix based on Fourier Transform matrix of size $L = 4$, the following Fourier matrix is obtained:

$$\mathbf{\Theta}_4^{FFT} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \tag{33}$$

This leads to the following global hermitian circulant matrix:

$$\mathbf{A}_4 = \frac{1}{4}\begin{bmatrix} a & b & c & b^* \\ b^* & a & b & c \\ c & b^* & a & b \\ b & c & b^* & a \end{bmatrix} \tag{34}$$

with

$$\begin{aligned} a &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \\ b &= \lambda_1 - \lambda_3 - j(\lambda_2 - \lambda_4) \\ c &= \lambda_1 - \lambda_2 + \lambda_3 - \lambda_4 \end{aligned} \tag{35}$$

For Vandermonde and Fourier matrices, the diagonal terms follow a χ^2_{16} law when an Alamouti MRC equalizer is chosen in a flat Rayleigh fading channel environment. The interference term noted c follows a χ^2_8 law difference and b follows a χ^2_4 law difference per dimension. If a ZF or an MMSE equalizer is used, this leads to null or quasi-null interferences respectively.

B. Complex Hadamard matrices based on SU(2) group

The global transmission/reception scheme with the complex Hadamard matrices based on SU(2) group is represented by:

$$\mathbf{A}_L = \frac{2}{L}\begin{bmatrix} \mathbf{A}^1_{L/2} + \mathbf{A}^2_{L/2} & \mathbf{A}^1_{L/2} - \mathbf{A}^2_{L/2} \\ \mathbf{A}^1_{L/2} - \mathbf{A}^2_{L/2} & \mathbf{A}^1_{L/2} + \mathbf{A}^2_{L/2} \end{bmatrix} \tag{36}$$

with

$$\begin{aligned} \mathbf{A}^1_{L/2} &= \mathbf{\Theta}_{L/2}^{Had^H}.\mathbf{\Lambda}^1_{L/2}.\mathbf{\Theta}_{L/2}^{Had} \\ \mathbf{A}^2_{L/2} &= \mathbf{\Theta}_{L/2}^{Had^H}.\mathbf{\Lambda}^2_{L/2}.\mathbf{\Theta}_{L/2}^{Had} \end{aligned} \tag{37}$$

$$\mathbf{\Lambda}_L = \begin{bmatrix} \mathbf{\Lambda}^1_{L/2} & 0 \\ 0 & \mathbf{\Lambda}^2_{L/2} \end{bmatrix} \tag{38}$$

where

$$\begin{aligned} \mathbf{\Lambda}^1_{L/2} &= diag(\lambda_1, \dots, \lambda_{L/2}) \\ \mathbf{\Lambda}^2_{L/2} &= diag(\lambda_{L/2+1}, \dots, \lambda_L) \end{aligned} \tag{39}$$

in the case of interleaving. As $\mathbf{\Theta}_L^{Had}$ is a unitary matrix, all the diagonal elements of $\mathbf{A}_L$ in equation (30) are equal to:

$$a_{ii} = \frac{2}{L}\sum_{l=0}^{L/2-1}\cos^2\eta.\lambda_{2l+1} + \sin^2\eta.\lambda_{2l+2} \, \forall i \in [1 \dots L] \tag{40}$$

while the interference terms are all difference between these λ_l's. One of these interference terms is:

$$a_{ik} = -\frac{2}{L}\cos\eta.\sin\eta.e^{-j(\theta_1+\theta_2)}\sum_{l=0}^{L/2-1}(\lambda_{2l+1} - \lambda_{2l+2}) \quad i,k \in [1 \dots L]_{i \neq k} \tag{41}$$

In [15] and in this paper, we found that the optimal Bit Error Rate (BER) performance results are obtained with pure real or pure imaginary interference, as for instance with the $\mathbf{\Theta}_2$ matrix and coefficients $\eta = \frac{\Pi}{4}$, $\theta_1 = \frac{5\Pi}{4}$ and $\theta_2 = \frac{3\Pi}{4}$. These coefficients are fixed and do not depend on a channel knowledge at the transmitter side. The reason why these coefficients are optimal will be detailed at the end of this chapter. Using $\mathbf{\Theta}_L^{Had}$ for $L = 4$ with the Alamouti scheme with interleaving the following diagonal channel matrix is obtained:

$$\mathbf{\Lambda}_4 = diag(\lambda_1, \lambda_2, \lambda_3, \lambda_4) \tag{42}$$

The resulting matrix is then:

$$\mathbf{A}_4 = \frac{1}{4}\begin{bmatrix} a+b & c+d & a-b & c-d \\ -c-d & a+b & -c+d & a-b \\ a-b & c-d & a+b & c+d \\ -c+d & a-b & -c-d & a+b \end{bmatrix} \tag{43}$$

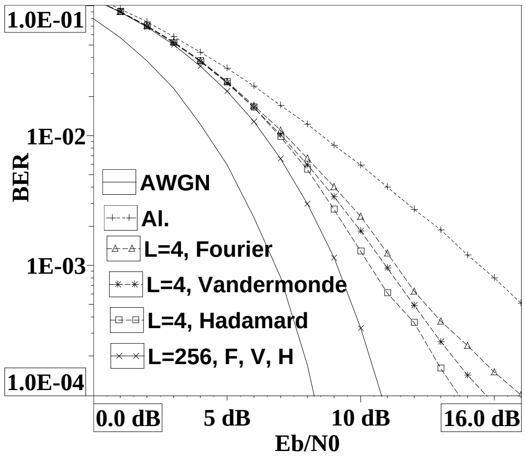


Fig. 2. Alamouti (Al.) associated without and with different linear precoding matrices for η=2 bps/Hz compared to the AWGN channel

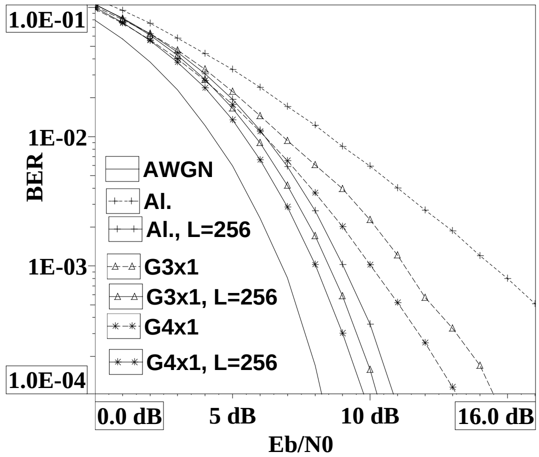


Fig. 3. Alamouti (Al.) and Tarokh (G3x1, G4x1) associated without and with linear precoding matrices for η=1 bps/Hz compared to the AWGN channel

with

$$\begin{aligned} a &= \lambda_1 + \lambda_2 \quad & b &= \lambda_3 + \lambda_4 \\ c &= \lambda_1 - \lambda_2 \quad & d &= \lambda_3 - \lambda_4 \end{aligned} \tag{44}$$

As for Vandermonde and Fourier matrices, the diagonal terms follow a χ^2_{16} law applying an Alamouti MRC equalizer in a flat Rayleigh fading channel environment. The interference terms follow a χ^2_8 law difference. With perfect channel estimation and a ZF equalizer, the interference terms are null as well as with an MMSE equalizer at high SNR.

Applying an Alamouti MRC equalizer in a flat Rayleigh fading channel environment with Vandermonde, Fourier or complex Hadamard linear precoding matrices of size $L \times L$, the diagonal elements follow a χ^2_{4L} law. However, the non-diagonal terms are different. Indeed, some of the interference terms of Fourier and Vandermonde matrices follow a χ^2_L law difference when the complex Hadamard matrix follow a χ^2_{2L} law difference. Therefore, it is expected that Fourier and Vandermonde matrices lead to worse results than complex Hadamard matrices based on SU(2) group for small size of L.

V. Simulation results

The simulations are carried out in a Rayleigh flat fading channel environment well adapted to OFDM-like modulation with Alamouti and Tarokh codes, MMSE equalizer and interleaving for different sizes of precoding matrices. In fact, the MMSE equalizer provides the best results compared to the MRC or the ZF equalizer. Figure 2 shows the performance of the linear precoded Alamouti system with Vandermonde, Fourier and complex Hadamard based SU(2) matrices for different sizes of precoding matrices ($L = 4$ and 256 representing a large scale for the linear precoding sizes) and $N_r = 1$. As expected, owing to a higher diversity order, the performance of the linear precoded OSTBC scheme performs better than the sole Alamouti scheme. With $L = 256$, where F, V, H acronym stands for Fourier, Vandermonde or Hadamard matrices, the maximum diversity order is reached. There is no noticeable difference between these 3 different matrices for this large size of precoding matrices. Moreover, one can see that for $L = 4$ the performance of the Vandermonde matrix and $\alpha = \frac{\Pi}{4}$ is slighty better than the performance of the Fourier matrix although these two matrices have the same global transmission/reception equations (34) and (35). Therefore, this improvement is only due to the shaping gain of the transmitted constellation for $L = 4$ as seen in Figure 1, because the number of constellation points with a Vandermonde matrix is higher than the number of constellation points with a Fourier matrix. One can see that for $L = 4$ the system with the complex Hadamard matrix outperforms the systems with Vandermonde or Fourier matrices. This can be explained by the complex interference terms (34)(35) for Vandermonde and Fourier matrices and pure real or imaginary interferences (43)(44) for complex Hadamard matrices. However, when using complex Hadamard matrices, the interferences are either pure real or pure imaginary interferences. Therefore, when using a linear decoder, complex Hadamard matrices perform better than Vandermonde or Fourier matrices when L is small. Moreover, the performance of the complex Hadamard matrix for $L = 4$ is very close to the ML decoder and the performance obtained with $L = 256$ is already reached with $L = 64$ [15]. This result is the same whatever the choice of the number of transmit antennas if the space diversity is exploited by transmitting symbols by several groups of transmit antennas alternatively.

Figure 3 shows the performance of different OSTBC with linear precoding for $L = 256$ and spectral efficiency η = 1 bps/Hz. To obtain this spectral efficiency, a Binary PSK (BPSK) is applied to the Alamouti code whereas QPSK is applied to Tarokh codes $\mathcal{G}_3$ and $\mathcal{G}_4$. The results show that the performance improves when linear precoding is applied whatever the OSTBC code because of an efficient exploitation of the diversity. In fact, the gain on the BER at 10^{-4} provided by linear precoding is about 9.2 dB, 4.3 dB and 3.2 dB for respectively the Alamouti, the $\mathcal{G}_3$ and the $\mathcal{G}_4$ Tarokh codes.

Figure 4 shows the performance of different OSTBC with linear precoding for $L = 256$ and spectral efficiency η = 3 bps/Hz. To obtain this spectral efficiency, a 8PSK is applied

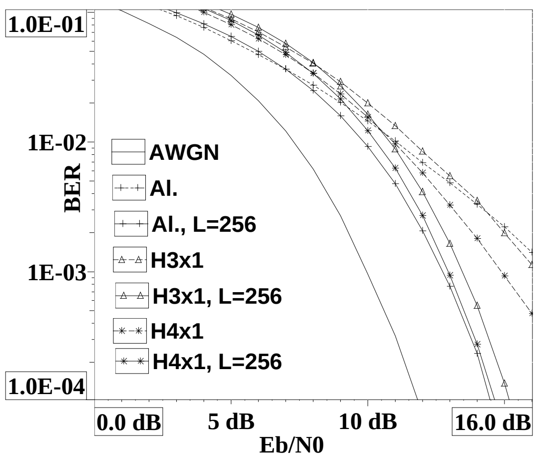


Fig. 4. Alamouti (Al.) and Tarokh (H3x1, H4x1) associated without and with linear precoding matrices for η=3 bps/Hz compared to the AWGN channel

to the Alamouti code, whereas 16QAM is applied to Tarokh codes $\mathcal{H}_3$ and $\mathcal{H}_4$. Contrary to η = 1 bps/Hz, the Alamouti code performs better than Tarokh codes with linear precoding $L = 256$ for η = 3 bps/Hz. This is due to the 16QAM constellation of the Tarokh codes less robust than 8PSK. For $\eta > 1$ bps/Hz, the Alamouti outperforms Tarokh codes with linear precoding owing to its larger rate allowing a more robust modulation to provide the same spectral efficiency. Similar conclusions have been drawn for OSTBC applied to Multi-Carrier Code Division Multiple Access (MC-CDMA) [17] confirming the analogy between single user linear precoded OSTBC and multi-user multicarrier OSTBC systems [10].

VI. CONCLUSION

The proposed combination of linear precoding and OSTBC applied by block of N_t antennas over mN_t total transmit antennas leads to an efficient exploitation of space-time-frequency diversity. We have demonstrated that this diversity increases with the length of the precoding matrix at a linear cost of complexity for the decoder. We have shown that unitary matrices lead to a maximization of the diversity order for the detected symbols and a minimization of the interference terms. With a ZF equalizer or with an MMSE equalizer, these interference terms are null or quasi-null when perfect channel estimation is performed. Thus, a simple linear decoder is a very good performance/complexity tradeoff. Moreover, the system described can be applied with other OSTBC codes and several antenna configurations, keeping the initial OSTBC rate as for instance the rate 1 for the Alamouti code and the rate 1/2 or 3/4 for the Tarokh codes whatever the number of transmit antennas and the size of the linear precoding matrix. The complex Hadamard matrix provides the best results for small size of L owing to the specific low interference terms. The three studied matrices provide the same performance due to the gaussian laws which have been reached when L is large. Simulation results with the specific linear precoders using OSTBC can be easily adapted to multi-carrier modulations. Therefore, these precoders can be applied to various MIMO transmissions in order to efficiently and simply exploit spatial, temporal and frequency diversities depending on the space-time-frequency channel characteristics.

REFERENCES

[1] G. J. Foschini, *Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas*, Bell Labs Technical Journal, Vol. 1, No. 2, 1996, pp. 41-59.

[2] S. M. Alamouti, *A Simple Transmit Diversity Technique for Wireless Communications*, IEEE Journal on Selected Areas in Communications, Vol. 16, No. 8, October 1998, pp. 1451-1458.

[3] V. Tarokh, H. Jafarkhani, A. R. Calderbank, *Space-Time Block Codes from Orthogonal Designs*, IEEE Transactions on Information Theory, Vol. 45, No. 5, July 1999, pp. 1456-1467.

[4] H. Jafarkhani, *A Quasi-Orthogonal Space-Time Block Code*, IEEE Transactions on Communications, Vol. 49, No. 1, 2001, pp. 1-4.

[5] O. Tirkkonen, A. Boariu, A. Hottinen *Minimal Non-Orthogonality rate one space-time Block Code for 3+ Tx Antennas*, IEEE International Symposium on Spread Spectrum Techniques and Applications (ISSSTA), 2000, pp. 429-432.

[6] M. O. Damen, K. Abed-Meraim, J-C. Belfiore, *Diagonal Algebraic Space-Time Block Codes*, IEEE Transactions on Information Theory, Vol. 48, No. 3, 2002, pp. 628-636.

[7] Y. Xin, Z. Wang, G. Giannakis *Space-Time Constellation-Rotating Codes Maximizing Diversity and Coding Gains*, IEEE GLOBECOM, San Antonio, 2001, pp. 455-459.

[8] J. Boutros, E. Viterbo, *Signal Space Diversity: A Power-and Bandwidth-Efficient Diversity Technique for the Rayleigh Fading Channel*, IEEE Transactions on Information Theory, Vol. 44, No. 4, Jul. 1998, pp. 1453-1467.

[9] C. Lamy, J. Boutros, *On Random Rotations Diversity and Minimum MSE Decoding of Lattices*, IEEE Transactions on Information Theory, vol. 46, n0 4, pp. 1584-1589, July 2000.

[10] A. Stamoulis, Z. Liu, G.B. Giannakis, *Space-Time Block Coded OFDMA with Linear Precoding for Multirate Services*, IEEE Transactions on Signal Processing, Vol. 50, No. 1, 2002.

[11] M.M. Da Silva, A. Correia, *Space Time Block Coding for 4 antennas with Coding Rate 1*, IEEE 7^th Symp. on Spread Spectrum Technologies and Applications, Prague, Czech Republic, Sep. 2-5 2002.

[12] M.O. Damen, K. Abed-Meraim, J.C. Belfiore, *Transmit Diversity using Rotated Constellations with Hadamard Transform*, Proc. Adaptive Systems for Signal Processing, Lake Louise, Alberta, Canada, Oct. 2000, pp. 396-401 .

[13] Y. Xin, Z. Wang, G. B. Giannakis, *Space-Time Diversity Systems Based on Unitary Constellation-Rotating Precoders*, IEEE Conference on Acoustics, Speech, Systems Signal Processing for Signal Processing, Lake Louise, Alberta, Canada, Oct. 2000, pp. 396-401.

[14] T. Jung, K. Cheun *Design of Concatenated Space-Time Block Codes Using Signal Space Diversity and the Alamouti Scheme*, IEEE Communications Letters, Vol. 7, No. 7, pp. 329-331, July 2003.

[15] V. Le Nir, M. Hélard *Reduced-Complexity Linear Space Time Block Coding and Decoding Schemes with Block Linear Precoding*, IEE Electronic Letter, Vol. 39, No.14, pp. 1066-1068, 10th July 2003.

[16] G. D. Forney Jr., G. Ungerboeck *Modulation and Coding for Linear Gaussian Channels*, IEEE Trans. Inform. Theory, Vol.44, pp. 2384-2415, October 1998.