Low complexity iterative receiver for Linear Precoded OFDM

P.-J. Bouvet, M. Hélard, *Member, IEEE*, and V. Le Nir
France Telecom R&D
4 rue du Clos Courtel, 35512 Cesson-Sévigné, France
Email: {pierrejean.bouvet,maryline.helard}@francetelecom.com

***Abstract*— Linear Precoded OFDM systems with channel coding have already been demonstrated to efficiently exploit frequency and time diversities of the transmission channel. Iterative receivers which iteratively perform channel decoding and deprecoding provide very good performance. In the literature, these iterative receivers are based on maximum likelihood based functions for both linear deprecoding and channel decoding leading to a high complexity for large size of precoding matrices or high modulation order. In this paper, a simple linear operation instead of a maximum likelihood based one is carried out for deprecoding allowing the use of large precoding matrices and thus an optimal exploitation of the signal diversity.**

I. Introduction

Orthogonal Frequency Division Multiplexing technique (OFDM) has widely been studied for wireless broadband multimedia applications over the last decade. The main advantages of OFDM are its robustness in the case of frequency selective fading channels and its performance for portable and mobile reception [1]. Due to the large number of orthogonal subcarriers, the symbol duration is much higher than the channel time dispersion, minimizing the Inter-Symbol Interference (ISI). Furthermore, residual ISI is suppressed thanks to the insertion of a guard interval larger than the channel time delay at the beginning of each symbol. On the other hand, Linear Precoding (LP) also called constellation rotation or signal shaping has already been demonstrated to efficiently exploit both frequency and time diversities when combined with OFDM (LP-OFDM) [2]. The studied linear precoding technique is based on linear combinations of complex symbols and thus do not require any channel knowledge at the transmitter side and therefore exploits signal-space diversity without bandwidth expansion. In [2], the deprecoding stage is based on Maximum Likelihood (ML) algorithm which complexity exponentially increases with the precoding length and the modulation order. In [3], a sub-optimal linear receiver based on Minimum Mean Square Error (MMSE) criterion is proposed in order to cope with interferences brought by transmission of linear precoded symbols By applying the turbo principle described in [4], interferences can efficiently be cancelled in an iterative process where deprecoding and channel decoding are performed with ML-based algorithms leading to high complexity for large precoding matrix or high modulation order. In this paper, an iterative receiver with an MMSE-based deprecoding is proposed. Simulation results over theoretical time and frequency fading channel show that performance close to the Gaussian curve is reached at high signal-to-noise ratio (SNR) when the diversity order brought by linear precoding is large. Low complexity at the receiver is conserved even for large precoding matrix size and high modulation order. Results over more realistic channels are also presented stressing on the influence of the size of the interleaver on performance.

II. Linear precoding

The principle of linear precoding is to linearly combine the L symbols of vector $\mathbf{x}$ with a complex unitary matrix $\mathbf{\Theta}_L$ of size $L \times L$ in order to bring diversity between each component of the resulting vector $\mathbf{s} = \mathbf{\Theta}_L \cdot \mathbf{x}$. By using ML receivers, authors of [2] show that a L-order diversity can be achieved. Moreover this diversity is brought without bandwidth expansion and without any channel knowledge at the transmitter. Nevertheless the use of ML receiver limits the exploitation of the signal diversity to low values of L. In [3], a non-iterative MMSE receiver with reduced complexity linear MMSE receiver for deprecoding is presented allowing the use of large matrix and thus better signal-shape diversity exploitation. In this paper, a unitary precoding matrix based on a Hadamard construction is chosen where:

$$\mathbf{\Theta}_L = \sqrt{\frac{2}{L}} \begin{bmatrix} \mathbf{\Theta}_{L/2} & \mathbf{\Theta}_{L/2} \\ \mathbf{\Theta}_{L/2} & -\mathbf{\Theta}_{L/2} \end{bmatrix} \tag{1}$$

with:

$$\mathbf{\Theta}_2 = \frac{1}{2} \begin{bmatrix} -1-j & -1+j \\ 1+j & -1-j \end{bmatrix}$$

III. Linear precoded OFDM with channel coding

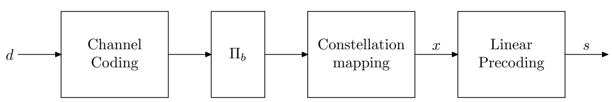
The LP-OFDM system including channel coding is described in Fig. 1(a). At the transmitter side, information bits $\mathbf{d}$ are first convolutional encoded, bit interleaved (Π_b) and then mapped to complex symbols $\mathbf{x}$ with variance σ_x^2 belonging to the constellation $\mathcal{A}$. The linear precoding represented by matrix $\mathbf{\Theta_L}$ is applied to $\mathbf{x}$. The precoded symbols $\mathbf{s}$ are OFDM modulated and transmitted to the channel. In practice OFDM modulation and demodulation are easily carried out by performing respectively Inverse Fast Fourier Transform (IFFT) and FFT operations of size N. Furthermore the insertion of a guard interval chosen greater than the delay spread of the channel guarantees the absence of inter-symbol interference. Thus the OFDM modulation, the multi-path fading channel and the OFDM demodulation can be represented by an equivalent flat fading channel (see Fig. 1(b)) on each subcarrier such as:

$$r_k = h_k \cdot s_k + n_k \tag{2}$$

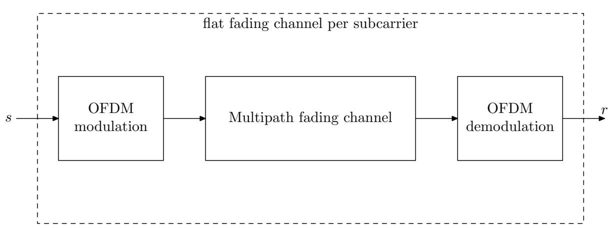
where s_k is the precoded symbol before OFDM modulation, h_k is the attenuation coefficient of the kth subcarrier on which the precoded symbol s_k is transmitted and n_k an equivalent additive white Gaussian noise sample with zero-mean and total variance σ_n^2. Assuming ideal time and frequency interleaving in the OFDM process, the $|h_k|^2$ samples follow an uncorrelated Rayleigh law with unit variance. By introducing $\mathbf{H} = \text{Diag}[h_1, h_2, \ldots, h_L] \in \mathbb{C}^{L\times L}$, we can write:

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{3}$$

with $\mathbf{n} = \{n_k\} \in \mathbb{C}^{L\times 1}$, $\mathbf{s} = \{s_k\} \in \mathbb{C}^{L\times 1}$ and $\mathbf{r} = \{r_k\} \in \mathbb{C}^{L\times 1}$. The concatenation of $\mathbf{H}$ and $\mathbf{\Theta}_L$ brings inter-element interference (IEI) terms that the receiver has to cope with. Note that matrix $\mathbf{H}$ does not depend on N, size of the FFT. In fact the FFT is performed on N precoded symbols available at the entrance of the OFDM modulator.



(a) Transmitter



(b) OFDM channel

Fig. 1. LP-OFDM representation

IV. Proposed iterative receiver

The proposed receiver iteratively performs deprecoding according to MMSE criterion and channel decoding as illustrated in Fig. 2. According to the turbo principle, two main functions exchange information learned from one stage to another at each iteration [5], [6]. The first function, called LP decoder, consists of a deprecoder and a demapper. The second function is a channel decoder.

A. LP decoding stage

1) MMSE receiver using a priori information: The LP decoder stage employs a linear interference canceller optimized under the MMSE criterion latter called MMSE-IC and that produces at iteration p vector $\tilde{\mathbf{x}}^{(p)}$ of equalized symbols from the previous vector $\hat{\mathbf{x}}^{(p-1)}$ made of estimated symbols. By defining the global matrix $\mathbf{C} = \mathbf{H}\mathbf{\Theta}_L$, let rewrite the expression of the receive vector as a function of the kth element of the transmitted vector:

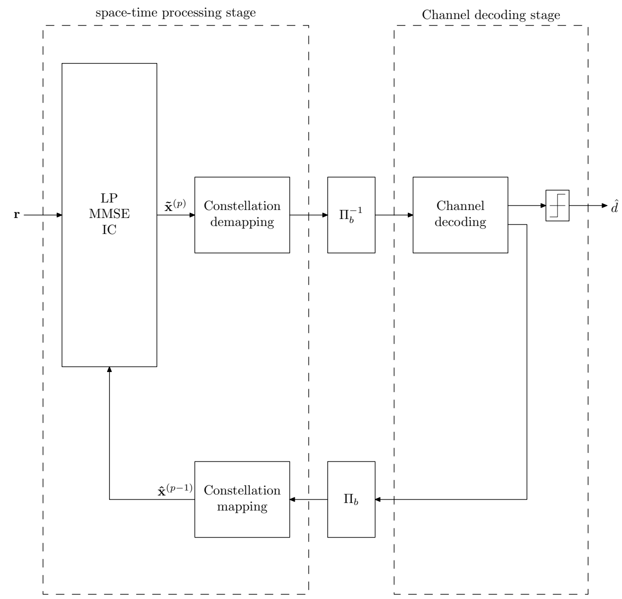


Fig. 2. Proposed receiver

$$\mathbf{r} = \underbrace{\mathbf{C}\mathbf{e}_k x_k}_{\text{useful term}} + \underbrace{\sum_{n\neq k} \mathbf{C}\mathbf{e}_n x_n}_{\text{interference terms}} + \underbrace{\mathbf{n}}_{\text{noise terms}} \tag{4}$$

where $\mathbf{e}_k$ is a L by 1 vector containing zero in all position except a 1 in position k. We propose a linear equalizer based on the MMSE principle and soft interference cancellation inspired from turbo-equalization concept [7]–[9]. The idea is to remove the interference terms brought by the linear precoding and then perform a linear filter $\mathbf{w}_k \in \mathbb{C}^{L\times 1}$ optimized under the MMSE criterion. The equalized symbol can be expressed as:

$$\tilde{x}_k^{(p)} = \mathbf{w}_k^H \left(\mathbf{r} - \mathbf{C}\hat{\mathbf{a}}_k^{(p-1)}\right) \tag{5}$$

with

$$\hat{\mathbf{a}}_k^{(p-1)} = \begin{bmatrix} \hat{x}_1^{(p-1)} & \ldots & \hat{x}_{k-1}^{(p-1)} & 0 & \hat{x}_{k+1}^{(p-1)} & \ldots & \hat{x}_L^{(p-1)} \end{bmatrix}^T$$

where $\hat{x}_k^{(p-1)}$ is the estimated vector of x_k given by previous iteration. The Wiener-Hopf solution is obtained by minimizing the following quantity:

$$\mathbf{w}_k^{opt} = \arg\min_{\mathbf{w}_k} \left|x_k - \tilde{x}_k^{(p)}\right|^2 \tag{6}$$

Exact solution of such equation can be found in [7], [8]:

$$\mathbf{w}_k^{opt} = \left(\mathbf{C}\mathbf{V}_k\mathbf{C}^H + \frac{\sigma_n^2}{\sigma_x^2}\mathbf{I}\right)^{-1} \mathbf{C}\mathbf{e}_k \tag{7}$$

where

$$\mathbf{V}_k = \sigma_x^2 \mathbf{I} - \sigma_{\tilde{x}}^2 \sum_{n=1, n\neq k}^{L} \mathbf{e}_n \mathbf{e}_n^T$$

and $\mathbf{I}$ is the $L\times L$ identity matrix whereas $\sigma_{\hat{x}}^2$ is the variance of the estimated symbol $\hat{x}_k$. However, each filter are determined by matrix inversion that increases the computational complexity. Sub-optimal solution can be found by assuming in Eq. (6), perfect estimation of transmitted symbol i.e.: $\hat{x}_k = x_k$. The corresponding solution is:

$$\mathbf{w}_k^{subopt} = \left(\mathbf{C}\mathbf{e}_k\mathbf{e}_k^T\mathbf{C}^H + \frac{\sigma_n^2}{\sigma_x^2}\mathbf{I}\right)^{-1}\mathbf{C}\mathbf{e}_k \quad (8)$$

Thus, the equalized symbol at iteration p can be easily rewritten in a matrix form:

$$\tilde{\mathbf{x}}^{(p)} = \left[\mathbf{D} + \frac{\sigma_n^2}{\sigma_x^2}\mathbf{I}\right]^{-1}\left[\mathbf{C}^H\mathbf{r} - \mathbf{J}\hat{\mathbf{x}}^{(p-1)}\right] \quad (9)$$

Where $\mathbf{D}$ and $\mathbf{J}$ are $L \times L$ complex matrices containing the diagonal and the off-diagonal elements of matrix $\mathbf{C}^H\mathbf{C}$ respectively. The equalization process now requires only matrix product, matrix sum and diagonal matrix inversion leading to reasonable complexity. Discussion on this approximation is given in the simulation results section.

2) First iteration equalization: At the first iteration because no prior information on symbols is available, a first equalized vector is obtained by using classical MMSE equalization (see [3]):

$$\tilde{\mathbf{x}}^{(1)} = \left[\mathbf{\Theta}_L^H\mathbf{H}^H\mathbf{H}\mathbf{\Theta}_L + \frac{\sigma_n^2}{\sigma_x^2}\mathbf{I}\right]^{(-1)}\mathbf{H}^H\mathbf{\Theta}_L^H\mathbf{r} \quad (10)$$

By using the unitary property of $\mathbf{\Theta_L}$, Eq. (10) can be simplified:

$$\tilde{\mathbf{x}}^{(1)} = \mathbf{\Theta}_L^H\left[\mathbf{H}^H\mathbf{H} + \frac{\sigma_n^2}{\sigma_x^2}\mathbf{I}\right]^{(-1)}\mathbf{H}^H\mathbf{r} \quad (11)$$

Since $\mathbf{H}$ is diagonal, matrix inversion operation is reduced to L scalar inversions.

3) Soft mapping/demapping: For an equalized sample $\tilde{x}_k$, the demapper produces a vector of q Logarithm Likelihood Ratios (LLRs) $[\Lambda_{1,k}^{eq}, \ldots, \Lambda_{q,k}^{eq}]$. Under the assumption that the output of the MMSE-IC follows a Gaussian law and according to the "max-log" approximation we obtain:

$$\begin{aligned}\Lambda_{i,k}^{eq} \quad = \quad & \frac{1}{\sigma_N^2 + \sigma_{IEI}^2}\left(\min_{x\in\mathcal{A}_0^i}\left|\tilde{x}_k - \mathbf{w}_k^H\mathbf{C}\mathbf{e}_k x\right|^2\right. \\ & \left. - \min_{x\in\mathcal{A}_1^i}\left|\tilde{x}_k - \mathbf{w}_k^H\mathbf{C}\mathbf{e}_k x\right|^2\right) \quad (12)\end{aligned}$$

where $\mathcal{A}_b^i$ denotes the subset of the constellation for which the i-th bit is equal to b. The respective variances of the residual noise and the inter-elements interferer terms, are calculated from Eq. (4) and (8):

$$\sigma_N^2 = \left\|\mathbf{w}_k^H\right\|^2\sigma_n^2$$
$$\sigma_{IEI}^2 = \left\|\mathbf{w}_k^H\sum_{n=1,n\neq k}^{L}\mathbf{C}\mathbf{e}_n\mathbf{e}_n^T\right\|^2(\sigma_x^2 - \sigma_{\hat{x}}^2)$$

Note that, for the first iteration, we set:

$$\sigma_{\hat{x}}^2 = 0$$
$$\mathbf{w}_k^H = \mathbf{e}_k^T\mathbf{\Theta}_L^H\left[\mathbf{H}^H\mathbf{H} + \frac{\sigma_n^2}{\sigma_x^2}\mathbf{I}\right]^{-1}\mathbf{H}^H$$

The processed LLRs are sent to the channel decoding stage via a de-interleaver $\mathbf{\Pi}_b^{-1}$. The updated LLRs coming from the output of the channel decoder are again interleaved and fed to the LP-decoding stage. The soft mapper provides the inverse transformation at the output of the channel decoder by converting the block of $[\Lambda_{1,k}^{dec}, \ldots, \Lambda_{q,k}^{dec}]$ calculated by the decoder into a soft symbol $\hat{x}_k$. If we denote $[b_1, \ldots, b_L]$ the bits that constitute a q-ary symbol belonging to constellation $\mathcal{A}$, the soft estimated symbol is given by the following expression [9]:

$$\hat{x}_k = \sum_{x\in\mathcal{A}} x \prod_{i=1}^{q}\left[\frac{1}{2} + \frac{2b^i - 1}{2}\tanh\left(\frac{\Lambda_{i,k}^{dec}}{2}\right)\right] \quad (13)$$

B. Channel decoding stage

The input of the channel decoding stage is the set of a priori LLRs of coded bits. The channel decoder processes the soft information and compute refined LLRs of coded bits and at the last iteration the LLR of the information bits. A forward-backward algorithm of the type of BCJR or a Soft Output Viterbi Algorithm (SOVA) can be implemented [10].

V. Channel and system parameters

A. Channel models

1) Theoretical channel: The coefficients are chosen i.i.d. Gaussian samples with unit variance. This channel corresponds to an optimal OFDM transmission over time and selective channel where the interleaver depth in the OFDM process is perfectly dimensioned. Performance over such channel provides the optimal performance that can be reached with the iterative receiver over a time and frequency selective channel.

2) Time variant Frequency fading channel: We consider the 18-tap BRAN E model specified by ETSI-BRAN [11] that represents a typical outdoor urban multi-path propagation characterized by a large delay and Doppler spread. For simulations, the mobile speed is set to 16.6 m/s.

B. System parameters

We choose a half-rate convolutional code with $(23, 35)_o$ as generator polynomials and a QPSK modulation leading to a raw spectral efficiency of 1 bit/s/Hz. All systems parameters are presented in Table I. OFDM parameters have been chosen according to the time and frequency coherence of the channel in order to reduce the inter carrier interference and the inter-symbol interference. Finally the interleaving depths are chosen relatively small in order to provide realistic simulation results. Moreover, in order to highlight the performance of the receiver, we plot the matched filter bound (MFB) of each system that

TABLE I
SYSTEM PARAMETERS

Carrier frequency	5 GHz
Sample Frequency	57.6 MHz
FFT Size	1024
Number of modulated subcarriers	768
Occupied bandwidth	42.75 MHz
OFDM Guard interval duration	3.73 s
Total OFDM symbol duration	21.52 s
Symbol interleaving	Random type, 768 symbols
Precoding length	4 or 64
Constellation mapping	QPSK
Bit Interleaving	Random type, 1432 bits
Channel coding	$1/2$-rate, $(23, 35)_o$

stands also for performance obtained with an ideal receiver that perfectly cancels the interference coming from linear precoding.

VI. SIMULATION RESULTS

Bit Error Rate (BER) performance, provided over theoretical Rayleigh channel and a BRAN E channel, is plotted for different sizes of linear precoding matrices and depending on the number of iterations at the receiver side. The curves labeled "no LP" stand for a classical OFDM system with channel coding but without linear precoding. The curves AWGN are obtained with only Additive White Gaussian Noise and channel coding. The curves named "opt" represents the respective matched filter bounds that could be theoretically obtained by a genie aided system where interference would be perfectly cancelled and data perfectly estimated.

A. Theoretical channel

Fig. 3 illustrates the BER performance without any iterative process. This receiver can be viewed as a linear MMSE one. This performance is also obtained after the first iteration of the iterative receiver. We can note that, as demonstrated in [3], that LP-OFDM outperforms OFDM if the SNR is greater than a threshold that depends on the channel code structure and the precoding length. In case of $(23, 35)_o$ half rate convolutional code as channel coder, this threshold is equal to 5.7 dB and 6.1 dB for $L = 4$ and $L = 64$ respectively. The degradation of LP-OFDM at the first iteration compared to OFDM is due to the interferences brought by the linear precoding. By increasing the precoding size, the asymptotical performance of LP-OFDM are improved owing to a better exploitation of the signal-space diversity but the degradation due to many interference terms becomes greater at low SNR.

However, as illustrated by Fig. 4 and 5 for respectively LP-OFDM with L=4 and L=64, these interferences are progressively removed during the iterative process leading to meaningful gains after few iterations. Two methods are used in order to implement the MMSE-IC, the first one called IC exact with a matrix inversion for calculations (see [7] or [8]),

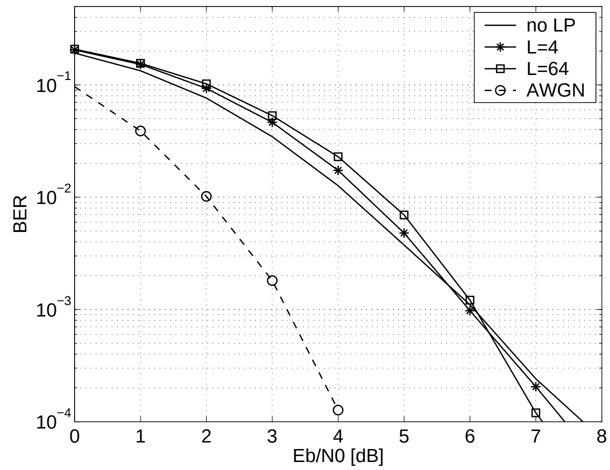


Fig. 3. BER performance of the LP-OFDM system with linear MMSE receiver over theoretical channel, QPSK, $1/2$-rate convolutional code, 1 bps/Hz

and a second one called IC approx that stands for simplified calculations given by Eq. (9) presented in part IV and requiring no matrix inversion. Better results are obtained with the IC exact at low SNR but at a higher complexity cost due to the matrix inversion. The larger the size of precoding matrix, the larger the interference terms and thus the greater the degradation between the 2 methods at low SNR. Nevertheless, the simplified method provides very good trade-off between performance and complexity and exactly same performance at BER=10^{-3}. There is about a 2.5 dB gain at 10^{-4} achieved by LP-OFDM with $L = 4$ after 4 iterations and a 3.5 dB gain at 10^{-4} achieved by LP-OFDM with $L = 64$ after 4 iterations. The performance of the system with $L = 64$ is very close to the Gaussian curve at high SNR, confirming the efficiency of the receiver and that time and frequency diversities are efficiently exploited.

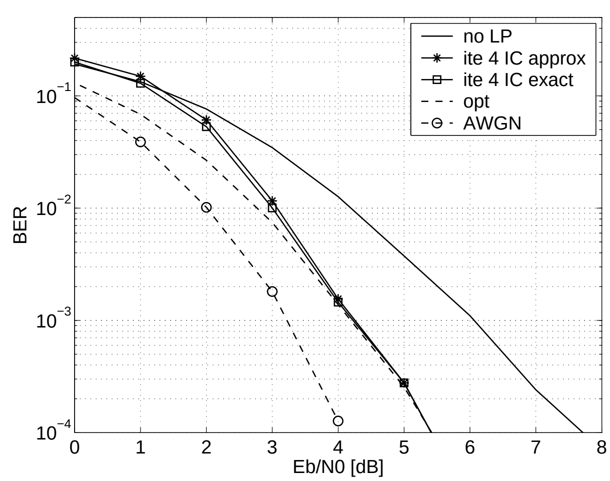


Fig. 4. BER performance of the LP-OFDM system with iterative receiver over theoretical channel with size L=4, QPSK, $1/2$-rate convolutional code, 1 bps/Hz

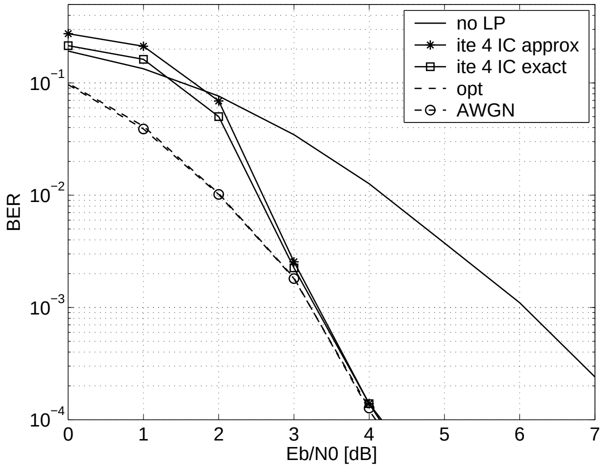


Fig. 5. BER performance of the LP-OFDM system with iterative receiver over theoretical channel with size L=64, QPSK, $1/2$-rate convolutional code, 1 bps/Hz

B. BRAN E channel

Results presented in Fig. 6 are obtained over a more realistic outdoor channel, frequency and time selective, as defined by BRAN ETSI with the BRAN E channel model. The MMSE IC carried out for these results is the simplest one, i.e. the so-called IC approx. The interleaving depth is chosen relatively small (1432 bits) in order to respect a realistic framing. Optimal performance are reached with a degradation smaller than 2 dB versus theoretical channel and an improvement greater than 2 dB and 3 dB for L= 4 and L=64 respectively versus an OFDM system. Increasing the size of the interleaver in respect with potential application should have led to a lower degradation compared to the Gaussian curve and thus to a greater improvement compared to the OFDM system for which no iterative process is performed.

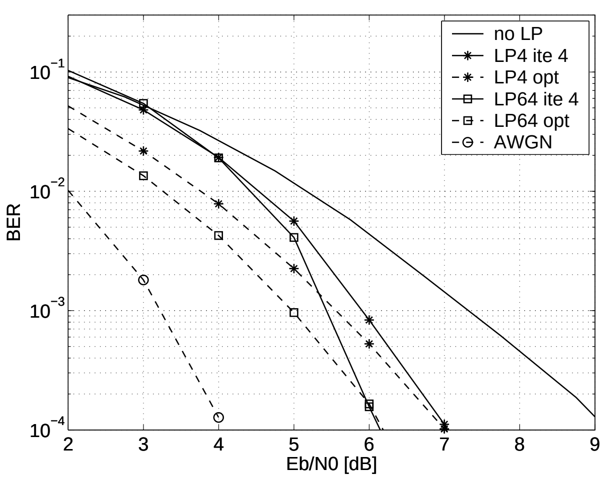


Fig. 6. BER performance of the LP-OFDM system with iterative receiver over Bran E channel with size L=64, QPSK, $1/2$-rate convolutional code, 1 bps/Hz

VII. Conclusion

The proposed iterative receiver for LP-OFDM system with channel coding offers very good BER performance results at a very low cost of complexity. In fact, the proposed iterative receiver does not require any matrix inversion and its complexity does not increase with the linear precoding matrix size, nor with the modulation order. The interferences brought by linear precoding are efficiently removed in the iterative process. In the other hand OFDM that conserves orthogonality is not improvable by such an iterative process. Over a theoretical channel, near Gaussian performance is obtained with a precoding size of 64 demonstrating that frequency, time and signal space diversities are all efficiently exploited. With more realistic channels, such as BRAN E channel, and a small interleaver, the improvement compared to OFDM remains important. Using a large size of precoding matrix allows a greater gain but a difference of only 1 dB is observed between $L = 4$ and $L = 64$. Finally performance over realistic channel could by improved by using larger size of interleaver

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