Autonomous Dynamic Spectrum Management for Coexistence of Multiple Cognitive Tactical Radio Networks

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Abstract—In this paper, dynamic spectrum management is studied for multiple cognitive tactical radio networks coexisting in the same area. A tactical radio network is composed of a transmitter which broadcasts the same information to its multiple receivers. First, we consider the problem of power minimization subject to a minimum rate constraint and a spectral mask constraint for a single tactical radio network with multiple receivers over parallel channels (parallel multicast channels). Then, we extend the iterative waterfilling algorithm to multiple receivers for the coexistence of multiple cognitive tactical radio networks, meaning that there is no cooperation between the different networks. The power allocation is performed autonomously at the transmit side assuming knowledge of the noise variances and channel variations of the network. Simulation results show that the proposed algorithm is very robust in satisfying these constraints while minimizing the overall power in various scenarios.

Index Terms—Cognitive tactical radio networks, dynamic spectrum management, iterative water-filling.

I. INTRODUCTION

The objective of this paper is to provide a distributed power allocation of multiple cognitive tactical radio networks coexisting in the same area. The transmitter of each tactical radio network broadcasts the same information to its group (voice, data...). This objective calls for a synergy between different areas:

- Cognitive radio [1], [2]: A wireless node or network can adapt to the environment by changing its transmission parameters (frequency, power, modulation strategy)
- Broadcast channel with only common information [3], [4], [5]: A tactical radio network is a network in which information is conveyed from one transmitter to multiple receivers. Most of the literature on broadcast channels covers the transmission of separate information to the different receivers or the transmission of both separate and common information to the different receivers over parallel channels [6], [7], [8], [9], [10], [11].
- Distributed multi-user power control [12], [13], [14]: Autonomous power allocation in the frequency domain by iterative waterfilling for interference channels. By considering the interference of the other users as noise, iterative updates of the power allocation for each user reach an equilibrium.

The waterfilling strategy has been initially designed for a single transmitter and a single receiver over multiple sub-channels [15]. The waterfilling strategy can maximize the rate

of the link subject to a power constraint (inner loop), but can also minimize the power subject to a rate constraint (outer loop). In the first part of the paper (Section II), we extend the waterfilling strategy to multiple receivers by considering parallel broadcast channels with only common information (parallel multicast channels) and assuming perfect channel state information (CSI) at the transmit side. In this case, the extended waterfilling strategy maximizes the minimum rate subject to a power constraint (inner loop) or minimizes the power subject to a minimum rate constraint (outer loop). However, finding a solution to these problems for parallel multicast channels is not straightforward. Moreover, standardization often defines a spectral mask that each transmitter has to satisfy. Therefore, we propose to use an utility function based on the weighted sum of the possible achievable rates to the receivers for the inner loop and to find the best set of weights that minimizes the power subject to a minimum rate constraint for all receivers and a spectral mask constraint.

In the second part of the paper (Section III), capitalizing on the previous results, we introduce an autonomous dynamic spectrum management algorithm based on iterative waterfilling [12] for multiple cognitive tactical radio networks. In the iterative waterfilling algorithm, each network considers the interference of all other networks as noise and performs a waterfilling strategy. The power spectrum of the network modifies the interference caused to all other networks. This process is performed iteratively until the power spectra of all networks converge. The main novelty of this paper is the extension of the iterative waterfilling algorithm to multiple receivers for the coexistence of multiple cognitive tactical radio networks. The transmitter of each tactical radio network takes into account the spectrum sensed by all its receivers and iteratively updates its power spectrum until all the constraints are satisfied in each network, i.e. minimum rate and a spectral mask constraints. Simulation results compare our strategy with the worst receiver strategy in Section IV.

II. SINGLE TACTICAL RADIO NETWORK

A. Minimization of power subject to a minimum rate constraint and a spectral mask constraint

Consider a T-receiver N_c parallel fading Gaussian broadcast channel as shown in Figure 1:

$$y_{it} = h_{it}x_i + n_{it} \quad t = 1 \ldots T, i = 1 \ldots N_c \quad (1)$$

where x_i is the transmitted signal, n_{it} represents a complex noise with variance σ_{it}^2 and h_{it} corresponds to the channel on receiver t and tone i. The primal problem for power minimization of a T-receiver N_c parallel fading Gaussian broadcast channel with only common information subject to a minimum rate constraint for all receivers R^{min} and a spectral mask constraint is

$$
\begin{aligned}
&\min_{(\phi_i)_{i=1 \ldots N_c}} \sum_{i=1}^{N_c} \phi_i \\
&\text{subject to } \sum_{i=1}^{N_c} \log_2(1 + \frac{|h_{it}|^2 \phi_i}{\Gamma \sigma_{it}^2}) \geq R^{min} \ \forall t \\
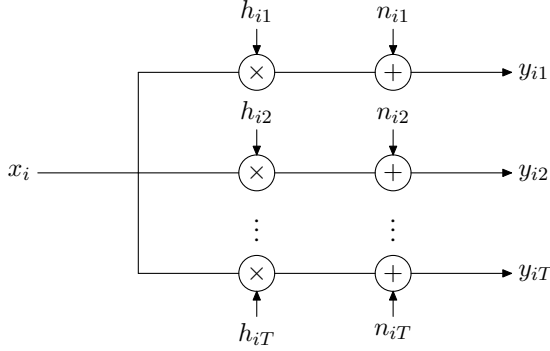&\phi_i \leq \phi_i^{mask} \ \forall i
\end{aligned}
\quad (2)
$$

Fig. 1. T-receiver N_c parallel fading Gaussian broadcast channel

with $\phi_i = E[|x_i|^2]$ the variance of the transmitted signal on channel i, ϕ_i^{mask} the mask constraint on sub-channel i, and Γ the SNR gap which measures the loss with respect to theoretically optimum performance [16]. The derivation of the modified Lagrangian function leads to a single variable search with T Lagrange multipliers [17]. Therefore, the optimal power allocation has an infinite set of solutions and the problem is intractable for $T > 1$. We propose a solution to this problem by defining an utility function which takes into account the possible achievable rates to the individual receivers. In the following, the weighted sum rate is chosen for this utility function as it allows to consider the achievable rates to the receivers with a certain flexibility owing to the weighting parameters. Therefore, an inner loop determines the power allocation maximizing the weighted sum rate subject to a total power constraint and a spectral mask constraint for a fixed set of weights. The minimum rate is then selected amongst the possible achievable rates to the receivers. Then, an outer loop minimizes the power such that a minimum rate constraint is achieved. This process is repeated for all set of weights and the set of weights exhibiting the least power determines the power allocation for power minimization subject to a minimum rate constraint. The primal problem for weighted sum rate maximization subject to a power constraint P^{tot} and a spectral mask constraint is:

$$\max_{(\phi_i)_{i=1\dots N_c}} \sum_{i=1}^{N_c} \sum_{t=1}^{T} w_t \log_2(1 + \frac{|h_{it}|^2 \phi_i}{\Gamma \sigma_{it}^2})$$
$$\text{subject to } \sum_{i=1}^{N_c} \phi_i = P^{tot} \qquad (3)$$
$$\phi_i \leq \phi_i^{mask} \; \forall i$$

with $\sum_{t=1}^{T} w_t = 1$. As the objective function is concave, the power allocation can be derived by the standard Karush-Kuhn-Tucker (KKT) condition [17]. By taking the derivative of the modified Lagrangian function with respect to ϕ_i, we can solve the KKT system of the optimization problem. The derivative with respect to ϕ_i is given by

$$\frac{\partial L(\lambda, (\beta_i, \phi_i)_{i=1\dots N_c})}{\partial \phi_i} = \frac{1}{\ln 2} \sum_{t=1}^{T} \frac{w_t}{\frac{\Gamma \sigma_{it}^2}{|h_{it}|^2} + \phi_i} - (\lambda + \beta_i) \quad (4)$$

with λ the Lagrange multiplier associated with the total power constraint, and β_i the Lagrange multipliers corresponding to the spectral mask constraint. Nulling the derivative gives

$$\frac{\partial L(\lambda, (\phi_i)_{i=1\dots N_c})}{\partial \phi_i} = 0 \Rightarrow \sum_{t=1}^{T} \frac{w_t}{\frac{\Gamma \sigma_{i1}^2}{|h_{it}|^2} + \phi_i} = \underbrace{\lambda \ln 2}_{\tilde{\lambda}} + \underbrace{\beta_i \ln 2}_{\tilde{\beta}_i} \quad (5)$$

From the previous formula, one can see that the power allocation depends on the number of receivers T. Let us derive the power allocation for different number of receivers:

- For a single receiver $T = 1$, the power allocation corresponds to Gallager's water-filling strategy for single-user parallel Gaussian channels [15] with additional spectral mask constraint given by:

$$\frac{\partial L(\lambda, (\phi_i)_{i=1\dots N_c})}{\partial \phi_i} = 0 \Rightarrow \phi_i = \left[\frac{1}{\tilde{\lambda} + \tilde{\beta}_i} - \frac{\Gamma \sigma_{i1}^2}{|h_{i1}|^2} \right]^+ \quad (6)$$

- For two receivers $T = 2$, the power allocation is a type of water-filling strategy given by the solution of a quadratic equation.

$$\frac{\partial L(\lambda, (\beta_i, \phi_i)_{i=1\dots N_c})}{\partial \phi_i} = 0$$
$$\Rightarrow \frac{w_1}{\underbrace{\frac{\Gamma \sigma_{i1}^2}{|h_{i1}|^2} + \phi_i}_{a_i}} + \frac{w_2}{\underbrace{\frac{\Gamma \sigma_{i2}^2}{|h_{i2}|^2} + \phi_i}_{b_i}} = \tilde{\lambda} + \tilde{\beta}_i \quad (7)$$

The quadratic equation to be solved is

$$(\tilde{\lambda} + \tilde{\beta}_i)\phi_i^2 + ((\tilde{\lambda} + \tilde{\beta}_i)(a_i + b_i) - (w_1 + w_2))\phi_i$$
$$+ (\tilde{\lambda} + \tilde{\beta}_i)a_i b_i - (w_1 b_i + w_2 a_i) = 0. \quad (8)$$

The discriminant is given by

$$\Delta = (\tilde{\lambda} + \tilde{\beta}_i)^2 (a_i - b_i)^2 + (w_1 + w_2)^2$$
$$- 2(\tilde{\lambda} + \tilde{\beta}_i)(a_i - b_i)(w_1 - w_2) . \quad (9)$$

The power allocation is given by the positive root

$$\phi_i = \left[\frac{1}{2(\tilde{\lambda} + \tilde{\beta}_i)} + \right.$$
$$\left. \sqrt{\frac{(w_1 + w_2)^2}{4(\tilde{\lambda} + \tilde{\beta}_i)^2} - \frac{(a_i - b_i)(w_1 - w_2)}{2(\tilde{\lambda} + \tilde{\beta}_i)} + \frac{(a_i - b_i)^2}{4}} - \frac{a_i + b_i}{2} \right]^+ . \quad (10)$$

In this formula, the power allocation for weighted sum rate subject to a power constraint and a spectral mask constraint takes into account the difference between the water-fill functions and the weights of the different receivers.

- For three receivers $T = 3$ and four receivers $T = 4$, the power allocation is a type of water-filling strategy given by the solution of a cubic and quartic equation respectively. Therefore, the power allocation can also be found analytically (the solution is not given in this paper due to space limitations). With $T > 4$, the power allocation is given by the solution of a polynomial equation with degree T. In general, the roots can't be expressed analytically but can be solved numerically.

The algorithm uses the weights to minimize the power subject to a minimum rate constraint and a spectral mask constraint.

Algorithm 1 Minimization of the power subject to a minimum rate constraint

1 n=0
2 for all $w_1,\ldots,w_T$, with $\sum\limits_{t=1}^{T} w_t = 1$
3   n=n+1
4   init $P = 10^{-9}$
5   init $pstep = 2$
6   init $p = 0$
7   init $R_t = 0 \; \forall t$
8   while $|min(R_1,\ldots,R_T) - R^{min}| > \epsilon$
9   init $\lambda = 10^{-9}$
10   init $step = 2$
11   init $b = 0$
12   init $\phi_i = 0 \; \forall i$
13   while $| \sum\limits_{i=1}^{N_c} \phi_i - P| > \epsilon$
14   Calculate $\phi_i \; \forall i$ according to (4)'s root
15   if $\sum\limits_{i=1}^{N_c} \phi_i - P < 0$
16          $b = b + 1$
17          $\lambda = \lambda/step$
18          $step = step - 1/2^b$
19   end if
20        $\lambda = \lambda * step$
21   end while
22   Individual rates $R_t = \sum\limits_{i=1}^{N_c} \log_2(1 + \frac{|h_{it}|^2 \phi_i}{\Gamma \sigma_{it}^2}) \; \forall t$
23   if $min(R_1,\ldots,R_T) - R^{min} > 0$
24        $p = p + 1$
25        $P = P/pstep$
26        $pstep = pstep - 1/2^p$
27   end if
28      $P = P * pstep$
29   end while
30    $P_n = P$
31 end for
32 $P^{min} = min(P_n)$

Algorithm 2 Modifications to **Algorithm 1** to take into account a spectral mask constraint

1 for $i = 1$ to N_c
2   init $\beta = 10^{-9}$
3   init $mstep = 2$
4   init $m = 0$
5   for $iteration = 1$ to 20
6   Calculate ϕ_i according to (4)'s root
7   if $\phi_i > \phi_i^{mask}$
8          $\phi_i = \phi_i^{mask}$
9   end if
10   if $\phi_i - \phi_i^{mask} < 0$
11        $m = m + 1$
12        $\beta = \beta/mstep$
13        $mstep = mstep - 1/2^m$
14   end if
15      $\beta = \beta * mstep$
16   end for
17 end for

Algorithm 1 provides the power allocation for power minimization subject to a minimum rate constraint R^{min} of a T-receiver N_c parallel fading Gaussian broadcast channel with only common information ($\beta_i = 0 \; \forall i$). The inner loop and the outer loop correspond to lines 13-21 and 8-29 respectively. To include a spectral mask constraint, we need to replace the line 14 with the modifications given in Algorithm 2.

III. Multiple cognitive tactical radio networks

In this Section, we consider the scenario in which N different cognitive radio networks can't cooperate with each other and wish to broadcast a common information to their network by sharing the same N_c parallel sub-channels. This scenario is particularly adapted to tactical radio networks in which N different networks coexist in a given area and broadcast a common information (voice, data...) to their group. With current technologies, if the legacy radios of the coalition nations share the same parallel sub-channels, the interference would increase and lead to a bad transmission. Cognitive radio enables the adaptation of the transmission parameters (transmit power, carrier frequency, modulation strategy) to these scenarios. Based on the results of Section II, we propose

a completely autonomous distributed power allocation. Considering N different networks and assuming that each network j has T_j receivers, the received data can be modeled as

$$y_{j,it} = h_{jj,it}x_{ji} + \sum_{k \neq j}^{N} h_{jk,it}x_{ki} + n_{j,it} \quad \begin{array}{l} i = 1 \ldots N_c, \\ j = 1 \ldots N, \\ t = 1 \ldots T_j \end{array}$$
(11)

where $n_{j,it}$ represents a complex noise with variance $\sigma_{j,it}^2$ and $h_{jk,it}$ corresponds to the channel from network k to j on receiver t and tone i. Similarly to Section II in which the initial problem of power minimization subject to minimum rate constraint is intractable for $T > 1$, we propose a way to solve the initial problem by defining an utility function (the weighted sum rate) which takes into account all the achievable rates of the receivers and to select the minimum rate in each network. The primal problem for the weighted sum rate maximization subject to a total power constraint and a spectral mask constraint per network (inner loop) is given by:

$$\max_{\substack{(\phi_{ij})_{j=1\ldots N} \\ i=1\ldots N_c}} \sum_{i=1}^{N_c} \sum_{j=1}^{N} \sum_{t=1}^{T_j} w_{jt}\log_2(1 + \frac{|h_{jj,it}|^2 \phi_{ij}}{\Gamma(\sigma_{j,it}^2 + \sum\limits_{k \neq j} |h_{jk,it}|^2 \phi_{ik})})$$

$$\text{subject to } \sum_{i=1}^{N_c} \phi_{ij} \leq P_j^{tot} \; \forall j$$

$$\phi_{ij} \leq \phi_{ij}^{mask} \; \forall i,j$$
(12)

This problem is highly non-convex and no closed-from solution can be derived. Even if a centralized cognitive manager was able to collect all the channel state information (CSI) within and between the different networks, it would require an exhaustive search over all possible ϕ_{ij}'s, or a more efficient genetic algorithm. To solve this problem, we propose a sub-optimal distributed algorithm based on the iterative water-filling algorithm initially derived for dynamic spectrum management in digital subscriber line (DSL) [12]. The iterative water-filling principle is extended to multiple cognitive tactical radio networks, in which each network considers the

interference of the other networks as noise and performs water-filling on its parallel multicast channels. Each update of one network's water-filling affects the interference of the other networks and this process is repeated iteratively between the networks until the power allocation of all networks converge and reach a Nash equilibrium. As the power updates between networks can be performed asynchronously, an iterative water-filling based algorithm for the coexistence between multiple cognitive tactical radio networks. Let us derive the modified Lagrangian function:

$$
\begin{aligned}
&L((\lambda_j)_{j=1...N}, (\beta_{ij}, \phi_{ij})_{i=1...N_c}^{j=1...N}) = \\
&\sum_{i=1}^{N_c} \left(\sum_{j=1}^{N} \sum_{t=1}^{T_j} w_{jt} \log_2(1 + \frac{|h_{jj,it}|^2 \phi_{ij}}{\Gamma(\sigma_{j,it}^2 + \sum_{k \neq j} |h_{jk,it}|^2 \phi_{ik})}) \right. \\
&\left. - \sum_{j=1}^{N} (\lambda_j + \beta_{ij})\phi_{ij} \right) + \sum_{j=1}^{N} \lambda_j P_j^{tot} + \sum_{i=1}^{N_c} \sum_{j=1}^{N} \beta_{ij} \phi_{ij}^{mask}
\end{aligned}
$$

(13)

in which the λ_j's and β_{ij}'s are the Lagrange multipliers. We assume that each transmitter has the knowledge of the noise variances and the channel variations in its own network j

$$
\begin{cases}
\sigma_{j,it}^2 + \sum_{k \neq j} |h_{jk,it}|^2 \phi_{ik} & \forall i, \forall t \\
h_{jk,it} & k = j, \forall i, \forall t
\end{cases}
$$

(14)

This knowledge can be acquired through a feedback channel from the receivers to the transmitter of each network assuming that the acquisition time is much lower than the coherence time of the channel fading. To this end, each terminal must be equipped with a spectrum sensing function to estimate the noise variances and a channel estimation function to estimate its channel variations. Then, by taking the derivative of the modified Lagrangian function with respect to ϕ_{ij}, we can solve the KKT system of the optimization problem:

$$
\begin{aligned}
&\frac{\partial L((\lambda_j)_{j=1...N}, (\beta_{ij}, \phi_{ij})_{i=1...N_c}^{j=1...N})}{\partial \phi_{ij}} = \\
&\frac{1}{\ln 2} \sum_{t=1}^{T_j} \frac{w_{jt}}{\Gamma(\frac{\sigma_{j,it}^2}{|h_{it}|^2} + \sum_{k \neq j} \frac{|h_{jk,it}|^2}{|h_{it}|^2} \phi_{ik}) + \phi_i} - (\lambda_j + \beta_{ij})
\end{aligned}
$$

(15)

For transmitter j, the power allocation is the solution given by the roots of (4) with the interference terms estimated at each receiver within the network j. For instance, with two receivers $T_j = 2$, the power allocation within the network j is given by (10) with the following modifications:

$$
\begin{cases}
a_i = \Gamma(\sigma_{j,i1}^2 + \sum_{k \neq j} |h_{jk,i1}|^2 \phi_{ik}) \\
b_i = \Gamma(\sigma_{j,i2}^2 + \sum_{k \neq j} |h_{jk,i2}|^2 \phi_{ik})
\end{cases}
$$

(16)

Therefore, a distributed power allocation of N different networks can be obtained by updating iteratively the powers of the different transmitters using the single-transmitter power allocation for minimizing the power subject to a minimum rate constraint and a spectral mask constraint. However, in Algorithm 2, the weight loop encompasses the outer loop to find which set of weights corresponds to the global minimum power satisfying a minimum rate constraint R^{min}. As the algorithm should be distributed and autonomous, the set of weights minimizing the power have to be determined for each network independently. To this end, we have to move the weight loop inside the outer loop by introducing a rule based on the rates. An adequate rule is to introduce a deviation metric (DM) which measures the dispersion of the rates. The DM must be computed within each network j for each set of weights n over the T_j receivers. The rule is given by the following formula:

$$
DM_j(n) = \frac{\sqrt{T_j \sum_{t=1}^{T_j} [(R_{jt}(n) - \frac{1}{T_j} \sum_{t=1}^{T_j} R_{jt}(n))^2]}}{\sum_{t=1}^{T_j} R_{jt}(n)}
$$

(17)

with $R_{jt}(n)$ the rate for the network j, receiver t and the set of weights n. This rule allows to achieve the global minimum power although the decision has to be taken inside the outer loop. It basically means that for a given power the closer the rates of the different receivers within a network, the less power will be needed to achieve the minimum rate constraint. This algorithm is referred to as Algorithm 3 in the simulation results.

IV. RESULTS

In the first set of simulations, we compare the algorithm for a single tactical radio network with the trivial case where the waterfilling is performed on the receiver with the worst channel conditions, i.e. the worst receiver strategy. Note that the worst receiver strategy can be seen as a special case of the presented algorithm in which $w_t = 0 \ \forall t$ except for the worst receiver. The log-distance path loss model is used to measure the path loss between the transmitter and the receivers [18], with bandwidth $\Delta f = 25$ kHz, $N_c = 4$ sub-channels, carrier frequency $f_c = 80$ MHz, path loss exponent $n = 4$, reference distance $d_0 = 20$ meters and thermal noise $\sigma_n^2 = 10^{-16}$. For the simulations, we use a square area of 1 km^2 in which the transmitter and $T = 2$ receivers are placed randomly using Monte Carlo trials. The SNR gap for an uncoded quadrature amplitude modulation (QAM) to operate at a symbol error rate 10^{-7} is $\Gamma = 9.8$ dB. The scenario considers a very strong noise ($\sigma_n^2 = 10^{-9}$) seen on the 4th sub-channel by the first receiver and on the 1st sub-channel by the second receiver. The different noises seen by the different receivers can be thought as sub-channel variations depending on the location, a sub-channel occupied by a primary or a secondary transmitter, a jammer etc.

The left part of Figure 2 shows the results of the power minimization subject to a minimum rate constraint per receiver ranging from $R^{min} = 2$ kbps to $R^{min} = 512$ kbps over 10^3 Monte Carlo trials for the locations of the transmitter and the receivers. Algorithm 2 provides a substantial gain compared to the worst receiver strategy. The right part of Figure 2 shows that the algorithm converges within 30 iterations (the number of iterations for convergence mainly depend on the starting point, in this case $P = 10^{-11}$). Since it is based on closed-form expressions, the algorithm has reasonable complexity for a low number of receivers as the search for the best set of weights require an exhaustive search over all possible weights.
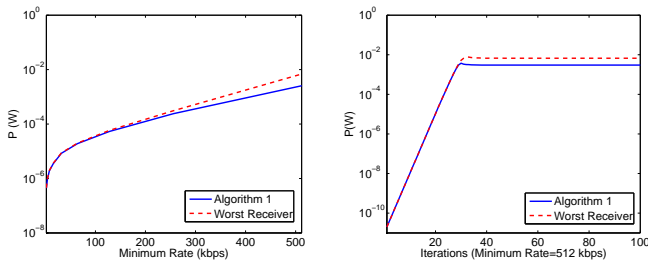
Fig. 2. Results on the power minimization subject to a minimum rate constraint for a single tactical radio network

Fig. 3. Results on the averaged power minimization subject to a minimum rate constraint averaged for the coexistence of two tactical radio networks

In the second set of simulations, we compare the iterative waterfilling based algorithm developed in Section III for N networks with the worst receiver strategy extended to multiple networks. Simulation results are performed with $N = 2$ networks, $T_j = 2$ receivers $\forall j$ and $N_c = 4$ sub-channels. We consider a scenario in which all receivers see a different noise σ_n^2 on their $N_c = 4$ sub-channels (similar to the first set of simulations). In the first network, a very strong noise ($\sigma_n^2 = 10^{-9}$) is seen on the 4th sub-channel by the first receiver and on the 1st sub-channel by the second receiver. In the second network, a very strong noise ($\sigma_n^2 = 10^{-9}$) is seen on the 3th sub-channel by the first receiver and the 2nd sub-channel by the second receiver. The left part of Figure 3 shows the results of the power minimization subject to a minimum rate constraint ranging from $R^{min} = 2$ kbps to $R^{min} = 512$ kbps over 10^3 Monte Carlo trials for the locations of the transmitter and the receivers. In this case, Algorithm 3 is the only strategy which provides a viable solution because the worst receiver strategy tends to utilize the maximum available power of 1 Watt. Although designed for the coexistence of multiple tactical radio networks, the convergence of Algorithm 3 is similar to Algorithm 1 for both networks (right part of Figure 2). The right part of Figure 3 shows that the deviation metric (DM) reduces as the algorithm converges. It can be seen that in practical scenarios in which the interference temperature varies along the sub-channel and the receiver locations, Algorithm 3 provides an efficient distributed strategy to find the power allocation of multiple networks in which each transmitter has to broadcast a common information to its receivers.

V. CONCLUSION

In this paper, dynamic spectrum management was studied for multiple cognitive tactical radio networks coexisting in the same area. First, we have considered the problem of power minimization subject to a minimum rate constraint and a spectral mask constraint for a single tactical radio network with multiple receivers over parallel channels (parallel multicast channels). Then, we have extended the iterative waterfilling algorithm to multiple receivers for the coexistence of multiple cognitive tactical radio networks assuming knowledge of the noise variances and channel variations of the network. Simulation results have shown that the proposed algorithm is very robust in satisfying these constraints while minimizing the overall power in various scenarios.
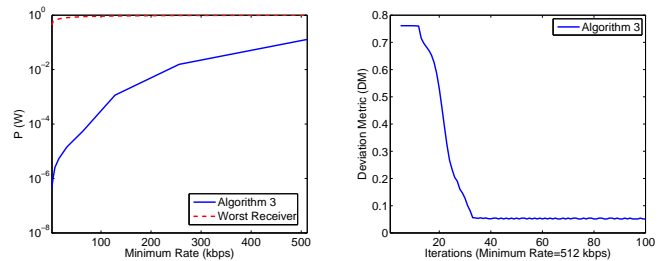
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