Volume of Influence for Magnetic Soils and Electromagnetic Induction Sensors

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***Abstract*—The concept of *volume of influence* for electromagnetic induction sensors is introduced and accurately defined. It enables to better understand the response of a magnetic soil to an electromagnetic induction sensor, as well as the effect of soil inhomogeneity on soil compensation. The volume of influence is first defined as the volume producing a fraction α of the total response of a homogeneous half-space. As this basic definition is not appropriate for sensor heads with intrinsic soil compensation, a generalized definition is then proposed. These definitions still do not yield a unique *volume of influence* and a constraint must be introduced to reach uniqueness. Two constraints are investigated: one yielding the *smallest volume of influence* and the other one the *layer of influence*. Those two specific *volumes of influence* have a number of practical applications which are discussed. The *smallest volume of influence* is illustrated for typical head geometries and we prove that, apart from differential heads such as the quad head, the shape of the *smallest volume of influence* is independent of the head geometry and can be computed from the far-field approximation. In addition, quantitative head characteristics are provided and show –among others– that double-D heads allow for a good soil compensation, assuming however approximate homogeneity over a larger volume of soil. The effect of soil inhomogeneity is further discussed and a worst-case *volume of influence* is defined for inhomogeneous soils.**

***Index Terms*—Electromagnetic induction, mine clearance, magnetic susceptibility, head sensitivity, metal detector, volume of influence.**

I. Introduction

ELECTROMAGNETIC Induction (EMI) sensors are widely used in many applications such as mine clearance [1], treasure hunting [2] and archaeological survey [3]. The soil can produce a significant response: either a signal of interest or clutter to be rejected. In all cases, it is essential to assess the Volume of Influence (VoI), i.e. the soil volume producing most of the soil response. If the soil response is the signal of interest, as in archaeological surveys, the knowledge of the VoI allows the assessment of the soil volume that can be investigated by the sensor. In mine action, the soil response is considered as clutter and the VoI can be used to objectively define the soil volume to characterize in order to model the soil response. In the same context, some detector heads are designed to compensate the soil response. This design is usually optimized for a homogeneous and flat soil (also called an homogeneous Half-Space (HS)) and the VoI enables the estimation of the soil volume that should be homogeneous for the compensation to be effective. In the same context again, when test lanes are built to evaluate EMI sensors for specific soils, the VoI allows the objective determination of the depth to which the original soil must be removed and replaced by the specific test soils.

In many applications, such as mine action, EMI detectors are used to detect metallic objects. A critical parameter is then the ratio of the target response to the soil response. Many authors [4]–[10] have proposed methods to evaluate the target response, but the influence of the soil has received less attention until recently. Therefore, in this paper, we concentrate on the soil response. Note however that the VoI should not be confused with the volume in which a metallic target can be detected.

The concept of VoI is not entirely new. It is for example discussed in [11] for capacitive probes. However, to the best knowledge of the authors, it has never been rigorously defined and quantified for EMI detectors. As will be shown, for such detectors, significant simplifications are possible to compute the soil response. As a result, the concept of VoI can be developed much further, for example by taking into account the effect of soil inhomogeneities. At first sight, the concept of VoI looks intuitive. However, to get an accurate and quantitative definition, what is called above 'most of the soil response' must be quantified. It seems natural to consider a homogeneous HS and to define the VoI as the volume producing a fraction of the total HS response. However this basic definition raises several issues. First, some heads are designed to compensate the soil response and the total HS response will then be close to zero. In that case, an arbitrary small volume fulfills the basic definition, which is clearly inappropriate. Indeed, even for such heads, the soil in an extended volume influences the response, which becomes apparent when soil inhomogeneity is present. We will therefore propose a generalized definition for the VoI which can be used even for heads with intrinsic soil compensation. For such heads, the new definition enables the determination of the soil volume that should be homogeneous for the compensation to be optimum. A second issue with the basic and intuitive definition of the VoI is that the corresponding volume is not unique. It is for example possible to find a soil layer (see Fig. 1 (b)) that fulfills the definition by increasing the depth of that layer until the required ratio of the HS response is reached. The resulting volume will be called the *Layer of Influence*. It is also possible to find a volume that fulfills the basic definition

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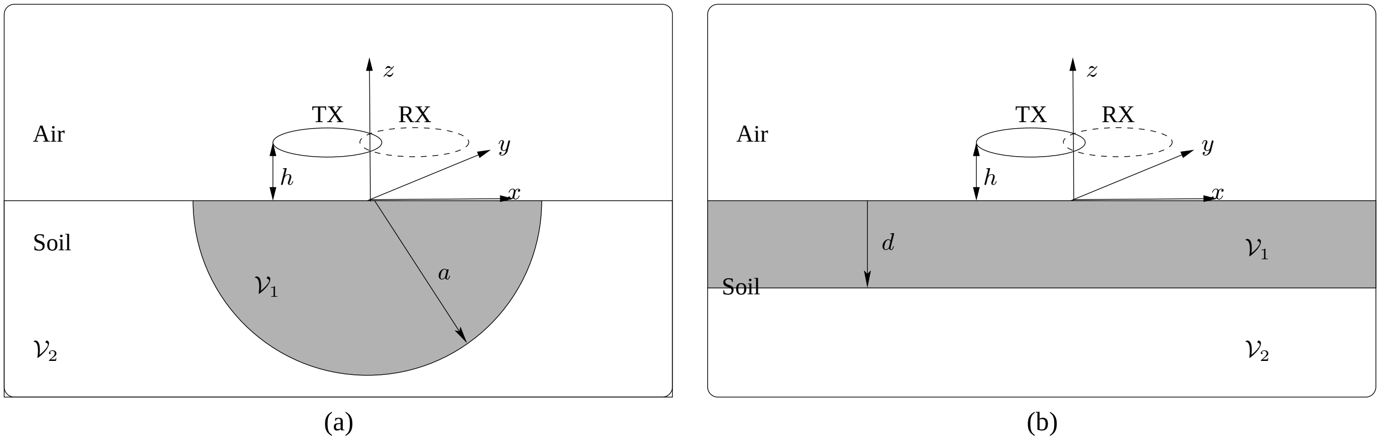


Fig. 1. Two shapes that may be used to define VoI (a) sphere of radius a and (b) layer of depth d leading respectively to the half-sphere of influence and to the *layer of influence*. TX (—) and RX (– –) coils are shown. h is the head height above the soil.

of the VoI by increasing the radius of a half-sphere (see Fig. 1 (a)) or by expanding any other shape of interest. Hence the shape of the VoI can be chosen arbitrarily and the size that yields a response equal to the chosen ratio of the total response can then be univocally computed. Imposing the shape of the VoI is a constraint that can be used to render the VoI unique. Other constraints can also be used. For example, imposing that the VoI be the *smallest possible*. This leads to the *smallest VoI*. As they are well suited for a number of practical applications, we will study in more detail two specific VoIs: the *smallest VoI* and the *Layer of Influence*.

The choice of the response ratio α is guided by the accuracy required for a given application. However, considering only homogeneous soils is a severe limitation for many applications. We will therefore consider a class of soil inhomogeneities and show that a worst-case VoI can be obtained by increasing the ratio α according to the expected soil inhomogeneity.

As explained above, to determine the VoI, the response of a soil volume with arbitrary shape needs to be computed, taking into account the geometry of the head (shape, size and relative position of the Transmit (TX) and Receive (RX) coils) . Analytic solutions are limited to elementary configurations such as a circular concentric head above a HS [12]–[16] and cannot be used to compute the VoI. Using generic numerical software would be computationally intensive, estimating the VoI requires computing the response of many volumes. Fortunately, the magnetic susceptibility dominates the response of most soils and their electric conductivity may be neglected[1] [13], [15]. Furthermore, the soil magnetic susceptibility is often quite small [17]–[23]. We will therefore restrict ourselves to those soils for which the solution presented in [24] can be used and provides an efficient means to compute the response of an arbitrary soil volume, with arbitrary inhomogeneities, for an EMI sensor with arbitrary head geometry. This solution expresses the soil response as an integral on the soil volume of the product of head sensitivity and magnetic susceptibility. We will show that with this solution, the shape and extent of the VoI can be efficiently computed for the head geometries used in practice. This will be illustrated for the commonly used head geometries [25] presented in Fig. 2.

The remainder of this paper is organized as follows. Section II presents the notations and conventions and Section III defines the VoI: first the basic definition in Section III-A then the generalized definition in Section III-B. Section III-C discusses the constraint that must be introduced in order to make the VoI unique and presents two examples: one yielding the *smallest VoI* and the other one the *layer of influence*. Section III-D describes how a worst-case VoI may be defined by taking the expected soil inhomogeneity into account and Section III-E discussed a number of practical applications for the *smallest VoI* and the *layer of influence*. Section IV discusses the shape of the *smallest VoI* and Section V presents numerical results. Finally, Section VI concludes the paper.

II. Conventions and notation

Vectors and scalars are denoted $\boldsymbol{x}$ and x respectively. $|x|$ is the absolute value of x. When presenting results, dimensionless quantities are used. The dimensionless quantity corresponding to the scalar x is denoted x. Volumes and surfaces are denoted $\mathcal{V}_\mathrm{x}$ and $\mathcal{S}_\mathrm{x}$ respectively. The union, intersection and subtraction of volumes $\mathcal{V}_1$ and $\mathcal{V}_2$ are denoted by $\mathcal{V}_1 \cup \mathcal{V}_2$, $\mathcal{V}_1 \cap \mathcal{V}_2$ and $\mathcal{V}_1 \setminus \mathcal{V}_2$ respectively. Magnetic fields are denoted $\boldsymbol{H}_\mathrm{s}^{(\mathrm{env})}$ where ‘s’ denotes the source and ‘env’ the environment in which the source is located.

When the same subscript ’x’ is used for a volume $\mathcal{V}_\mathrm{x}$ and a surface $\mathcal{S}_\mathrm{x}$, the surface is defined as the boundary of the volume. The response of a volume of soil $\mathcal{V}_\mathrm{x}$ is denoted by V_x.

Harmonic sources and fields are considered; their time variation $e^{j\omega t}$ is omitted for the sake of clarity.

III. Definitions of the VoI

We first simply and intuitively define the VoI for a uniform HS. The definition is then extended and the effect of inhomogeneities is discussed. In addition, to obtain a unique VoI, a constraint must be introduced. Two constraints are discussed,

[1]Conductivity may have a significant effect only for very conductive soils, such as beaches saturated with sea water, or for large coils, when the coil size is of the order of the skin depth.

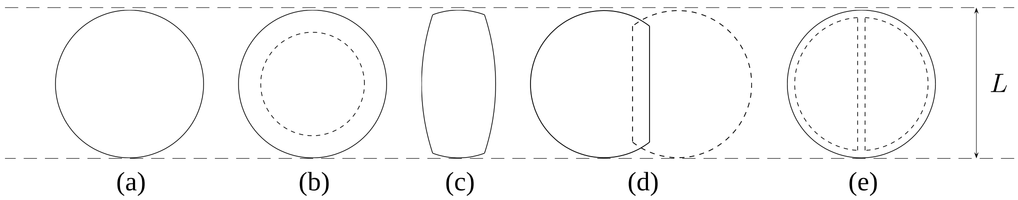


Fig. 2. Head configurations considered. TX — and RX – – coils for (a) circular, , (b) concentric, (c) elliptic, (d) double-D and (e) quad heads. For circular and elliptic heads, a single coil is used for RX and TX. L is the characteristic head length.

yielding two specific volumes of influence: the *smallest VoI* and the *layer of influence*.

A. Basic definition

At a given position of the sensor head, the VoI ($\mathcal{V}_{\text{inf}}^{\alpha}$) is defined as the volume producing a fraction α of the total soil response V_{HS} for an uniform HS. The fraction α is chosen according to the application and the expected inhomogeneities, as will be explained in Section III-D.

For the soils with negligible conductivity and low magnetic susceptibility which are commonly encountered and considered in this paper, the response of an arbitrary region $\mathcal{V}_{\text{reg}}$ of an homogeneous HS can be expressed as [24, Equ. 10]:

$$V_{\text{reg}} = j\omega\mu_0 I_{\text{TX}}\chi \int_{\mathcal{V}_{\text{reg}}} S \, \mathrm{d}V \tag{1}$$

where ω is the angular frequency, χ the soil susceptibility and S the head sensitivity which can be expressed as:

$$S = \frac{\boldsymbol{H}_{\text{TX}}^{(\text{fs})} \cdot \boldsymbol{H}_{\text{RX}}^{(\text{fs})}}{I_{\text{TX}} I_{\text{RX}}} \tag{2}$$

where $\boldsymbol{H}_{\text{TX}}^{(\text{fs})}$ and $\boldsymbol{H}_{\text{RX}}^{(\text{fs})}$ are the magnetic fields that would be produced by the TX and RX coils in free space, if they were carrying currents I_{TX} and I_{RX} respectively. The sensitivity maps of the heads presented in Fig. 2 can be found in [24].

The volume $\mathcal{V}_{\text{inf}}^{\alpha}$ can the be found by searching for a volume $\mathcal{V}_{\text{reg}}$ for which the response is:

$$V_{\text{reg}} = \alpha V_{\text{HS}} \tag{3}$$

where V_{HS}, the HS response, could be computed using (1), which unfortunately requires integration over a large volume. Alternatively, for a HS, it is more efficient to use the following expression [24, Equ. 16]:

$$V_{\text{HS}} = j\omega \frac{\chi}{2} I_{\text{TX}} M_{\widehat{\text{TX}},\text{RX}} \tag{4}$$

where χ is the soil susceptibility, I_{TX} the current in the transmit coil and $M_{\widehat{\text{TX}},\text{RX}}$ the mutual induction coefficient between the RX coil and the mirrored TX coil ($\widehat{\text{TX}}$).

According to (1) and (4), the fraction $V_{\text{reg}}/V_{\text{HS}}$ is independent[2] of ω, χ and I_{TX}. Therefore, the VoI is only function of the detector head geometry, position and orientation above the soil. In most cases, the head is kept horizontally above the ground during the scanning. Restricting ourselves to such a configuration, for a given head shape, the VoI is only function of the head characteristic length L (see Fig. 2) and of the head height above the soil h (see Fig. 1). One easily checks that two configurations are in electromagnetic similitude if the dimensionless height $\mathsf{h} = h/L$ is kept constant. Therefore, the shape of the VoI and its dimensionless size (size normalized with characteristic length L) do not depend on the head dimension, they only depend on the shape and the dimensionless height h of the head above the ground.

[2] At frequencies used by EMI sensors and in soils with negligible conductivity, the Magneto-Quasi-Static (MQS) approximation holds and the magnetic field appearing in (2) is independent of the frequency [24, Equ. 12].

B. Generalized definition

For single-coil heads, according to (2), the sensitivity is always positive and the basic definition of the VoI is appropriate. However, for two-coils heads, the sensitivity sign may change. This sign reversal leads to total or partial soil compensation inherent to some head geometries. For a differential head, such as the quad head presented in Fig. 2 (e), the soil compensation is perfect for a homogeneous HS and the corresponding soil response is zero. Therefore, an arbitrary small volume obeys the basic definition of the VoI, which is obviously not acceptable.

To avoid this pitfall, we note that the total soil response is

$$V = |V_+| - |V_-| \tag{5}$$

where V_+ (V_-) is the contribution of the soil volume $\mathcal{V}_+$ ($\mathcal{V}_-$) for which the sensitivity (2) is positive (negative). If V_- is not negligible compared to V_+, both soil volumes $\mathcal{V}_-$ and $\mathcal{V}_+$ significantly influence the total response. To isolate the two contributions, we consider a Half-Space with Holes (HSH) which is a specific kind of inhomogeneity. A HSH is defined as a soil of which the susceptibility only takes one of two values at each location: an arbitrary but constant value χ or zero. If the HSH is such that the susceptibility is χ inside $\mathcal{V}_+$ and zero outside, the total response is $V = V_+$. Similarly, the total response will be V_- for an HSH defined on $\mathcal{V}_-$. Applying the basic definitions on those two HSHs, one gets the positive and negative volumes of influence. The positive VoI ($V_{\text{inf},+}^{\alpha}$) is defined as the part of $\mathcal{V}_+$ for which:

$$V_{\text{inf},+}^{\alpha} = \alpha V_+ \tag{6}$$

and the negative VoI is similarly defined by replacing '+' by '-' in (6). As already mentioned, both $\mathcal{V}_+$ and $\mathcal{V}_-$ may significantly contribute to the total response. We therefore define the VoI as the union of the positive and negative volumes of influence:

$$\mathcal{V}_{\text{inf}}^{\alpha} = \mathcal{V}_{\text{inf},+}^{\alpha} \cup \mathcal{V}_{\text{inf},-}^{\alpha} \tag{7}$$

For heads with intrinsic soil compensation, this generalized definition is also appropriate to assess the volume of soil that should be homogeneous for an efficient compensation. This is discussed in more details in Section III-D. In addition, for single-coil heads, $\mathcal{V}_-$ is empty and the generalized definition is identical to the basic definition presented in Section III-A.

As apparent from Section IV (see [24] for more details), $\mathcal{V}_-$ is usually a small volume close to the head while $\mathcal{V}_+$ is a large volume extending towards infinity. Therefore, it is advantageous to calculate V_+ according to

$$V_+ = V_{\mathrm{HS}} - V_- \tag{8}$$

where V_{HS} and V_- are computed according to (4) and (1) respectively.

In the above discussion, we assumed that the sensitivity is positive far away from the head. This is always possible for the co-planar coils considered in this paper if the two coils have a non-vanishing dipolar moment. Indeed for such coils, the dipolar moments are parallel and (A.1) can then be applied to compute the far field sensitivity. From this expression, it is apparent that the sensitivity does not change sign in the far field. It can therefore be made positive by an appropriate choice of the positive coil currents I_{TX} and I_{RX}, which is a question of convention. Indeed changing the positive direction for one of the coil currents changes the sign of the sensitivity everywhere. Apart from the RX coil of the quad head, all coils presented in Fig. 2 have non-vanishing dipolar moments and the positive coil current was chosen to ensure that the sensitivity is positive far away from the head. For the quad-head, the positive direction of the coil currents is not important as the head symmetry imposes positive and negative volumes with identical shape and size.

C. Introduction of constraints

There exists an infinity of volumes which satisfy the definition of the VoI. It is for example possible to generate a soil layer (see Fig. 1 (b)) that fulfills the basic definition by increasing the depth of that layer until the required ratio of the HS response is reached. It is also possible to generate a volume that fulfills the basic definition by increasing the radius of a half-sphere (see Fig. 1 (a)) or by expanding any other shape of interest. As will be shown, for the general definition, a similar but slightly more complex procedure can be adopted to find the extent of the VoI for an arbitrary generating shape. Hence, to get a unique VoI, a constraint must be introduced. Imposing the shape of the VoI is a possible solution; imposing that the VoI should be the smallest possible is another one.

1) Shape defined VoIs: We now show how an arbitrary generating shape can be used to define a specific VoI satisfying the generalized definition given in Section III-B. To indicate that the VoI considered is shape defined, it will be denoted $\mathcal{V}^{\alpha}_{\mathrm{inf\_shape}}$. According to the definition, the VoI is the union of a positive and a negative VoI. The positive VoI must only include soil regions where the sensitivity is positive. Such a 'positive' volume can easily be obtained, keeping the idea of expending a generating shape, by taking the intersection of the resulting volume with the soil volume in which the sensitivity is positive.

Formally speaking, starting from a shape-defined volume $\mathcal{V}^{\beta}_{\mathrm{shape}}$, where the subscript 'shape' is a placeholder for the chosen shape and the superscript β defines the size of the volume, the above mentioned 'positive' volume is defined by:

$$\mathcal{V}^{\beta}_{\mathrm{shape},+} = \mathcal{V}^{\beta}_{\mathrm{shape}} \cap \mathcal{V}_+ \tag{9}$$

where $\mathcal{V}_+$ is the part of the lower HS in which the sensitivity is positive. The corresponding positive VoI ($\mathcal{V}^{\alpha}_{\mathrm{inf\_shape},+}$) can then be computed for a given response ratio α, according to the general definition (6), by computing the response $V^{\beta}_{\mathrm{shape},+}$ of $\mathcal{V}^{\beta}_{\mathrm{shape},+}$ as a function of the size β and finding the value of β for which $V^{\beta}_{\mathrm{shape},+} = \alpha V_+$.

Note that for a given shape and response ratio α, β always exists and is unique. Indeed, by construction, the volume $\mathcal{V}^{\beta}_{\mathrm{shape},+}$ grows from zero to $\mathcal{V}_+$ and the sensitivity is positive everywhere inside $\mathcal{V}^{\beta}_{\mathrm{shape},+}$. Therefore, the response $V^{\beta}_{\mathrm{shape},+}$ is a monotonic function increasing from zero to V_+ as β varies from 0 to ∞, and the value of the parameter β for which $V^{\beta}_{\mathrm{shape},+} = \alpha V_+$ always exists and is unique.

The negative VoI ($\mathcal{V}^{\alpha}_{\mathrm{inf\_shape},-}$) can be found similarly and is also unique. Therefore, a unique shape defined VoI ($\mathcal{V}^{\alpha}_{\mathrm{inf\_shape}}$) is obtained as the union of the positive and the negative VoIs: $\mathcal{V}^{\alpha}_{\mathrm{inf\_shape}} = \mathcal{V}^{\beta}_{\mathrm{shape},+} \cup \mathcal{V}^{\beta}_{\mathrm{shape},-}$.

An example of shape-defined VoI is the *layer of influence* which is obtained by using as generating volume ($\mathcal{V}^{\beta}_{\mathrm{shape}}$) a layer of soil extending from the surface to a given depth d. This is illustrated in Fig. 3 for a double-D head and for a response ration $\alpha = 0.95$. The parameter β is then the depth d and the corresponding values for the positive and negative *layers of influence* are called the positive and negative *depth of influence* and they are denoted d_+ and d_-. Note that the *layer of influence*, which is the union of the positive and the negative layers of influence is not a full layer; some holes are apparent (in white on the figure). To understand the origin of those holes, note that the sensitivity is positive in the dark gray region and negative elsewhere in the lower HS. Hence, the sensitivity is negative in the region appearing in white which is therefore not part of the positive layer of influence. This white area is further below d_- and is therefore also not part of the negative *layer of influence*; it forms a hole in the layer of influence. This hole is however small (and will become smaller for larger values of α) and considering the hole as part of the VoI does not significantly enlarge the VoI. This is acceptable for most applications and the *layer of influence* can then efficiently be visualized as a full layer extending from the surface to a depth d_+. This parameter d_+ can then be used to fully characterize the VoI and will be called the *depth of influence*.

We have shown that holes may appear in a shape shape-defined VoI and that therefore, rigorously speaking, its shape differs from that of the generating volume $\mathcal{V}^{\beta}_{\mathrm{shape}}$. However, we saw that for the previous example, those holes may be ignored for most applications. This is expected to remain that case for most heads and for most VoIs. Indeed, $\mathcal{V}_-$ (the volume where the sensitivity is negative) is in general a small volume located close to the head whereas $\mathcal{V}_+$ extends towards infinity. It is therefore $\mathcal{V}_+$ which contributes the most to the VoI,

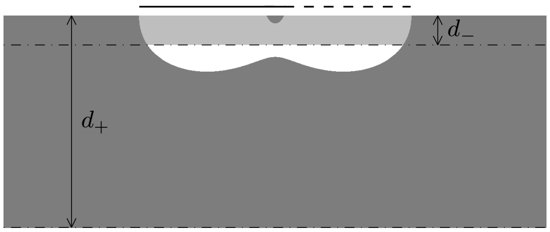


Fig. 3. *Layer of influence* for the double-D head and for a response ration $\alpha = 0.95$. Vertical cut trough the center of the head is illustrated and shows the projection of TX (—) and RX (– –) coils, the border of generating volume (– · –), the positive (dark gray) and negative (light gray) layer of influence with corresponding positive and negative *depths of influence* d_+ and d_-. The holes in the *layer of influence* appear in white.

especially for a large response ratio α, and using $\mathcal{V}_-$ instead of the negative VoI only yields a small enlargement the VoI. When this is done, the holes disappear, the shape of the of VoI becomes identical to that of the generating volume and the VoI can efficiently be characterized by a single parameter β_+ obtained by searching for the positive VoI.

2) Smallest VoIs: As already mentioned, another constraint that can be used to get a unique VoI is to impose that the VoI be the smallest possible. This constraint can be translated into a constraint similar to that of the shape-defined VoIs discussed above because the *smallest VoI* can be shown to be bound by an equi-sensitivity surface. It can therefore be computed as described above using the volume bound by an equi-sensitivity surface $\mathcal{V}^{\beta}_{\text{equi}}$ (with β the corresponding sensitivity) for $\mathcal{V}^{\beta}_{\text{shape}}$. The only difference is that by changing the parameter β, it is not only the size, but also the shape of the volume, that changes.

To prove that the *smallest VoI* is bound by an an equi-sensitivity surface, we now show that any other volume of the same size has a smaller response. This other volume can be constructed from $\mathcal{V}^{\beta}_{\text{equi}}$ by removing a part of it and adding a volume (of the same size as the removed part) outside $\mathcal{V}^{\beta}_{\text{equi}}$. By construction, the sensitivity is everywhere smaller in the added volume than in the removed part and the resulting response is smaller, which concludes the proof.

D. Effect of soil inhomogeneity

1) Effect on the VoI: Once a shape is chosen, the procedure described in Section III-C can still be used to compute the VoI in presence of any soil inhomogeneity. The only difference with the case of a homogeneous soil is that the space-dependent susceptibility must be used in the integral (1) to compute the response of the parametric volume. We have shown in Section III-C that for a homogeneous soil, the *smallest VoI* is bound by an equi-sensitivity. Following the same reasoning it appears that for an inhomogeneous soil, the *smallest VoI* is bound by a surface for which the product of sensitivity with magnetic susceptibility ($S\chi$) is constant. This being said, except for very specific applications, the precise soil inhomogeneity is unknown and defining a representative inhomogeneity may be difficult. Using a worst-case approach is then a possible alternative.

For this, let us consider an inhomogeneous soil for which the susceptibility ranges, at each point, from $\chi_{\min}$ to $\chi_{\max} = \rho\chi_{\min}$ where ρ is defined as the inhomogeneity ratio and let us analyze the effect of this inhomogeneity on the positive VoI (the results are identical for the negative VoI). To find the positive VoI, we need to find the value of the parameter β for which $V^{\beta,\text{i}}_{\text{shape},+} = \alpha^{\text{i}} V^{\text{i}}_+$ where the superscript 'i' denotes the worst inhomogeneity, i.e. yielding the largest VoI or, in other words, the smallest α^{i} for a given β. For a given β, the fraction α^{i} will be the smallest if the soil susceptibilities are $\chi_{\min}$ and $\chi_{\max}$, respectively inside and outside the volume $\mathcal{V}^{\beta}_{\text{shape},+}$. The corresponding fraction α^{i} is given by

$$\alpha^{\text{i}} = \frac{V^{\beta,\text{i}}_{\text{shape},+}}{V^{\text{i}}_+} = \frac{V^{\beta}_{\text{shape},+}}{V^{\beta}_{\text{shape},+} + \rho V^{\beta}_{\text{out},+}} \tag{10}$$

where $V^{\beta}_{\text{shape},+}$ and $V^{\beta}_{\text{out},+}$ are the responses respectively of a soil volume $\mathcal{V}^{\beta}_{\text{shape},+}$ and of a soil volume $\mathcal{V}^{\beta}_{\text{out},+} = \mathcal{V}_+ \setminus \mathcal{V}^{\beta}_{\text{shape},+}$ with a uniform susceptibility $\chi_{\min}$.

Equation (10) could be used directly to determine the (positive) VoI by computing α^{i} as a function of β in order to determine the value of β corresponding to the chosen response ratio α^{i}. A more convenient approach is to first cast the chosen response ratio α^{i} into a response ratio α that would be obtained with the searched for VoI in presence of an homogeneous HS. The routine developed for the homogeneous case can then be used to determine the VoI in presence of inhomogeneities but using the computed α. To find the relation between α and α^{i}, we note that for a volume $\mathcal{V}^{\beta}_{\text{shape},+}$, the fraction α that would be obtained for a homogeneous HS can be computed by using[3] $\rho = 1$ in (10). It is then apparent that the searched for relation is:

$$\alpha = \frac{\alpha^{\text{i}}\rho}{1 - \alpha^{\text{i}} + \alpha^{\text{i}}\rho} \tag{11}$$

The latter expression shows that, for a given inhomogeneity ratio ρ, the VoI can be computed as in the homogeneous case, but α must now be computed from the chosen response ratio α^{i}. For example, if the VoI is defined as the volume producing 99% of the total response ($\alpha^{\text{i}} = 0.99$) and if the soil susceptibility is assumed to be, at most, ten times bigger outside the VoI than inside ($\rho = 10$), then the corrected fraction is $\alpha = 0.999$, yielding as expected a larger VoI in the presence of the worst inhomogeneity.

2) Effect on soil compensation: Soil inhomogeneities also have an effect on the intrinsic head compensation, if any. A motivation for defining the VoI as the union of the positive and negative volumes of influence (7) is that the compensation is nearly as good as for a homogeneous HS if the soil is homogeneous in the VoI. To quantify the effect of inhomogeneities outside the VoI on the compensation, we define the compensation ratio as:

$$\gamma^{\text{i}} = \frac{|V^{\text{i}}_-|}{V^{\text{i}}_+} \tag{12}$$

[3]the susceptibility of the homogeneous HS needs not to be specified because it factors out from the expression of α.

where again the superscript 'i' denotes the worst inhomogeneity, i.e. in this case, yielding the worst compensation ($\gamma^{\rm i}$ minimum). As we have assumed[4] that $V_+^{\rm i} \geqslant |V_-^{\rm i}|$, the compensation ratio ranges from zero (no compensation) to 100% (perfect compensation). Let us start from a VoI defined for a homogenous HS and yielding a response ratio α. We then consider inhomogeneities outside the VoI, keeping the soil susceptibility constant ($\chi = \chi_{\rm in}$) inside the VoI, and we assume that the susceptibility outside the VoI varies between $\chi_{\rm min} = \rho_{\rm min}\chi_{\rm in}$ and $\chi_{\rm max} = \rho_{\rm max}\chi_{\rm in}$. The worst inhomogeneity occurs then when $\chi = \chi_{\rm min}$ in $\mathcal{V}_- \setminus \mathcal{V}^{\alpha}_{{\rm inf},-}$ and $\chi = \chi_{\rm max}$ in $\mathcal{V}_+ \setminus \mathcal{V}^{\alpha}_{{\rm inf},+}$. Indeed, in this case $V_-^{\rm i}$ is minimum and $V_+^{\rm i}$ is maximum, yielding the minimum compensation ratio. The responses $V_-^{\rm i}$ and $V_+^{\rm i}$ occurring under the worst inhomogeneity are then related to their homogenous counterparts by:

$$V_+^{\rm i} = \alpha V_+ + \rho_{\rm max}(1-\alpha)V_+ \quad (13a)$$
$$V_-^{\rm i} = \alpha V_- + \rho_{\rm min}(1-\alpha)V_- \quad (13b)$$

with V_+ and V_- the response of $\mathcal{V}_+$ and $\mathcal{V}_-$ for a homogeneous HS with susceptibility $\chi_{\rm in}$. The compensation ratio can then be expressed as:

$$\gamma^{\rm i} = \frac{(\alpha + \rho_{\rm min}(1-\alpha))}{(\alpha + \rho_{\rm max}(1-\alpha))}\frac{|V_-|}{V_+} \quad (14)$$

Defining the corresponding compensation ratio for a homogeneous soil as $\gamma = |V_-|/V_+$, the degradation of the compensation due to the worst-case inhomogeneity can be quantified by the degradation factor $\delta = \gamma^{\rm i}/\gamma$ which according to (14) can be expressed as:

$$\delta = \frac{\alpha + (1-\alpha)\rho_{\rm min}}{\alpha + (1-\alpha)\rho_{\rm max}} \quad (15)$$

Inverting this expression yields

$$\alpha = \frac{\rho_{\rm min} - \delta\rho_{\rm max}}{\rho_{\rm min} - 1 + \delta - \delta\rho_{\rm max}} \quad (16)$$

which can be used to translate a requirement on the efficiency of the soil compensation in presence of soil inhomogeneities (δ) into a response ratio α from which the VoI can be determined. For example, if $\rho_{\rm min} = 0.1$, $\rho_{\rm max} = 10$ and if we require that $\delta = 0.99$, then one must use the VoI corresponding to $\alpha = 0.999$. In other words, if the soil is homogeneous in the VoI defined by $\alpha = 0.999$ and if outside that volume, the susceptibility ranges from one tenth to ten times the susceptibility inside the VoI, the compensation will be degraded by one percent in the worst case.

E. Smallest VoI and layer of influence

The *smallest VoI* and the *layer of influence* are two specific VoIs that were defined in Section III-C. They are quite useful in a number of practical applications:

- the *smallest VoI* is useful to define the soil volume to be characterized in order to model the soil response *at a given location*. Indeed, by definition, measuring the magnetic susceptibility distribution inside any VoI allows the calculation of the response with an accuracy that depends on the response fraction α. This fraction may further be modified to take into account the expected soil inhomogeneity according to (11). By choosing the *smallest VoI*, this calculation requires the minimum number of soil samples. Sampling the *layer of influence* also allows the calculation of the soil response with the same accuracy but requires more soil sample measurements because the *layer of influence* is a much larger volume.
  The *smallest VoI* is also useful to visualize the volume that should be homogeneous to achieve a good compensation. The response ratio α should then be computed according to (16). Requiring that the *layer of influence* be homogeneous would yield the same result but this is a much more stringent requirement, since the *layer of influence* is a much larger volume than the *smallest VoI*.
- the *layer of influence* is useful to define the soil volume to be characterized in order to model the soil response *for all points of the scanning plane* with an accuracy that depends on the fraction α and on the expected soil inhomogeneity. Indeed, following the same reasoning than above for the *smallest VoI*, the required accuracy is reached for the head location used to calculate the *layer of influence*. In addition, by symmetry, the result is valid for all points of the scanning plane[5]. Note that it is also appropriate to sample the soil in the volume defined as the union of the smallest VoIs (or the union of any other VoIs), one for each point of the scanning plane, but the resulting volume is larger. As will be confirmed in Figs. 7 and 8, the depth of the *smallest VoI* ($L_{\rm z}$, see Fig. 4) is indeed larger than the *depth of influence*.
  The *layer of influence* is also useful to build a test lane. It can then be used to objectively define the *depth of influence* to which the native soil should be replaced by a soil of interest in order for the underlying native soil to have an influence negligible up to a given tolerance. The expected native and test soil susceptibilities should then be used in (11) to compute the *depth of influence*. Again, considering the union other VoIs would also be appropriate but would require to replace the native soil with the soil of interest to a depth $L_{\rm z}$ which is larger than the *depth of influence*, hence demanding more work than necessary.

IV. Shape of smallest the volumes of influence

Analyzing the shape of the *smallest VoI* may enable a better understanding of the head behavior. Therefore we present in this section the shape of representative volumes of influence for various heads and fractions α. It is interesting to note that for large values of α, the shape of most volumes of influence becomes similar. This is discussed in Section IV-B in the light of the far-field approximation .

[4]The results can be transposed to the case $V_+^{\rm i} \leqslant |V_-^{\rm i}|$ by interchanging $V_+^{\rm i}$ and $|V_-^{\rm i}|$, with similar conclusions.

[5]For a general inhomogeneity, the *layer of influence* may depend on the location of the head. However, with the worst-case approach considered in Section III-D, the *layer of influence* remains indeed the same for all point of the scanning plane.

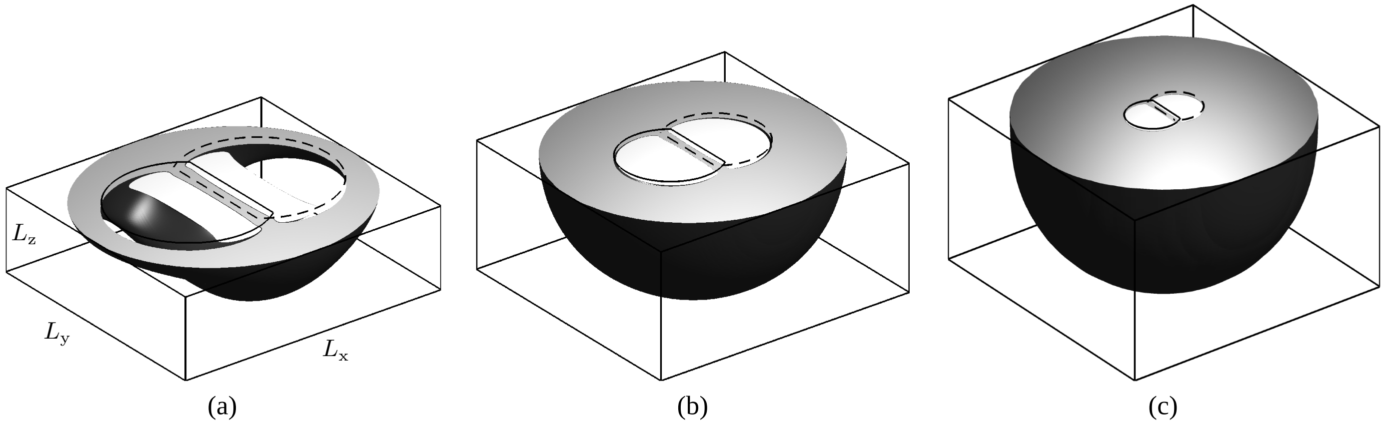


Fig. 4. Smallest VoI and corresponding bounding box for the double-D head with dimensionless height $\mathsf{h} = 0.05$. (a) $\alpha = 0.5$, (b) $\alpha = 0.90$ and (c) $\alpha = 0.99$. L_x, L_y and L_z are the dimensions of the bounding box along the corresponding axes.

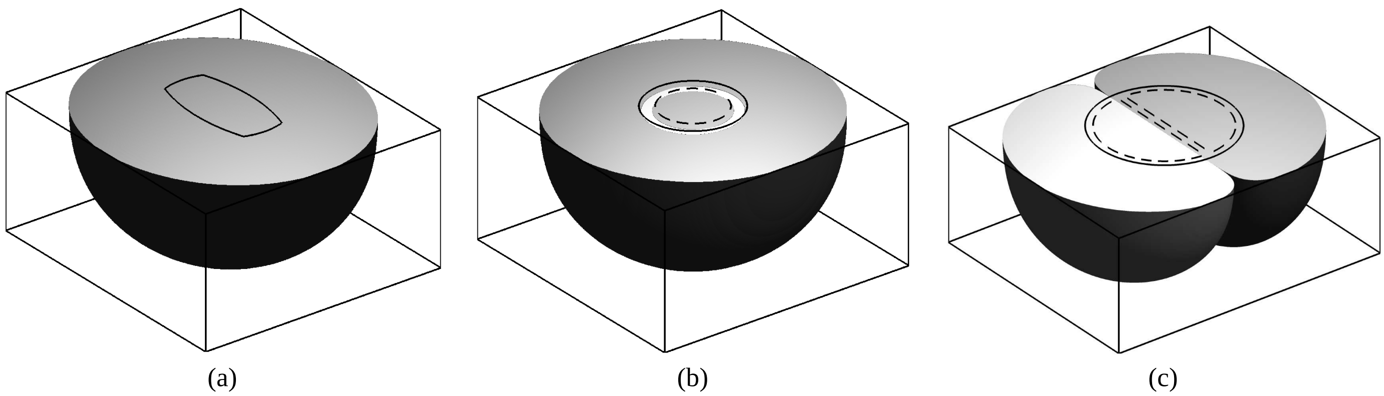


Fig. 5. Smallest VoI and corresponding bounding box for the elliptic (a), concentric (b) and quad (c) heads with dimensionless height $\mathsf{h} = 0.05$ and $\alpha = 0.99$.

A. Exact shape

Fig. 4 presents the *smallest VoI* for a double-D head and a fraction α of 0.5, 0.90 and 0.99 together with the *smallest VoI* bounding box. The dimensions of the bounding box along the axes are denoted by L_x, L_y and L_z. In all cases, the dimensionless head height above the soil h amounts to 0.05 and the soil is assumed to be a homogeneous HS. The positive VoI is plotted in gray and the negative one in white. The shape for $\alpha = 0.5$ is the most complex one and may appear difficult to visualize in 3D. The positive volume looks like a twin washbowl with a hole in the bottom and the negative volume looks like two thin plates partly covering the washbowl. Note that the negative VoI is close to the head and small compared to the positive VoI. For large values of α, the *smallest VoI* has a similar shape for all heads considered in this paper, except for the quad head. This is illustrated[6] in Fig. 5 and a theoretical justification is presented in Section IV-B. Note that for large values of α and except for some small holes which are irrelevant in practice, the *smallest VoI* is bounded by the air-soil interface and by an approximate ellipsoid denoted $\mathcal{S}^{\alpha}_{\mathrm{inf,out}}$.

[6]The VoI of a circular detector is not shown to save space but it is similar to the concentric case, except for the negative volume which does not exist.

B. Approximate shape

The surface $\mathcal{S}^{\alpha}_{\mathrm{inf,out}}$ is far away from the detector. Therefore, on that surface, the field can be approximated by the first term of a multipole expansion. We call this the far-field approximation, although it must not be confused with the far-field approximation commonly used at higher frequencies and for which the field decreases as $1/R$ and can be characterized by a radiation pattern. For horizontal coils with a dipolar term, Appendix A develops the expression of the far-field sensitivity. According to (A.1), a far-field equi-sensitivity surface is given by:

$$R = K(1 + 3\cos^2\theta)^{\frac{1}{6}} \tag{17}$$

where θ and R are the coordinates of a point on the equi-sensitivity surface in a spherical coordinate system with z oriented vertically and the origin taken in the middle of the head (see Fig. 6). K is a scaling factor related to the chosen sensitivity value. From (17), it appears that the equi-sensitivity surface exhibits a symmetry of revolution around the z axis and that its shape is invariant; only its size changes with K. This invariant shape is illustrated in Fig. 6 together with a half-circle for comparison purpose; it is valid for most heads and for large response ratio α. A noticeable exception is the quad-head for which the dipolar moment of the RX coil is zero. The quadrupole term must then be taken into account, which yields a different shape (see Fig. 5 (c)).

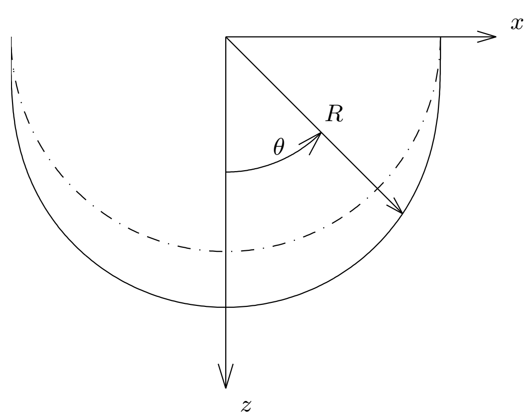


Fig. 6. Shape of the far-field equi-sensitivity (—), and half-circle (– · –) for comparison.

	V_+/χ	V_-/χ	γ	$\mathsf{V}_{\mathrm{HS}}/\chi$	v_-
Circular	0.4285	0	na	0.4285	0
Concentric	0.2124	-0.0011	0.0052	0.2113	0.0035
Elliptic	0.2716	0	na	0.2716	0
Double-D	0.0381	-0.0389	0.9807	-0.0007	0.2615
Quad	0.1486	-0.1486	1	0	∞

TABLE I
DIMENSIONLESS POSITIVE (V_+), NEGATIVE (V_-) AND HALF-SPACE (V_{HS}) RESPONSE (NORMALIZED BY χ) AS WELL AS DIMENSIONLESS NEGATIVE VOLUME (v_-) AND COMPENSATION RATIO (γ) FOR VARIOUS HEADS AND DIMENSIONLESS HEAD HEIGHT $\mathsf{h} = 0.05$. WITH THE CHOSEN NORMALIZATION, RESULTS ARE INDEPENDENT OF χ AND OF THE FREQUENCY.

To assess the accuracy of the approximated shape[7] of the *smallest VoI* obtained with the far-field approximation, we note that according to (17), the dimensions of the *smallest VoI* bounding box obey[8] $L_{\mathrm{x}} = L_{\mathrm{y}}$ and $2L_{\mathrm{z}} = 4^{\frac{1}{6}}L_{\mathrm{x}}$. Therefore, the xy ($\epsilon_{\mathrm{xy}} = \frac{L_{\mathrm{x}}^{\inf}}{L_{\mathrm{y}}^{\inf}} - 1$) and z ($\epsilon_{\mathrm{z}} = \frac{L_{\mathrm{z}}}{4^{\frac{1}{6}}(L_{\mathrm{x}}+L_{\mathrm{y}})} - 1$) errors will be small when the far-field shape approximation is accurate. This will be illustrated in Section V.

V. NUMERICAL RESULTS

In this section, numerical characteristics for the heads of Fig. 2 are provided. To avoid any dependence on the frequency, on the TX current or on the head dimension L, dimensionless quantities are used: the dimensionless response, $\mathsf{V} = V/(j\omega\mu_0 L I_{\mathrm{TX}})$, dimensionless lengths such as the dimensionless bounding box along x ($\mathsf{L}_{\mathrm{x}} = L_{\mathrm{x}}/L$) or the dimensionless head height ($\mathsf{h} = h/L$) and the dimensionless volumes $\mathsf{v} = v/L^3$. To assess the importance of the negative volume, Table I presents the response of the positive and negative volumes, V_+ and V_-, the corresponding compensation ratio $\gamma = |V_-|/V_+$ and the HS response $\mathsf{V}_{\mathrm{HS}} = \mathsf{V}_+ + \mathsf{V}_-$. Note that the negative volume is significant for the double-D and quad heads and that it is the existence of that volume that allows for a significant compensation of the soil response. The dimensionless volume v_- of the negative volume is also indicated. Note that the latter is very small for the concentric head, relatively small for the double-D head and infinite for the quad head. The double-D head is quite specific in that respect, as the magnitude of the sensitivity is high in the small-negative volume and compensates the response of the infinite positive volume where the sensitivity is smaller on the average.

Fig. 7 presents the size of the VoI as a function of α for the heads considered in this paper. For the *smallest VoI*, the size of the dimensionless bounding box L_{x}, L_{y} and L_{z} are shown. For the *layer of influence*, the dimensionless *depth of influence* $\mathsf{d} = d/L$ is shown. In all cases, the dimensionless head height is $\mathsf{h} = 0.05$. In addition Table II presents the same parameters together with the far-field approximation errors (ϵ_{xy} and ϵ_{z}) for α equal to 50, 90 and 99%. Note that the VoI is significantly bigger for the double-D head than for other heads. This indicates that for a double-D head, the soil compensation will be optimum only if the soil is homogeneous in a large volume as indicated by the size of the VoI. Note also that the *depth of influence* is always smaller than the depth of the VoI (L_{z}). This was already mentioned in Section III-C and confirms that choosing the right constraint to obtain the best VoI for the application may result in a significant gain. For example, if the objective is to build a test lane by replacing the native soil by a soil of interest, then the native soil should be replaced down to the *depth of influence* computed for a response ratio α chosen according to the needed test accuracy.

To assess the effect of the head height on the VoI, Fig. 8 presents the dimensionless size of the VoI as a function of the dimensionless head height h for $\alpha = 0.99$. Note that the size of the negative volume becomes zero for a sufficient height as the volume in which the sensitivity is negative, is small and close to the head, thus moving completely outside the soil volume. Note also that the positive volume becomes larger with increasing height. This may sound counter-intuitive but it can be understood by analyzing the spatial distribution of the head sensitivity. When lifting the head, the total response decreases but the sensitivity spatial distribution is such that a given volume close to the soil surface contributes less, proportionally to the total response, than when the head is closer to the soil.

VI. CONCLUSIONS

Basically, the Volume of Influence (VoI) is the volume that produce most of the total soil response. It should not be confused with the volume in which a metallic target can be detected. The concept of VoI is not entirely new but, to the best knowledge of the authors, it has never been rigorously defined and quantified for Electromagnetic Induction (EMI) detectors. For such detectors, significant simplifications are possible to compute the response of most soils (those for which the response is dominated by the magnetic susceptibility, which

[7]only the shape of the *smallest VoI* can be estimated using the far-field approximation, not its size. Indeed, the size of the VoI depends on of the sensitivity everywhere in the soil, hence also close to the head where the far-field approximation is very inaccurate. To compute the size of the VoI, the exact shape of the coils must still be used.

[8]L_{x} and L_{y} are obtained by computing R for $\theta = \pi/2$. L_{z} is obtained by computing R for $\theta = 0$. In addition, the bounding box along x and y is twice the radius while along z it is only once the radius.

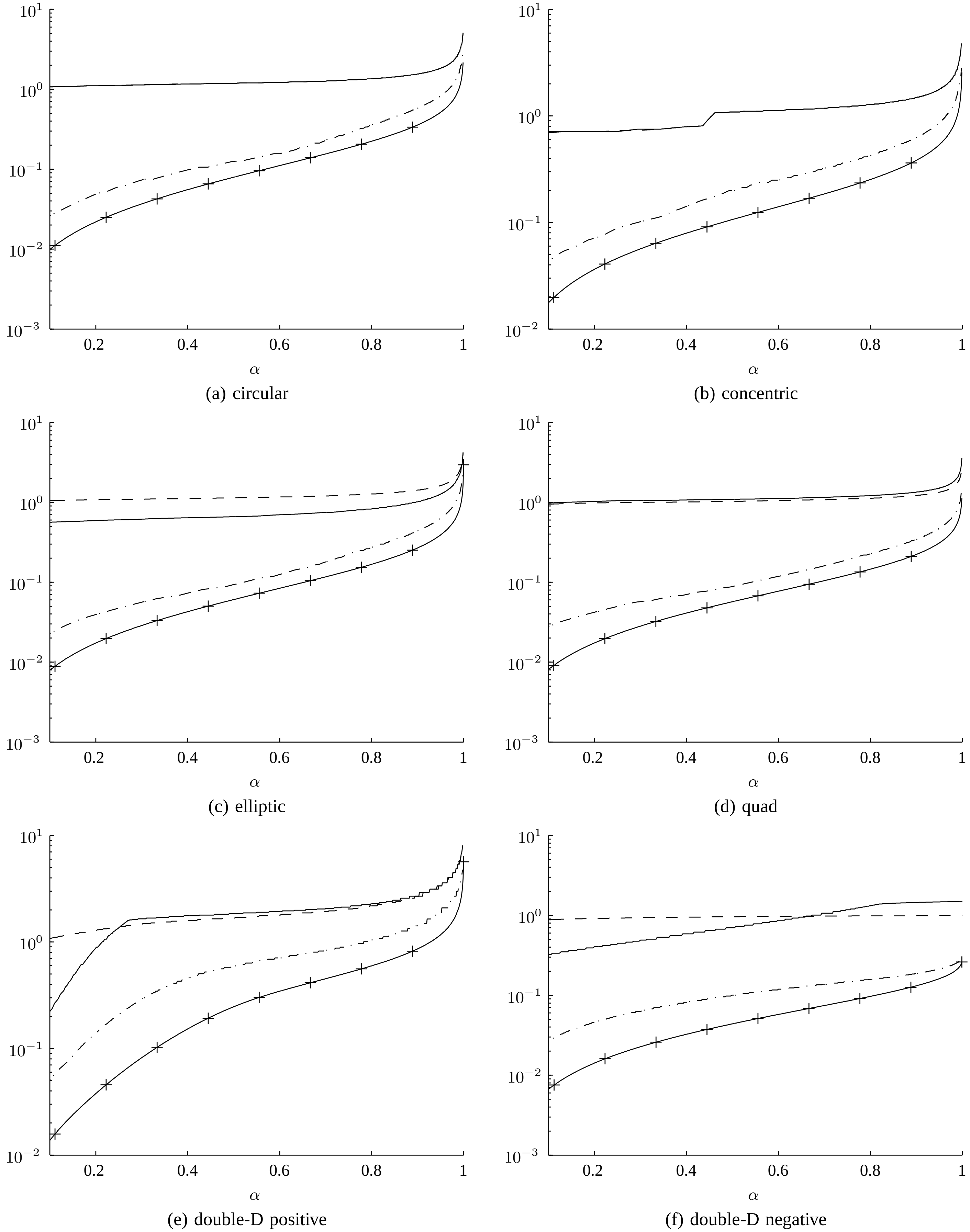
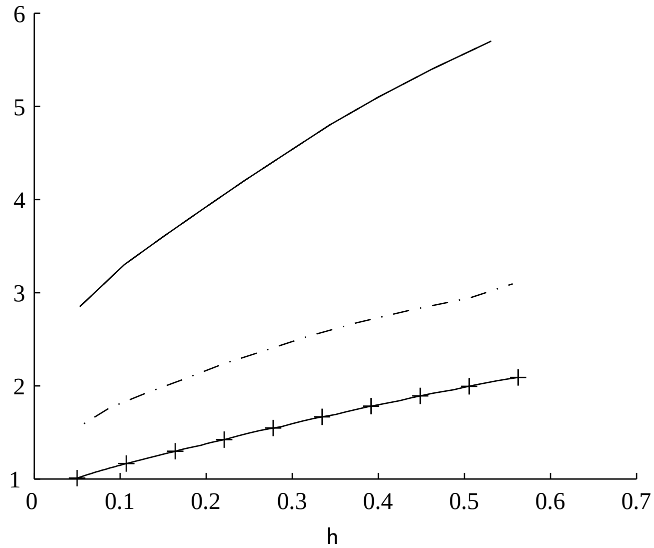
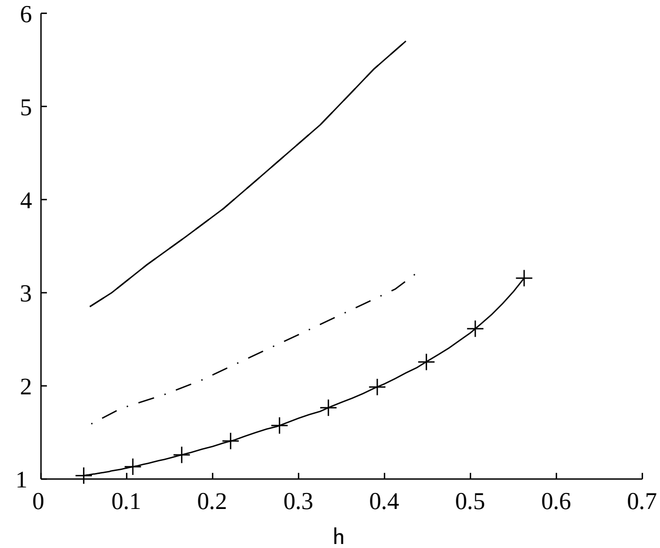


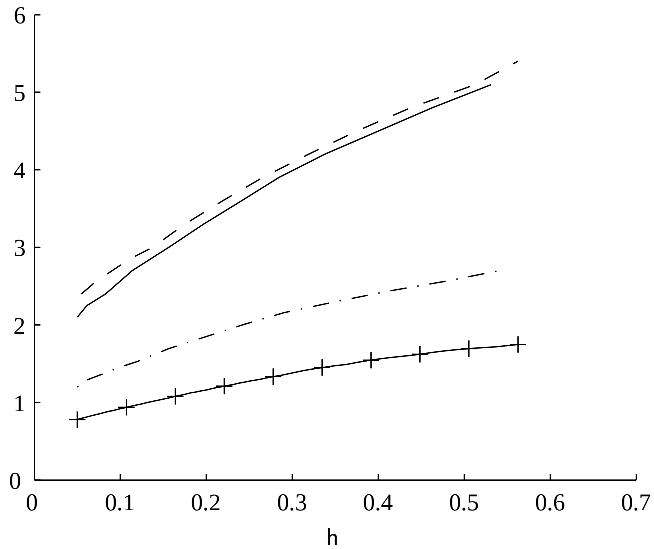
Fig. 7. Size of positive (a-e) and negative (f) VoIs versus fraction α for circular (a), concentric (b), elliptic (c), quad (d) and double-D (e,f) heads. To characterize the *smallest VoI*, the size of the dimensionless bounding box L_x (—), L_y (– –) and L_z (– · –) are shown. To characterize *the layer of influence*, the dimensionless *depth of influence* d (+) is shown. In all cases, the dimensionless head height is $h = 0.05$.



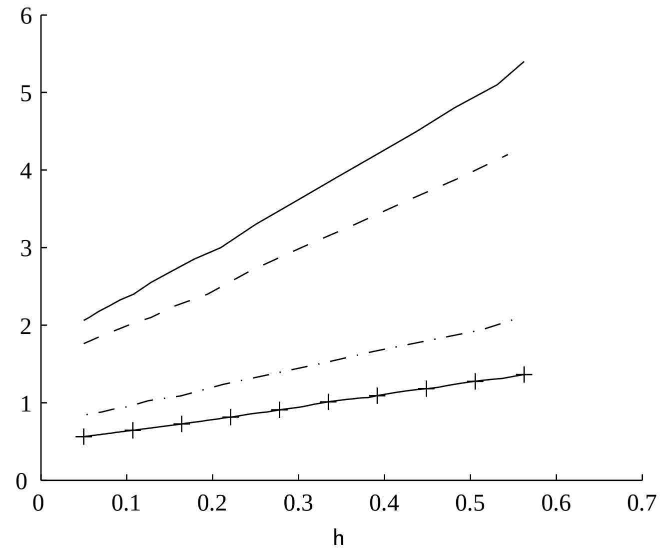
(a) circular



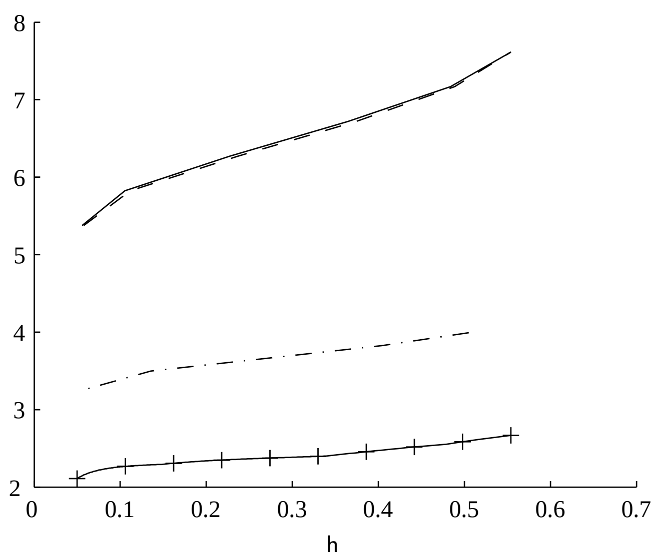
(b) concentric



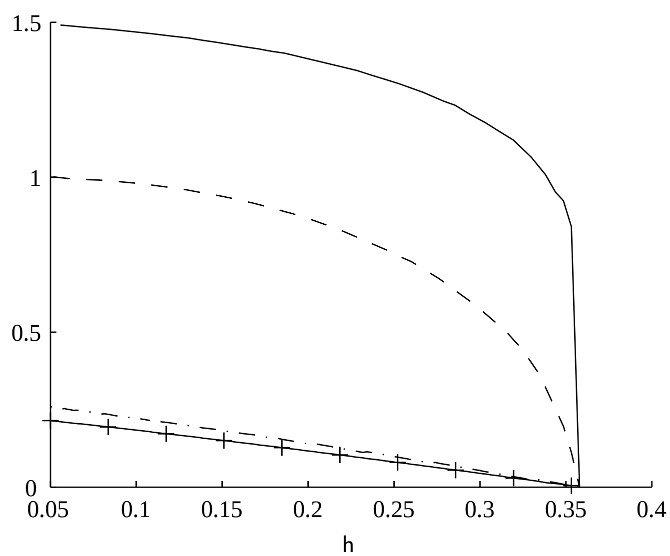
(c) elliptic



(d) quad



(e) double-D positive



(f) double-D negative

Fig. 8. Size of positive (a–e) and negative (f) VoIs versus dimensionless height h for circular (a), concentric (b), elliptic (c), quad (d) and double-D (e,f) heads. To characterize the *smallest VoI*, the size of the dimensionless bounding box L_x (—), L_y (– –) and L_z (– · –) are shown. To characterize the *layer of influence*, the *depth of influence* (+) is shown. In all cases, $\alpha = 0.99$.

	α	$\mathsf{v}^{\alpha}_{\mathrm{inf},+}$	$\mathsf{v}^{\alpha}_{\mathrm{inf},-}$	L_x	L_y	L_z	d_+	d_-	ϵ_{xy}	ϵ_z
Circular	50	0.0976	na	1.191	1.191	0.1232	0.0794	na	0	-0.8357
	90	0.7338	na	1.554	1.554	0.5814	0.3572	na	0	-0.4059
	99	7.105	na	2.846	2.846	1.584	1.009	na	0	-0.1164
Concentric	50	0.0922	0.0009	1.089	1.089	0.201	0.1067	0.0042	0	-0.7069
	90	0.7068	0.0023	1.485	1.485	0.6271	0.383	0.0108	0	-0.3297
	99	7.286	0.0031	2.841	2.841	1.627	1.036	0.0155	0	-0.0907
Elliptic	50	0.044	na	0.6623	1.137	0.0932	0.0607	na	-0.4176	-0.8355
	90	0.3409	na	1.018	1.419	0.4387	0.2687	na	-0.2826	-0.4283
	99	3.456	na	2.146	2.378	1.237	0.7798	na	-0.0976	-0.1317
Double-D	50	0.4054	0.0297	1.844	1.691	0.5952	0.2468	0.0439	0.0904	-0.4653
	90	5.257	0.1253	2.767	2.678	1.416	0.8617	0.1314	0.0333	-0.1744
	99	53.49	0.2076	5.427	5.386	3.249	2.111	0.2143	0.0076	-0.046
Quad	50	0.027	0.027	1.093	1.033	0.0893	0.0571	0.0571	na	na
	90	0.1519	0.1519	1.343	1.218	0.345	0.2228	0.2228	na	na
	99	0.8926	0.8926	2.079	1.768	0.8385	0.5628	0.5628	na	na

TABLE II
DIMENSIONLESS VOLUME OF POSITIVE ($\mathsf{v}^{\alpha}_{\mathrm{inf},+}$) AND NEGATIVE ($\mathsf{v}^{\alpha}_{\mathrm{inf},-}$) VOI, DIMENSIONLESS BOUNDING BOX OF *smallest VoI* (L_x, L_y AND L_z), DIMENSIONLESS POSITIVE (d_+) AND NEGATIVE (d_-) *depth of influence* AND FAR-FIELD APPROXIMATION ERRORS (ϵ_{xy} AND ϵ_z) FOR VARIOUS HEADS AND DIMENSIONLESS HEAD HEIGHT $\mathsf{h} = 0.05$

in addition is small) and this allows the concept of VoI to be developed much further.

After showing that the basic and intuitive definition of the VoI has some limitations for heads with intrinsic soil compensation, we proposed a generalized definition in which the VoI is the union of a positive and a negative VoI, each being defined as in the basic case, but accounting only for the areas in which the sensitivity is positive or negative. We showed that this generalized definition can be used for all heads and that for heads with intrinsic soil compensation, it allows one to estimate the volume that should be homogeneous for a given efficiency of compensation.

Next, we showed that to yield a unique VoI, a constraint must be introduced. We have proposed two constraints, one yielding the *smallest VoI* and the other one yielding the *layer of influence*. We showed that those two specific VoI are very useful for a number of applications. For example, in the scope of mine action, if test lanes are built to evaluate EMI sensors for specific soils, the *layer of influence* allows the objective definition of the depth to which the actual soils must be removed and replaced by those specific test soils. Also, the *smallest VoI* is the volume of soil that should be characterized if the aim is to predict the response of a detector at a given location.

Then, the effect of soil inhomogeneity has been investigated. More precisely, we have considered a soil for which the magnetic susceptibility is in a range $[\chi_{\min}, \chi_{\max}]$ at each location and we have shown how a worst-case VoI (i.e. valid for the worst-case soils under consideration) can be obtained by increasing the fraction α taking into account the expected range of magnetic susceptibilities. Similarly, we have quantified the effect of soil inhomogeneity on the efficiency of the head soil compensation. This allowed us to translate a requirement on quality of the compensation in a worst-case situation into a response fraction α and, hence, to compute the corresponding VoI.

Further, visualizing the *smallest VoI* may help to understand the detector behavior. For example, in archaeological survey, it defines the soil region that can be investigated by the detector. Similarly, in the scope of mine action, it defines the soil region that may be at the origin of a false alarm. In the same context, it defines the volume of soil that should be homogeneous for a given efficiency of soil compensation. Therefore, the shape of the *smallest VoI* has been illustrated for a number of typical head geometries. It was noted that for small values of the response fraction α (below 0.9), the shape can be arbitrary. However for most applications, large values of the fraction α need to be considered. For such values, the shape of the *smallest VoI* becomes identical for most heads, only its size changes. A noticeable exception is the quad head, for which the shape of the *smallest VoI* is significantly different compared to the other heads considered in this paper. This shape similarity has been explained by resorting to the far-field approximation of the magnetic field, for which only the dipolar moment of the coils is considered. The specific case of the quad head is then explained by the fact that one needs to account for the quadrupole moment, since the receiving coil has no dipolar moment.

Finally, the size of the VoI has been computed and compared for various heads and this yielded practical conclusions. For example, the *smallest VoI* is much larger for the double-D head than for the other heads considered and therefore, the compensation of the double-D head will be optimal only if the soil is homogenous in a large region below the coil. We also noted that the size of the VoI strongly depends on the head height. This size becomes larger when the head is lifted above the ground. In addition to the size of the VoI, the study of some other head characteristics allows for a better understanding of the head behavior. For example, the compensation ratio (γ) is much larger for the double-D head than for the concentric head, and this explains why the former nearly perfectly compensates the response of a homogeneous soil while the latter only partly compensates the soil response.

APPENDIX A
FAR FIELD APPROXIMATION OF SENSITIVITY

We consider a head composed of two horizontal coils which have non-vanishing magnetic moments m_{TX} and m_{RX} both along the vertical direction. Far away from the head, the distance between the coils can be neglected and the fields may be approximated by that of two collocated magnetic dipoles. Introducing the magnetic field of a dipole [26, Equ. 11.7] in (2) yields:

$$S^{\mathrm{FF}}(\boldsymbol{R}) = \frac{1}{16\pi^2}\frac{m_{\mathrm{TX}}m_{\mathrm{RX}}}{I_{\mathrm{TX}}I_{\mathrm{RX}}}\frac{1+3\cos^2\theta}{R^6} \qquad \text{(A.1)}$$

where $\boldsymbol{R}$ is the vector from the common location of the TX and RX dipoles to the field point, R is its norm and θ the angle between that vector and the common dipoles direction.

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