Reduced-complexity Space Time Block Coding and Decoding Schemes with Block Linear Precoding

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Abstract: Space-Time-Block-Coding (STBC) offers a good performance/complexity trade-off to exploit spatial diversity in multi-antenna systems. In this paper, we combine a particular linear precoder and the Alamouti STBC to improve the space-time diversity using simple linear algorithms. Our system presented with 4-transmit antennas may be applied to other STBC codes and several antenna configurations.

Introduction: STBC was demonstrated to be a good trade-off between performance and complexity to exploit spatial diversity in multi-antenna systems. The initial 2-transmit antenna system proposed by Alamouti [1] has rate 1. Then, Tarokh [2] extended STBC to 3 or 4 transmit antennas but resulting in rate 1/2 and 3/4 complex orthogonal codes. Since, many studies were carried out either to find new schemes adapted to more antennas or to combine the initial Alamouti scheme to 4-transmit antennas leading to a rate 1 non orthogonal STBC [3, 4]. In parallel, linear precoding was demonstrated to efficiently exploit time diversity in Single Input Single Output (SISO) [5] and Mutiple Input Multiple Output (MIMO) with orthogonal STBC systems [6]. In [7], the non orthogonal STBC proposed in [3] is combined with linear precoding, requiring a Maximum Likelihood (ML) decoder. New non orthogonal STBC codes are under studies as for instance in [8, 9]. In this letter, we combine a particular linear precoding and the orthogonal Alamouti STBC to improve the space-time diversity increasing with the size of the precoding matrix. At the receiver, a simple linear decoder offers a good trade-off between performance and complexity thanks to low interference terms. We present simulation results for a 4-transmit antenna system including either a classical ML receiver or a linear one. We finally show how to extend the proposed scheme to various MIMO configurations with $N = 2^n$ transmit antennas.

Alamouti STBC: The Alamouti STBC code for $N_t = 2$ transmit and 1 receive antennas is represented by:

$$\mathbf{r} = \mathbf{H}.\mathbf{s} + \mathbf{n} \tag{1}$$

where $\mathbf{r} = [r_1 \quad r_2]^T$ is the received signal over two consecutive symbol durations, $\mathbf{s} = [s_1 \quad s_2]^T$ is the transmitted signal, $\mathbf{n} = [n_1 \quad n_2]^T$ is the additive white gaussian noise, $\mathbf{H} = \mathbf{H}_{12} = \begin{bmatrix} h_1 & h_2 \\ -h_2^* & h_1^* \end{bmatrix}$ is the equivalent channel matrix for the 2 successive symbol durations over 2 antennas, and h_i is the channel response of transmit antenna i. Applying the Alamouti Maximum Ratio Combining (MRC) decoding leads to:

$$\hat{\mathbf{s}} = \mathbf{\Lambda}..\mathbf{s} + \mathbf{n}' \qquad (2)$$

where $\hat{\mathbf{s}} = [\hat{s}_1 \quad \hat{s}_2]^T$ is the estimated symbol vector after decoding, $\mathbf{\Lambda} = \mathbf{\Lambda}_2 = \mathbf{H}_{12}.\mathbf{H}_{12}^{\mathbf{H}} = \lambda_{12}.\mathbf{I}_2$, where $(.)^{\mathrm{H}}$ stands for the transconjugate, $\mathbf{I}_2$ the identity *2x2* matrix, $\lambda_{12} = |h_1|^2 + |h_2|^2$ and $\mathbf{n}' = \mathbf{H}_{12}^{\mathbf{H}}.\mathbf{n}$.

STBC state of the art for 4-transmit antennas: Most STBC codes of rate 1 for 4-transmit antenna systems can be expressed by (1) where $\mathbf{r} = [r_1 \quad r_2 \quad r_3 \quad r_4]^T$ is the received signal and $\mathbf{s} = [s_1 \quad s_2 \quad s_3 \quad s_4]^T$ the transmitted one. For the STBC proposed in [3, 4], **H** is a 4x4 matrix equals to either $\mathbf{H} = \begin{bmatrix} \mathbf{H}_{12} & \mathbf{H}_{34} \\ -\mathbf{H}_{34}^* & \mathbf{H}_{12}^* \end{bmatrix}$ or $\mathbf{H} = \begin{bmatrix} \mathbf{H}_{12} & \mathbf{H}_{34} \\ \mathbf{H}_{34} & \mathbf{H}_{12} \end{bmatrix}$. $\mathbf{H}_{ij} = \begin{bmatrix} h_i & h_j \\ -h_j^* & h_i^* \end{bmatrix}$ is the equivalent channel matrix for 2 successive symbol durations over 2 antennas i and j, h_i and h_j are the channel responses of transmit antenna i and j respectively. In [7], the linear precoding is applied to the first previous matrix. In fact, all these STBC require ML decoders due to high interference terms.

The proposed scheme for 4-transmit antennas: In our 4-transmit antenna system, we first apply Alamouti STBC alternatively to antennas 1 and 2 and then to antennas 3 and 4. Thus, we obtain the following equivalent matrix:

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{12} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{34} \end{bmatrix} \tag{3}$$

of size *4x4*. At the reception, we get (2) with $\mathbf{\Lambda} = \mathbf{\Lambda}_4 = \mathbf{H}.\mathbf{H}^{\mathbf{H}} = diag(\lambda_{12}, \lambda_{12}, \lambda_{34}, \lambda_{34})$ and $\lambda_{ij} = |h_i|^2 + |h_j|^2$ leading to a *2* channel diversity order. Before this space-time code, we apply a linear precoding represented by a $L\,x\,L$ unitary matrix $\mathbf{\Theta_L} = \sqrt{\frac{2}{L}} \cdot \begin{bmatrix} \Theta_{L/2} & \Theta_{L/2} \\ \Theta_{L/2} & -\Theta_{L/2} \end{bmatrix}$ based on Hadamard construction with $\mathbf{\Theta}_2 = \begin{bmatrix} e^{i\theta_1}\cos\eta & e^{i\theta_2}\sin\eta \\ -e^{-i\theta_2}\sin\eta & e^{-i\theta_1}\cos\eta \end{bmatrix}$ the general matrix of the $SU(2)$ group ($\mathbf{\Theta}_2^{-1} = \mathbf{\Theta}_2^H$ and $\det\mathbf{\Theta}_2 = 1$) where η, θ_1 and θ_2 are parameters to be further optimized.

For instance, for $L = 4$ and $\mathbf{\Theta}_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$, we obtain the global transmission and reception system described by the following matrix:

$$\mathbf{A}_4 = \mathbf{\Theta}_4.\mathbf{\Lambda}_4.\mathbf{\Theta}_4^H = \frac{1}{2}\begin{bmatrix} \lambda_{12}+\lambda_{34} & 0 & \lambda_{12}-\lambda_{34} & 0 \\ 0 & \lambda_{12}+\lambda_{34} & 0 & \lambda_{12}-\lambda_{34} \\ \lambda_{12}-\lambda_{34} & 0 & \lambda_{12}+\lambda_{34} & 0 \\ 0 & \lambda_{12}-\lambda_{34} & 0 & \lambda_{12}+\lambda_{34} \end{bmatrix}$$

where $\mathbf{\Theta}_4^H$ is the linear decoding. Diagonal elements of $\mathbf{A_4}$ are all equal to $\frac{1}{2}\sum_{l=1}^{L=4}|h_l|^2$. Therefore, thanks to linear precoding, the exploited channel diversity order increases from *2* to *4*. Moreover, interference terms are either null or proportional to $\frac{1}{2}\left(\sum_{l=1}^{2}|h_l|^2 - \sum_{l=3}^{4}|h_l|^2\right)$ and smaller than those of systems described in the bibliography.

If we apply a LxL precoding matrix to our 4 transmit antenna system, at the reception, we get (2) with $\mathbf{\Lambda} = \mathbf{\Lambda}_L = \mathbf{H}.\mathbf{H}^{\mathbf{H}} = diag(\lambda_{12}, \lambda_{12}, ..., \lambda_{(L-1)L}, \lambda_{(L-1)L})$ where $\mathbf{H}$ is a LxL matrix expressed by:

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{12} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \mathbf{H}_{(L-1)L} \end{bmatrix}.$$

Thus, the global transmission/reception scheme is given by:

$$\mathbf{A_L} = \mathbf{\Theta_L}.\mathbf{\Lambda}_L.\mathbf{\Theta}_{\mathbf{L}}^H \qquad (4)$$

All the diagonal elements of $\mathbf{A_L}$ are equal to $\frac{2}{L}\sum_{l=1}^{L}|h_l|^2$ while interference terms are equivalent to $\frac{2}{L}\left(\sum_{l=1}^{L/2}|h_l|^2 - \sum_{l=L/2+1}^{L}|h_l|^2\right)$. Moreover, for flat independent Rayleigh channels, the diagonal terms tend to a non-centered gaussian law while the interference terms tend to a centered gaussian law when L increases.

We can note that **H** is the same for all $N_t=2^n$ antenna systems, with $n \geq 1$, when $L=2^m$ with $m \geq n$. Since the global formulation (4) does not depend on N_t, our linear precoding can be applied to several multiple antenna systems and various transmit antennas $N_t = 2^n$ with $1 \leq n \leq m$. Thus, the presented STBC for four transmit antennas with linear precoding of size *LxL* is similar to the STBC for two transmit antennas with linear precoding of size $\frac{L}{2} \times \frac{L}{2}$ with uncorrelated channels between transmit and receive antennas.

Simulations results: We carried out simulations over flat independent Rayleigh channels. Depending on the choice of $\mathbf{\Theta_2}$ based matrix, we have demonstrated by simulation that the best performance leads to interferer terms either pure real or pure imaginary which is obtained for instance with $\eta = \frac{\pi}{4}$, $\theta_1 = \frac{5\pi}{4}$ and $\theta_2 = \frac{3\pi}{4}$ and a Minimum Mean Square Error (MMSE) equalizer for the STBC decoding.

In Figure 1, we present BER performance obtained for systems with $N_t=4$ transmit and $N_r=1$ receive antennas, with either a linear *LIN* or a *ML* decoder, and different *LxL* size of precoding matrix as presented in (4). We observe that the specified system with *L=4* leads to better performance than the sole Alamouti scheme without precoding as described in (3). In fact, all diagonal terms $\frac{1}{2}\sum_{l=1}^{L=4}|h_l|^2$ follow a χ_8^2 chi-square law while the interference terms in $\frac{1}{2}\left(\sum_{l=1}^{2}|h_l|^2 - \sum_{l=3}^{4}|h_l|^2\right)$ follow a law of χ_4^2 difference. We can notice that for *L=4,* the penalty when using linear decoder denoted *LIN* instead of the *ML* decoder is very small. We observed that this penalty diminishes when L increases thanks to the form of interference terms null or of the form $\frac{2}{L}\left(\sum_{l=1}^{L/2}|h_i|^2 - \sum_{l=L/2+1}^{L}|h_i|^2\right)$ and the curves match at *L=64*. We also see that the diversity increases with L following a χ_{2L}^2 law for diagonal terms. In fact, the slope of the curve corresponding to *L=64* is almost parallel to the gaussian curve. The curve *L=16* could have been obtained with *2*, *4*, *8* or *16* antennas under the assumption of independent Rayleigh channels every 2 symbol durations. The performance of the system with *L=64* is very close to the asymptotic performance.

Conclusion: The proposed combination of linear precoding and Alamouti STBC applied by block of *2* antennas leads to an efficient exploitation of space-time diversity that increases with the length of the precoding matrix at a linear cost of complexity. In fact, our system described with 4-transmit antennas may be applied to other STBC codes and several antenna configurations.

Moreover, the greater L, the smaller the interference terms. Thus, a simple linear decoder is sufficient when using a STBC combined with linear precoding.

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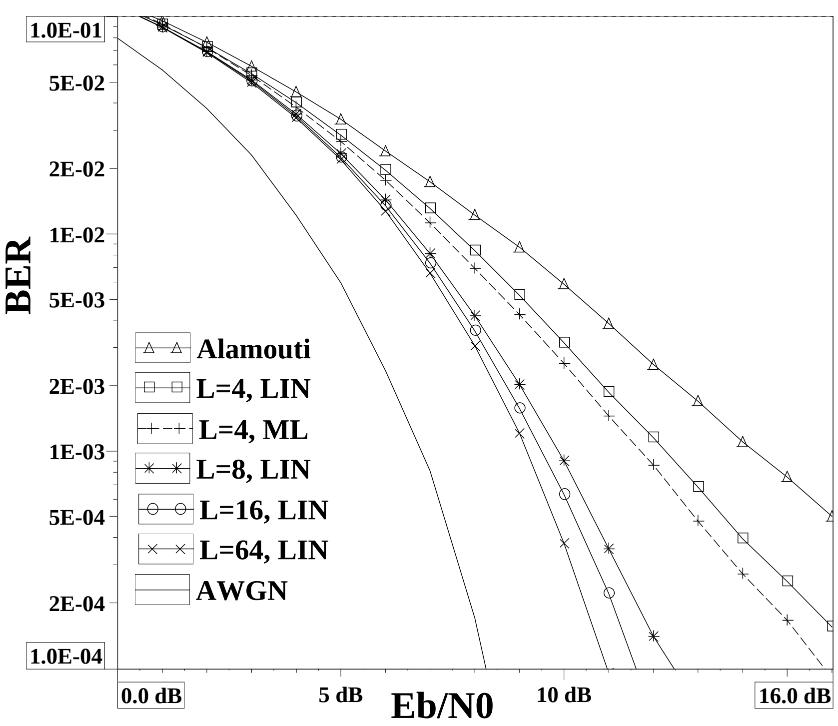
Figure captions :

Fig. 1: Bit Error Performance of different 4-transmit antenna systems including STBC and linear precoding.

Figure 1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5E-02
2E-02
1E-02
5E-03
2E-03
1E-03
5E-04
2E-04
1.0E-04
BER
Alamouti
L=4, LIN
L=4, ML
L=8, LIN
L=16, LIN
L=64, LIN
AWGN
0.0 dB
5 dB
10 dB
16.0 dB
Eb/N0