Battery-Aware Power Allocation for Lifetime Maximization of Wireless Sensor Networks

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***Abstract*— We consider a wireless sensor network deployed to observe a physical phenomenon. The sensors amplify and forward their observations to a remote fusion center via single hop. The objective is to maximize the operational lifetime of the network such that the estimate of the underlying source at FC satisfies a certain fidelity criterion given by the maximum tolerable estimation distortion. Each sensor is powered by a battery which limits its lifetime and consequently of the network also. Recent studies show that the battery discharge behavior is dependent on the load current: higher current leads to higher losses inside the battery due to the non-linearities of the electrochemical reaction in the battery. This work presents a power allocation design where goal is to maximize the network lifetime incorporating the non-linear discharging behavior of the batteries. The design is based on the knowledge of the instantaneous channel gains as well as when we only know the channel statistics. The numerical examples illustrate that the impact of battery-aware power scheduling on the network life is substantial.**

I. Introduction

A wireless sensor network (WSN) consists of spatially distributed sensors that cooperatively monitor physical or environmental conditions - temperature, vibration, pressure, motion or pollutants. Here we assume that the sensors transmit their noisy observations via single-hop to a remote fusion center (FC) which forms the global estimate of the underlying source. Each sensor node is powered by a battery with limited energy which is assumed to be non-rechargeable and irreplaceable because the nodes may be deployed in a hostile or inaccessible terrain. The objective is to keep the network operational to the maximum possible time such that the estimation distortion is less than a maximum tolerable threshold. As each sensor is powered by a non-rechargeable battery, the objective of network lifetime maximization can be achieved by minimizing the consumed energy in each observation instance. To this end, minimal-energy estimation techniques have been considered in [1]- [3]. There are other schemes which deal with the minimal-energy data collection at a central access point from the sensors [4]. Very recently Li et al. in [5] presented a joint source coding and routing design for lifetime maximization in a multihop network. All these schemes, assume ideal battery with fixed usable capacity. However, recent studies show that the discharge behavior of the battery is not ideal. Due to non-linearities, in addition to the energy delivered to the load, some energy is locked inside the battery which is unavailable [6], [7]. Consequently the total apparent consumed energy is always greater than the actual energy delivered to the load. Moreover, some of the locked energy is recovered by introducing rest periods. The battery is dead when the apparent consumed energy reaches a certain threshold value.

The authors would like to thank the Walloon region ministry DGTRE framework program COSMOS/TSARINE and EU project FP7 NEWCOM++ for the financial support and the scientific inspiration.

In this work, we first develop a theoretical model for the energy consumption of the battery which takes into account the nonlinear discharge behavior and the recovery effect. Subsequently from this model, we introduce a cost function which takes into consideration the load history for a particular sensor. Then based on the cost function we propose a battery-aware power allocation for the network-lifetime maximization in a single-hop network such that the estimation distortion does not exceed a maximum tolerable limit. The proposed scheme assumes perfect knowledge of the fading channel gains from the sensors to the FC in each observation instance. However, in practical systems due to resource constraints we may only know the statistics of the channels. To this end, we develop an expression for the average estimation distortion assuming the channel gains are independently Rayleigh distributed and subsequently we propose the power-allocation scheme.

II. Preliminaries

A. System Model

We consider the sensor network comprising N sensors deployed to observe a source s. Each sensor node amplifies and forwards the noisy observation to the fusion center (FC) via some orthogonal multiple-access scheme, e.g. FDMA. The received signal is $z_i(n) = h_i(n)\alpha_i(n)(s(n) + n_i(n)) + w_i(n)$, $\forall i$, where $n_i(n)$ is the observation noise, $w_i(n)$ is the receiver noise, the $|h_i(n)|$ is the channel gain and $\alpha_i(n)$ is the amplifying factor for sensor i at discrete time n. We assume that the observation and the receiver noises are respectively zero-mean and independently distributed across sensors and time with variances $\sigma^2_{n_i}$ and $\sigma^2_{w_i}$, $\forall i$. The channel gains from the sensors to the FC are independently flat fading. The task of the FC is to estimate the source $s(n)$ based on the received observations from the sensors. We assume that the parameter $s(n)$ is deterministic and we have the second order statistics of the observation and the communication noises. The estimate $\hat{s}(n)$ is based on BLUE estimation rule [8]:

$$\hat{s}(n) = \left(\sum_{k=1}^{N} \frac{|h_k(n)|^2 \alpha_k^2(n)}{|h_k(n)|^2 \alpha_k^2(n) \sigma^2_{n_k} + \sigma^2_{w_k}} \right)^{-1}$$

$$\sum_{k=1}^{N} \frac{|h_k(n)|\alpha_k(n) z_k(n)}{|h_k|^2 \alpha_k^2(n) \sigma_{n_k}^2 + \sigma_{w_k}^2} \quad (1)$$

with the associated variance of the estimation error given by

$$D = \left(\sum_{k=1}^{N} \frac{|h_k(n)|^2 \alpha_k^2(n)}{|h_k(n)|^2 \alpha_k^2(n) \sigma_{n_k}^2 + \sigma_{w_k}^2} \right)^{-1}. \quad (2)$$

Let $P_k(n) = \alpha_k^2(n)$ denotes the power drawn from the battery and $\zeta_k(n) := \frac{|h_k(n)|^2}{\sigma_{w_k}^2}$ defines the channel SNR of sensor k. Now the distortion D can be written as

$$D = \left(\sum_{k=1}^{N} \frac{P_k(n)}{P_k(n)\sigma_{n_k}^2 + \frac{1}{\zeta_k(n)}} \right)^{-1}. \quad (3)$$

Note that for $\zeta_k(n) \rightarrow \infty \ \forall k \wedge n$ (i.e. ideal communication channels), the estimation distortion becomes $D_0 = \left(\sum_{k=1}^{N} \frac{1}{\sigma_{n_k}^2} \right)^{-1}$. The estimator which achieves this distortion is called clairvoyant estimator and is used as a performance benchmark.

B. Battery Aware Cost Function

As pointed out in [9] and the references therein, the battery tends to provide more usable capacity at a low discharge current. Generally, the deliverable capacity of a fully charged battery decreases/degrades from the normalized usable capacity as the discharge current is increased. This motivates us to operate the sensors at the low discharge current to increase the lifetime of the individual sensors and consequently of the entire network. The battery model in [6] , [7] shows that when a load is applied to a real battery, the apparent charge lost from the battery comprises two components: (*i*) the actual charge delivered to the load and (*ii*) a charge which is locked inside the battery called unavailable charge. The unavailable charge is dependent on the battery parameters as well as on the load current. The higher the load current, the higher is the unavailable charge. The unavailable charge also increases with the load duration.

The battery model in the sequel is based on the work in [6] and is obtained by solving the partial differential equations of the electrochemical process for a simple 1-dimensional cell model based on the Fick's laws and Faraday's law with associated initial and boundary conditions.

Assume that all sensors start sampling the observations at the same time instance and with equal sampling frequency. Let T_s be the sampling period and the corresponding sampling instances are

$$t = 0,\ T_s,\ 2T_s,\ 3T_s,\ \ldots,\ nT_s,\ (n+1)T_s,\ \ldots. \quad (4)$$

We assume a step-wise constant power drawn from the battery of sensor k corresponding to the sampling periods, i.e.

$$P_k(1),\ P_k(2),\ P_k(3),\ \ldots,\ P_k(n),\ P_k(n+1),\ \ldots. \quad (5)$$

Let the total initial energy available from the battery of sensor k be E_k^*. Following theorem describes relationship between the available energy and consumed energy.

Theorem 1: At sensor k at the end of n^{th} observation cycle, the remaining available energy is

$$E_{r_k}(n) = E_k^* - E_{c_k}(n),$$

where

$$E_{c_k}(n) = \underbrace{2 \sum_{j=1}^{n} \frac{P_k(j)}{\phi} \sum_{m=1}^{\infty} \frac{1 - e^{-\beta^2 m^2 T_s}}{\beta^2 m^2} e^{-\beta^2 m^2 (n-j) T_s}}_{:= E_{c_k}^{(u)}(n)} + \underbrace{\sum_{j=1}^{n} \frac{P_k(j)}{\phi} T_s}_{:= E_{c_k}^{(l)}(n)} = E_{c_k}^{(u)}(n) + E_{c_k}^{(l)}(n) \quad (6)$$

denotes the total apparent consumed energy at sensor k up to and including the n^{th} observation sample. In (6), ϕ denotes the conversion efficiency of a DC-DC converter and β is a parameter characterizing the nonlinear characteristics of the battery. The higher the value of β, the smaller is the unavailable energy which means that the battery discharge behavior is more close to an ideal battery. Note that we have assumed that the circuit energy consumption and the energy consumed in the sensing process is much less than the transmission energy and hence negligible. Nevertheless, these energies can be easily included in the battery model.

Proof: See appendix I for proof of Theorem 1.

The apparent consumed energy $E_{c_k}(n)$ comprises two components: $E_{c_k}^{(l)}(n)$ is the actual energy delivered to the load and $E_{c_k}^{(u)}(n)$ is the energy which is locked inside the battery and is called unavailable energy. However, introducing rest period of duration ΔT_s, part of $E_{c_k}^{(u)}(n)$ is recovered and available energy increases $E_{r_k}(n+\Delta) = E_{r_k}(n) + \delta E_{c_k}^{(u)}(n)$ with $\delta \in [0 \ 1]$ and $\Delta \in \mathcal{Z}^+$, where $\mathcal{Z}^+$ is a set of positive integers. Note the analogy to the energy consumed in a resister and the energy stored in a capacitor. Fig. 1 plots the consumed and remaining energy of a battery with $E^* = 10^3 \, J$, $T_s = 1\, sec$, $\beta = 0.06\,(sec)^{-0.5}$ and load profile $P = [24.77\, dBm,\ n = 1, \ldots, 800]$, $P = [0,\ n = 801, \ldots, 1300]$, $P = [26.99\, dBm,\ n = 1301, \ldots, 1800]$, $P = [0,\ n = 1801, \ldots, 2800]$ and $P = [26.02\, dBm,\ n = 2801, \ldots, 3800]$. The figure shows that the battery survives up to n_e. We can also observe the recovery effect due to the rest periods.

A sensor is considered to be dead if the remaining energy is less that a certain threshold value. Therefore, the life of the sensor node k is defined as

$$\mathcal{SL}_k = \{ n \mid E_{r_k}(n) \geq E_{th} \wedge E_{r_k}(n+1) < E_{th} \},\ \forall k. \quad (7)$$

Based on the definition of lifetime for single sensor, we can define the network lifetime as follows: the network is considered non-functional if any of the sensors is dead [4], i.e. $\mathcal{NL} = \mathit{min}\{\mathcal{SL}_1, \ldots, \mathcal{SL}_N\}$ or the network is dead if a certain percentage of the total sensors is dead. Note that these definitions of the network lifetime are very conservative to the least and utterly inappropriate from the functionality point of view because even if a single sensor or a portion of the sensors is dead, the network comprising the remaining sensors may still be able to perform the sensing with acceptable fidelity.

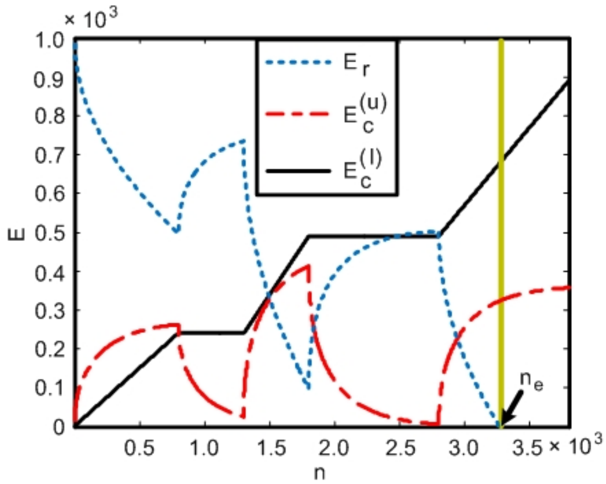

Fig. 1: Energy consumption model.

From the functional point of view the sensor network is dead if it is unable to achieve the target distortion i.e.

$$\mathcal{NL} = \{n \,|\, D(n) \leq D_{max} \wedge D(n+1) > D_{max}\}\,. \quad (8)$$

In our work we adopt the functional definition of the network lifetime according to which the network life is measured in terms of observation samples or observation cycles successfully processed, where success is measured in terms of achieved distortion not exceeding a certain maximum acceptable level D_{max}.

In order to extend the network lifetime, it is equivalent to maximize the remaining energy or minimize the consumed energy of the batteries during each observation cycle while taking into account the load history. For this purpose, we define the following cost function for sensor k:

$$f_k(n) = E_{c_k}(n) + \nu_k(n)P_k(n), \quad (9)$$

where $\nu_k(n)$ denotes the price of using power $P_k(n)$ and is further explained in the next section. The system-wide cost function is defined as follows:

$$f(n) = \sum_{k=1}^{N} f_k(n). \quad (10)$$

III. Power Allocation for Network Lifetime Maximization

A. Power Allocation with Perfect Channel State Information

We base our power allocation design for network lifetime maximization on the following optimization problem:

$$\begin{aligned} \min_{P_k(n),\,\forall k} \quad & f(n) \\ \textit{s.t.} \quad & \sum_{k=1}^{N} \frac{P_k(n)}{P_k(n)\sigma_{n_k}^2 + \frac{1}{\zeta_k(n)}} \geq \frac{1}{D_{max}}, \\ & E_{c_k}(n) \leq E_k^*,\; P_k(n) \geq 0,\; \forall k \wedge n. \end{aligned} \quad (11)$$

To solve the given problem, we first ignore the constraint $E_{c_k}(n) \leq E_k^*$, $\forall k \wedge n$ which we will later incorporate in the solution and solve the relaxed problem using method of Lagrangian multipliers. The Lagrangian function associated with the problem is

$$L\left(P_k(n), \lambda(n), \mu_k(n)\right) = f(n) - \sum_{k=1}^{N} \mu_k(n)P_k(n) + \lambda(n)\left(\frac{1}{D_{max}} - \sum_{k=1}^{N} \frac{P_k(n)}{P_k(n)\sigma_{n_k}^2 + \frac{1}{\zeta_k(n)}}\right). \quad (12)$$

Since the given problem is convex, the KKT conditions are sufficient for optimality. According to these conditions, $\mu_i(n) > 0$ for $P_i(n) = 0$ and in this case, the sensor is switched-off. For $P_i(n) > 0$, the Lagrangian multiplier $\mu_i(n) = 0$. For this particular case, setting the derivative of (12) w.r.t. $P_i(n)$ to zero and solving for $P_i(n)$ we get

$$P_i(n) = \frac{1}{\sigma_{n_i}^2 \zeta_i(n)} \left(\sqrt{\frac{\lambda(n)\zeta_i(n)}{a_i(n) + \nu_i(n)}} - 1\right)^+, \quad (13)$$

where $\lambda(n)$ is Lagrangian multiplier associated with the distortion constraint, $(x)^+ = \max(x, 0)$ and $a_i(n)$ is

$$a_i(n) = \frac{1}{\phi}T_s + 2\frac{1}{\phi}\sum_{m=1}^{\infty} \frac{1 - e^{-\beta^2 m^2 T_s}}{\beta^2 m^2}. \quad (14)$$

Let $\tilde{E}_{c_i}(n) := E_{c_i}^{(l)}(n-1) + E_{c_i}^{(u)}(n)$ defines the apparent consumed energy of sensor i at the end of n^{th} sample if $P_i(n) = 0$. Note that $\tilde{E}_{c_i}(n) \leq E_{c_i}(n-1)$ due to the recovery effect. We assume that if $E_i^* - \tilde{E}_{c_i}(n) \leq \epsilon$ then $P_i(n) = 0$, where $\epsilon > 0$. Without loss of generality, we assume

$$\tilde{E}_{c_1}(n) \leq \tilde{E}_{c_2}(n) \leq \ldots \leq \tilde{E}_{c_N}(n) \quad (15)$$

and let $\mathcal{B}$ be defined as an index-set of the potential sensors which can participate in the estimation of n^{th} observation sample, i.e. $\mathcal{B} = \{k \mid E_k^* - \tilde{E}_{c_k}(n) > \epsilon\}$. Eq. (13) shows that for any $i \in \mathcal{B}$ we have $P_i(n) = 0$ whenever $\sqrt{\frac{\lambda(n)\zeta_i(n)}{a_i(n)+\nu_i(n)}} \leq 1$. Without loss of any generality, we have

$$\frac{\lambda(n)\zeta_1(n)}{a_1(n) + \nu_1(n)} \geq \frac{\lambda(n)\zeta_2(n)}{a_2(n) + \nu_2(n)} \geq \ldots \geq \frac{\lambda(n)\zeta_K(n)}{a_K(n) + \nu_K(n)}. \quad (16)$$

Let us define the following set

$$\mathcal{A} = \left\{ j \,\middle|\, j \in \mathcal{B} \wedge \sqrt{\frac{\lambda(n)\zeta_j(n)}{a_j(n) + \nu_j(n)}} > 1 \right\}, \quad (17)$$

where $M = \max(\mathcal{A})$ such that $P_\kappa(n) > 0$ holds for $1 \leq \kappa \leq M$.

Since the given optimization problem is convex, the optimum transmit powers occur at the distortion constraint boundary, i.e. the constraint is active. Therefore, the $\lambda(n)$ must satisfy the distortion constraint with equality and is given by

$$\sqrt{\lambda(n)} = \frac{\sum_{j=1}^{M} \frac{1}{\sigma_{n_j}^2}\sqrt{\frac{a_j(n)+\nu_j(n)}{\zeta_j(n)}}}{\sum_{j=1}^{M} \frac{1}{\sigma_{n_j}^2} - \frac{1}{D_{max}}} = \frac{A^{(M)}(n)}{B^{(M)}(n)}. \quad (18)$$

Let us define $g(M) = \sqrt{\frac{\zeta_M(n)}{a_M(n)+\nu_M(n)}} \frac{A^{(M)}(n)}{B^{(M)}(n)} - 1$ for $1 \leq M \leq K$, where $K = \max(\mathcal{B})$. From (17), we can see that $g(M) > 0$ and $g(M+1) \leq 0$. Moreover, if $M = K$ then $g(M+1) = 0$. For $\kappa \in \mathcal{A}$, we have $P_\kappa(n) > 0$. The corresponding total consumed energy $E_{c_\kappa}(n)$ can be calculated from (6) which is greater than $\tilde{E}_{c_\kappa}(n)$. Furthermore, if for $P_\kappa(n) > 0$, the consumed power $E_{c_\kappa}(n)$ is greater than the battery capacity E^*_κ then we clamp the $P_\kappa(n)$ such that $E_{c_\kappa}(n) = E^*$ with the resultant transmit power $P'_\kappa(n)$ which is less than $P_\kappa(n)$. Furthermore, we change the distortion constraint as follows:

$$\sum_{j\in\mathcal{A}\backslash\kappa} \frac{P_j(n)}{P_j(n)\sigma^2_{n_j} + \frac{1}{\zeta_i(n)}} = \frac{1}{D'_{max}} \tag{19}$$

where $\frac{1}{D'_{max}} = \frac{1}{D_{max}} - \frac{P'_\kappa(n)}{P'_\kappa(n)\sigma^2_{n_\kappa} + \frac{1}{\zeta_\kappa(n)}}$ and reallocate power to $\mathcal{A}\backslash\kappa$ sensors likewise until the distortion and the battery capacity constraints are satisfied. Note that $\mathcal{A}\backslash\kappa$ means all elements of set $\mathcal{A}$ except κ.

Eq. (13) shows that the sensors with better observation quality, i.e. low observation noise variance are required to transmit with higher power. Consequently, they will die earlier than the sensors with higher observation noise variances. Note that the observation quality of the remaining sensors may not be good enough to achieve the target distortion. This fact motivates us to keep the good sensors alive by introducing augmented cost function with the power price $\nu_k(n)$ in (9). We propose the power price factor to be

$$\nu_k(n) = \sum_{j=1}^{n-1}\left(\frac{P_k(j)}{\phi}T_s + 2\frac{P_k(j)}{\phi} \sum_{m=1}^{\infty}\frac{1-e^{-\beta^2 m^2 T_s}}{\beta^2 m^2}e^{-\beta^2 m^2 (n-j)T_s}\right) \tag{20}$$

which is the apparent energy consumed due to the powers drawn from the battery up to and including the $(n-1)^{th}$ observation cycle (cf. (6)). In this way the sensor which has already consumed most of its battery capacity will increasingly transmit with less power and hence will remain alive.

B. Power Allocation with Knowledge of Channel Statistics

The network lifetime maximization problem presented in the preceding section requires instantaneous channel state information of the communication channels from the sensors to the FC. However, in practical systems due to constraints, only channel statistics may be available. We now present a scheme for network lifetime maximization based on the knowledge of channels distributions.

Theorem 2: Given that the communication channels from the sensors to the FC are independently Rayleigh distributed with zero mean, the estimation distortion can be lower bounded as follows:

$$D^{LB}_{avg} = \left(\frac{1}{D_0} - \frac{1}{2}\sum_{i=1}^{N}\frac{\log\left(1+2\sigma^2_{n_i}\bar{\zeta}_i P_i\right)}{\sigma^4_{n_i}\bar{\zeta}_i P_i}\right)^{-1}, \tag{21}$$

where $D_0 = \left(\sum_{i=1}^{N}\frac{1}{\sigma^2_{n_i}}\right)^{-1}$ denotes the MSE achieved by the clairvoyant BLUE estimator and $\bar{\zeta}_i = \frac{\sigma^2_{g_i}}{\sigma^2_{w_i}}$ is channel SNR with $\sigma^2_{g_i}$ specifying the variance of the Rayleigh fading channel of sensor i.

Proof: See Appendix II.

Corollary 1: In the low channel SNR regime, the average distortion D^{LB}_{avg} approaches to infinity, i.e. $\lim_{\bar{\zeta}_i\to 0,\,\forall i} D^{LB}_{avg} = \infty$; and in high channel SNR regime, it converges to the MSE of clairvoyant estimator, i.e. $\lim_{\bar{\zeta}_i\to\infty,\,\forall i} D^{LB}_{avg} = D_0$. The proof follows directly by substituting the identities

$$\lim_{\bar{\zeta}_i\to 0,\,\forall i} \frac{1}{2\sigma^2_{n_i}\bar{\zeta}_i P_i}\log\left(1+2\sigma^2_{n_i}\bar{\zeta}_i P_i\right) = 1, \tag{22}$$

$$\lim_{\bar{\zeta}_i\to \infty,\,\forall i} \frac{1}{2\sigma^2_{n_i}\bar{\zeta}_i P_i}\log\left(1+2\sigma^2_{n_i}\bar{\zeta}_i P_i\right) = 0, \tag{23}$$

in (21) respectively .

We now consider the following optimization problem:

$$\begin{aligned} \min_{P_i(n),\,\forall i} \quad & f(n) \\ \textit{s.t.} \quad & \frac{1}{D_0} - \frac{1}{2}\sum_{i=1}^{N}\frac{\log\left(1+2\sigma^2_{n_i}\bar{\zeta}_i P_i(n)\right)}{\sigma^4_{n_i}\bar{\zeta}_i P_i(n)} \geq \frac{1}{D^{avg}_{max}}, \\ & E_{c_i}(n) \leq E^*_i,\ P_i(n) \geq 0,\ \forall i \wedge n, \end{aligned} \tag{24}$$

where the cost function $f(n)$ is defined in (10). The given optimization problem is convex (see Appendix III).

Once again to find the solution we use Lagrangian multipliers method. Moreover, we ignore the constraint $E_{c_i}(n) \leq E^*_i,\ \forall i \wedge n$ which we can later incorporate in the solution. Since the given optimization problem is convex, therefore the KKT conditions are sufficient for optimality of the solution. Taking the derivative of the Lagrangian function and setting it to zero, we get for $P_k(n) > 0$ as follows:

$$\frac{\log\left(1+2\sigma^2_{n_k}\bar{\zeta}_k P_k(n)\right)}{P^2_k(n)} - \frac{2\sigma^2_{n_k}\bar{\zeta}_k}{P_k(n)\left(1+2\sigma^2_{n_k}\bar{\zeta}_k P_k(n)\right)} = \frac{2\sigma^4_{n_k}\bar{\zeta}_k}{\lambda(n)}\left[a_k(n)+v_k(n)\right]. \tag{25}$$

The closed form solution of (25) seems hard to find. Therefore, we resort to numerical methods to compute the power allotted to the sensors $k = 1,\ldots,N$. Note that $\lambda(n)$ is Lagrangian multiplier which should be determined such that to satisfy the distortion constraint with equality.

Homogeneous Sensor Network: For a homogeneous sensor network (i.e. $\sigma^2_{n_i} = \sigma^2_n$ and $\bar{\zeta}_i = \bar{\zeta},\ \forall i$), the solution to the power allocation problem becomes trivial with $P_i = P,\ \forall i$. In this case, the lower bounded average distortion becomes

$$D^{LB}_{avg} = \frac{1}{\frac{N}{\sigma^2_n} - \frac{N^2}{2\sigma^4_n\bar{\zeta}P_{tot}}\log\left(1+\frac{2\sigma^2_n\bar{\zeta}P_{tot}}{N}\right)}, \tag{26}$$

where we have assumed that $P_{tot} = NP$.

Corollary 2: For finite σ^2_n, $\bar{\zeta}$ and P_{tot}, we have $\lim_{N\to\infty} D^{LB}_{avg} = \frac{1}{\bar{\zeta}P_{tot}}$. For proof, note that as $N\to\infty$ then $\log\left(1+\frac{2\sigma^2_n\bar{\zeta}P_{tot}}{N}\right) \approx \frac{2\sigma^2_n\bar{\zeta}P_{tot}}{N} - \frac{2\sigma^4_n P^2_{tot}\bar{\zeta}^2}{N^2}$ [cf. $\log(1+x) \approx x - \frac{x^2}{2}$, $|x| \leq 1$] based on which from (26) we get the desired relation. This shows that for large number of sensors, the average distortion converges to the value $\frac{1}{\bar{\zeta}P_{tot}}$ regardless of the observation noise variances σ^2_n, which means adding more sensors for given P_{tot} does not decrease the average distortion. It has been proved in [2] that in such a case adding more sensors improve the outage probability.

IV. Numerical Examples and Discussion

Consider a wireless sensor network comprising $N = 50$ sensor nodes uniformly distributed in a 500×500 grid with FC at the center. The observation noise variance is $\sigma_{n_i}^2 = 0.01$ and the receiver noise variance is $\sigma_{w_i}^2 = -90\,dBm$, $\forall i$. The channel gain is $|h_i|^2 = G_0 d^{-\alpha} g_i$, $\forall i$, where d_i is distance between sensor i and FC; α denotes the pathloss exponent; G_0 depends on the carrier frequency and gains of the transmit and receive antennas; g_i's are i.i.d. Rayleigh-fading random variables with unit variance. We assume $\alpha = 2$ and $G_0 = -30\,dB$. The initial battery energy is $E_i^* = 10^3\,J$, $\forall i$, $T_s = 1\,sec$ and $\beta = 0.06\,(sec)^{-0.5}$. The network lifetime is averaged over $Tr = 10^3$ independent realizations. The examples in the sequel assume perfect CSI. Due to space constraint, the examples for statistical channels knowledge are not included here.

We compare the average lifetime achieved by our proposed power allocation design with a scheme which does not take into consideration the nonlinear battery discharge effects (i.e. assume $E_{c_i}^{(u)}(n) = 0$ and $\nu_i(n) = 0$, $\forall i \wedge n$ in the power allocation design) when allocating power to the sensors for a given target distortion. Henceforth the two schemes are respectively denoted as battery-aware scheme (BAS) and battery-unaware scheme (BUS). Fig. 2 plots the network lives (in terms of average number of samples $NL_{avg} = \frac{1}{Tr}\sum_{l=1}^{Tr} \mathcal{NL}_l$ for $\mathcal{NL}$ defined in (8)) achieved by the two schemes when the g_i's are i.i.d. Rayleigh fading. Similarly Fig. 3 compares the lives when channels are fixed, i.e. $g_i = 1$, $\forall i$. In both cases, we can see that the percentage increase in the lifetime achieved by BAS compared to BUS is substantial. These examples, illustrate the importance of considering the battery discharge behavior in the power allocation for network lifetime maximization.

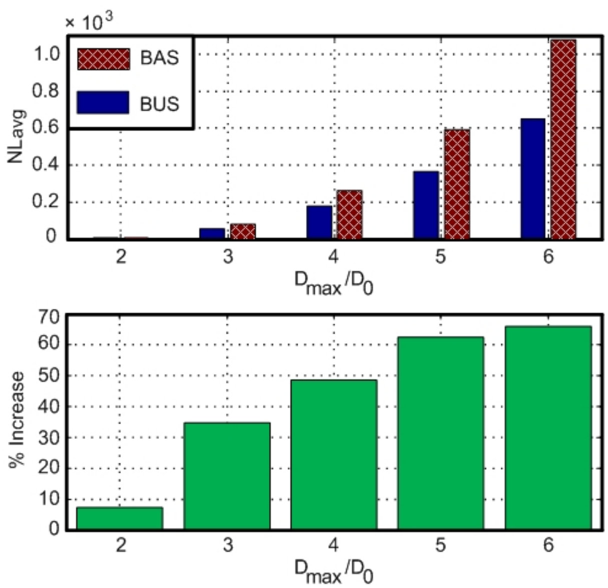

Fig. 2: Comparison of network lives for fading channels.

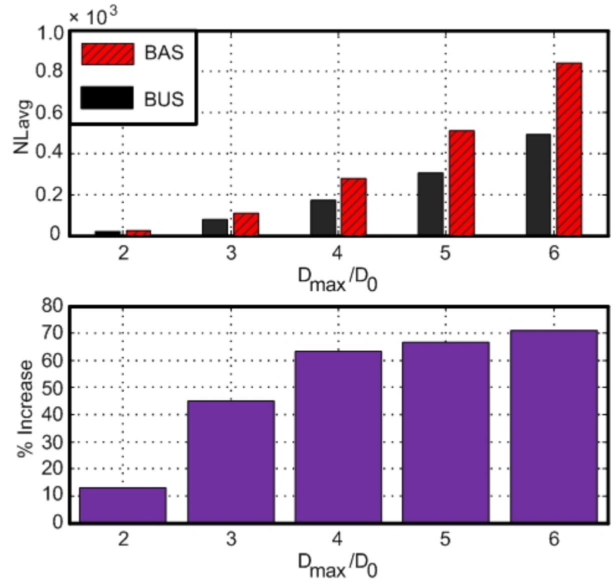

Fig. 3: Comparison of network lives for fixed channels.

V. Conclusions

In this contribution we have proposed a joint power allocation design for lifetime maximization of a single-hop wireless sensor network where each sensor is powered by a battery with limited capacity. Herein, we considered two cases: perfect knowledge of the instantaneous channel gains is available and when we only know the distribution of the channels. For the later case, we proposed a lower bound on the average distortion. Our proposed design takes into consideration the non-linearities of the battery discharging process. We have shown that the battery-aware power scheduling impacts the network lifetime to a substantial extent compared to a scheme which ignores the non-linear discharging behavior of the batteries in the power allocation scheme.

Appendix I
Proof of Theorem 1

In [6], Rakhmatov et al. obtained following expression for remaining concentration $C_k(t)$ of electroactive species at the electrode-electrolyte interface at time t by solving the electrochemical and diffusion equations with the associated initial and boundary conditions for a simple one-dimensional battery model based on the Fick's laws and Faraday's law:

$$C_{r_k}(x,t) = C_k^* - \frac{1}{vFAw}\left(\int_0^t i_k(\tau)\,d\tau + 2\int_0^t i_k(\tau)\sum_{m=1}^{\infty} e^{-\beta^2 m^2 (t-\tau)} d\tau\right), \quad (27)$$

where C_k^* is initial concentration, A is area of the electrode, F is Faraday's constant, w is width of the electrolyte region, v is number of electrons, $i_k(t)$ is the load current at time t for the battery of sensor k and β is a parameter characterizing nonlinear characteristics of the battery. Assume a step-wise constant load current corresponding to the sampling instances given in (4), i.e.

$$i_k(t) = I_k(1),\ I_k(2),\ \ldots,\ I_k(n),\ I_k(n+1),\ \ldots. \quad (28)$$

Then solving (27), we can show that the remaining charge at the end of the n^{th} sample is given by

$$Q_{r_k}(n) = Q_k^* - \sum_{j=1}^{n} I_k(j)T_s - 2\sum_{j=1}^{n} I_k(j) \sum_{m=1}^{\infty} \frac{1-e^{-\beta^2 m^2 T_s}}{\beta^2 m^2} e^{-\beta^2 m^2 (n-j)T_s}, \quad (29)$$

where $Q_k^* = vFAwC_k^*$. The battery is considered to be dead when $Q_{r_k}(n) < Q_{th}$ where Q_{th} is a threshold value.

Usually to provide a specific supply voltage V to the sensor-node electronics a DC-DC converter is used. Assume ϕ denotes the efficiency of the converter, then the current $I_k(n)$ in the observation sample n draws power $P_k(n)$ from the battery as follows:

$$P_k(n) = \phi V I_k(n), \quad (30)$$

substituting this in (29) and noting that the energy E can be written as $E = QV$, we get (6).

APPENDIX II
PROOF OF THEOREM 2

The instantaneous distortion D can be written as

$$D = \frac{1}{\sum_{i=}^{N} \frac{\zeta_i P_i}{\zeta_i P_i \sigma_{n_i}^2 + 1}} = \frac{1}{\sum_{i=}^{N} u_i} \quad (31)$$

and the average distortion is

$$D_{avg} = E_{\{u_1,\ldots,u_N\}}\left[\frac{1}{\sum_{i=}^{N} u_i}\right]. \quad (32)$$

Since D is a convex function of the random variables u_i's, therefore applying Jensen's inequality we get the following lower bound on the average distortion:

$$D_{avg} \geq \frac{1}{E_{\{u_1,\ldots,u_N\}}\left[\sum_{i=}^{N} u_i\right]} = \frac{1}{\sum_{i=}^{N} E\left[u_i\right]}. \quad (33)$$

Assuming that the channel gain $|h_i|$ is Rayleigh distributed with zero mean and variance $\sigma_{g_i}^2$, we can show that

$$E\left[u_i\right] = \frac{P_i \sigma_{n_i}^2 - \frac{1}{\bar{\zeta}_i} e^{\frac{1}{\sigma_{n_i}^2 \bar{\zeta}_i P_i}} Ei\left(1, \frac{1}{\sigma_{n_i}^2 \bar{\zeta}_i P_i}\right)}{P_i \sigma_{n_i}^4}, \quad (34)$$

where $Ei\left(1, \frac{1}{\sigma_{n_i}^2 \bar{\zeta}_i P_i}\right) = \int_1^{\infty} \frac{e^{\frac{-t}{\sigma_{n_i}^2 \bar{\zeta}_i P_i}}}{t} dt$ denotes exponential integral which can be bounded as

$$\frac{1}{2} e^{-x} \log\left(1 + \frac{2}{x}\right) < Ei(1,x) < e^{-x} \log\left(1 + \frac{1}{x}\right), \quad (35)$$

for $x > 0$ and $x = \frac{1}{\sigma_{n_i}^2 \bar{\zeta}_i P_i}$ [10]. From (33), (34) and (35), we get

$$D_{avg} \geq D_{avg}^{LB} = \frac{1}{\frac{1}{D_0} - \frac{1}{2}\sum_{i=1}^{N} \frac{\log\left(1+2\sigma_{n_i}^2 \bar{\zeta}_i P_i\right)}{\sigma_{n_i}^4 \bar{\zeta}_i P_i}}, \quad (36)$$

where $D_0 = \left(\sum_{i=1}^{N} \frac{1}{\sigma_{n_i}^2}\right)^{-1}$ and log denotes natural logarithm.

III. CONVEXITY OF OPTIMIZATION PROBLEM IN (24)

The objective function is affine and hence convex. Now for the problem to be convex, the inequality constraint

$$g(P_i) = \frac{1}{D_{max}^{avg}} - \frac{1}{D_0} + \frac{1}{2}\sum_{i=1}^{N} \frac{\log\left(1 + 2\sigma_{n_i}^2 \bar{\zeta}_i P_i\right)}{\sigma_{n_i}^4 \bar{\zeta}_i P_i} \leq 0 \quad (37)$$

must be convex. To this end, it suffices to show that all second order derivatives of $g(P_i)$ are non-negative, i.e. $\frac{\partial^2 g}{\partial P_i^2} \geq 0$ and $\frac{\partial^2 g}{\partial P_i \partial P_j} \geq 0$ for all i and j. Note that $\frac{\partial^2 g}{\partial P_i \partial P_j} = 0, \quad \forall i \wedge j$. Moreover,

$$\frac{\partial^2 g}{\partial P_i^2} = \frac{1}{\sigma_{n_i}^4 \bar{\zeta}_i P_i}\left[\frac{\log\left(1 + 2\sigma_{n_i}^2 \bar{\zeta}_i P_i\right)}{P_i} - \frac{2\sigma_{n_i}^2 \bar{\zeta}_i\left(1 + 3\sigma_{n_i}^2 \bar{\zeta}_i P_i\right)}{\left(1 + 2\sigma_{n_i}^2 \bar{\zeta}_i P_i\right)^2}\right] \quad (38)$$

$\forall i$, which is non-negative only if the term in square-brackets is non-negative, i.e.

$$\log\left(1 + 2x_i\right) \geq \frac{6x_i^2 + 2x_i}{4x_i^2 + 4x_i + 1}, \quad (39)$$

where $x_i = \sigma_{n_i}^2 \bar{\zeta}_i P_i$. Note that $\lim_{x_i \to \infty} \log\left(1 + 2x_i\right) = \infty$ and $\lim_{x_i \to \infty} \frac{6x_i^2 + 2x_i}{4x_i^2 + 4x_i + 1} = \frac{3}{2}$. Therefore condition in (39) holds as $x_i \to \infty$. Similarly, note that $\lim_{x_i \to 0} \log\left(1 + 2x_i\right) = 0$ and $\lim_{x_i \to 0} \frac{6x_i^2 + 2x_i}{4x_i^2 + 4x_i + 1} = 0$, which shows that (39) holds for $x_i \to 0$ as well. The given condition also holds for any $0 \leq x_i \leq \infty$. Therefore the given optimization problem is convex.

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