Quantization and Power Allocation in Wireless Sensor Networks with Correlated Data

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***Abstract*— This work addresses the problem of joint quantization and power allocation in wireless sensor networks where sensors observe a source, quantize their observations and transmit to a fusion center (FC) which reconstructs the source using linear minimum mean-squared error (LMMSE) estimation rule. The sensors employ scalar quantizers to quantize the observations. We formulate the reconstruction distortion without imposing any statistical structure on the quantization noise and without making any simplifying assumption about the contribution of the channel errors to the reconstruction distortion. Based on the formulation, we outline a solution to the problem of joint quantization and power allocation based on minimization of the distortion subject to a constraint on the network transmit power. We illustrate the effectiveness of the proposed solution with some numerical examples.**

I. Introduction

Wireless sensor networks consist of spatially distributed sensors that cooperatively monitor physical or environmental conditions. The networks are characterized by limited energy, bandwidth and computational complexity. In this work, our objective is to reconstruct the underlying source subject to resource constraints so that the overall distortion (e.g. mean squared error) be minimized.

We consider a system where individual sensors quantize and transmit their noisy observations of a common source, via some orthogonal multiple access scheme, e.g. TDMA or FDMA, to a remote fusion center (FC). The FC reconstructs the underlying source based on a linear minimum mean squared error (LMMSE) estimation rule. The sensors have partial and correlated observations of the source. The correlation exists where sensors measure data in the same geographical location, e.g. acoustic sensors that are sensing a common event produce measurements that are correlated. In addition, observation noise and communication channel may not have same conditions across all sensors. Therefore, independent quantization and transmission of the observations is not an optimal strategy.

A number of quantization and power allocation schemes has been proposed over the years to estimate a source in sensor networks, see [1]- [5] and references therein. In these works an unknown parameter is estimated by a set of distributed sensors nodes using BLUE or MLE estimation rules based on the quantized sensors observations. Therein, to model the estimation distortion, the authors assume some kind of statistical structure about the quantization noise and the contribution of the channel errors to the estimation distortion. Moreover, they do not exploit the spatial correlation and in some cases assume ideal communication channels or the homogenous sensor noise. To this end, in this work, we present a joint design of quantization and power allocation in the sensor network which takes into account the spatial correlation and cross-correlations of the observations, the observation quality and the communication channels to the FC. The joint quantization and power allocation scheme is based on minimization of the reconstruction distortion subject to a constraint on the network transmit power. The sensors use scalar quantizers to quantize their observations. The distortion is formulated based on LMMSE estimation rule wherein we do not impose any statistical structure on the quantization noise. Moreover, this formulation does not make any simplifying assumption about the contribution of the channel errors to the total distortion.

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II. System Model

Consider the system model shown in Fig. 1 in which N spatially distributed sensors observe an unknown zero-mean real Gaussian random source $s \sim \mathcal{N}(0, \sigma_s^2)$, and communicate with the fusion center (FC) via orthogonal multiple access channels. Each sensor has a partial and noisy observation of the source, and sends a quantized version of it to the FC. The FC collects the signals from all sensors and reconstructs the source. The $s_i \sim \mathcal{N}(0, \sigma_{s_i}^2)$ and $n_i \sim \mathcal{N}(0, \sigma_{n_i}^2)$ respectively denotes the partial observation of the source s and the noise corrupting this observation such that the noisy observation at sensor i is

$$x_i = s_i + n_i, \; i = 1, \dots, N, \tag{1}$$

where n_i is independent across sensors and is also independent of s and $\{s_i\}_{i=1}^N$.

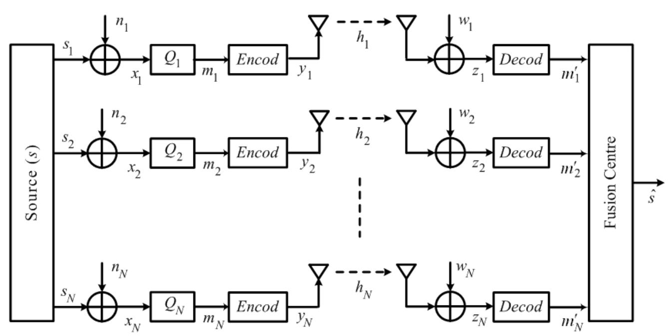

Fig. 1: Block diagram of the system.

In order to keep the exposition tractable, we assume that the sensors employ scalar quantization scheme to quantize their observations and the encoding of the quantization indices does not consider entropy coding, that is fixed length coding is used. At sensor i, the quantization function $\mathcal{Q}_i$ can be viewed as a mapping which maps x_i to one of the finite set of rational numbers $\{m_{1_i}, \dots, m_{M_i}\}$ as follows:

$$\begin{aligned} &\mathcal{Q}_i : x_i \rightarrow m_i \\ &m_i = m_{k_i}, \text{ for } u_{k_i} < x_i \leq u_{k_i+1}, k_i = 1, \dots, M_i, \end{aligned} \tag{2}$$

where u_{k_i}'s are quantization interval boundaries and m_i's are quantization values (also called representation or reconstruction values). At sensor i, the index k_i corresponding to the quantized value m_{k_i} is encoded according to some labeling rule, e.g. natural binary code, and then the resulting bits are transmitted to the FC using a digital modulation scheme, e.g. BPSK, PAM, QAM. Without considering entropy coding, we require $L_i = \log_2 M_i$ bits to encode the M_i quantization indices. The *Encod* block, in Fig. 1, performs the functions of encoding the quantization indices and the modulation of the resulting bits. The *Decod* block at the FC performs converse functions (demodulation and mapping of the received bits to the quantized values) of the corresponding *Encod* block. In this work we do not consider channel coding.

The sensors transmit the quantization indices to the FC via orthogonal channels where each channel experiences flat fading independent over time and across sensors. The fading channels $\{h_i\}_{i=1}^N$ between sensors and the FC are $h_i \sim \mathcal{CN}(0, \sigma_{h_i}^2), \forall i$ with gain factors $\{g_i = |h_i|\}_{i=1}^N$ which are Rayleigh distributed. We assume that the channels $\{h_i\}_{i=1}^N$ are perfectly known at FC and do not change during the estimation of each observation sample. The $w_i \sim \mathcal{CN}(0, 2\sigma_{w_i}^2)$ denotes the receiver noise which is independent across the sensors and is also independent of s, $\{s_i\}_{i=1}^N$ and $\{n_i\}_{i=1}^N$.

We assume that the source s, the observation s_i at sensor i and the observation s_j at sensor j are jointly Gaussian distributed having zero mean and covariances $\mathrm{Cov}\{s, s_i\} = \sigma_s \sigma_{s_i} \rho_{si}$, $\mathrm{Cov}\{s, s_j\} = \sigma_s \sigma_{s_j} \rho_{sj}$ and $\mathrm{Cov}\{s_i, s_j\} = \sigma_{s_i} \sigma_{s_j} \rho_{ij}$, $\forall i$ and $\forall j$. Note that ρ_{si} specifies correlation between s and s_i, and ρ_{ij} specifies correlation between s_i and s_j. We use the power exponential model to specify these correlation coefficients, see [6] and reference therein. Moreover, we assume that the samples of s, s_i, n_i and w_i are independent in time.

Assuming that the sensor observations $\{x_i\}_{i=1}^N$ are available at the FC then the optimal estimator in the mean-squared error sense is the conditional mean of s given $\{x_i\}_{i=1}^N$, that is $\hat{s}_0 = \mathbb{E}[s|x_i, \forall i]$, where $\mathbb{E}$ denotes the mathematical expectation operator. Under the jointly Gaussian assumption of s and $\{x_i\}_{i=1}^N$, the conditional mean estimator turns out to be linear and is called linear minimum mean-squared error estimator (LMMSEE) which can be written as $\hat{s}_0 = \mathbf{c}^T(\mathbf{C_s} + \mathbf{C_n})^{-1}\mathbf{x}$ with the associated MSE distortion

$$D_0 = \sigma_s^2 - \mathbf{c}^T(\mathbf{C_s} + \mathbf{C_n})^{-1}\mathbf{c}, \tag{3}$$

where $\mathbf{c} = \mathbb{E}[\mathbf{x}s]$, $\mathbf{C_s} = \mathbb{E}[\mathbf{s}\mathbf{s}^T]$ and $\mathbf{C_n} = \mathbb{E}[\mathbf{n}\mathbf{n}^T]$ with $\mathbf{x} = [x_1, \ldots, x_N]^T$, $\mathbf{s} = [s_1, \ldots, s_N]^T$ and $\mathbf{n} = [n_1, \ldots, n_N]^T$ [7]. The estimator which achieves the distortion D_0 is called a clairvoyant estimator and is used as a performance benchmark. Note that the distortion achieved by any estimator (designed to minimize the mean-squared estimation error) based on the quantized sensor observations is lower bounded by D_0.

In the sequel, firstly we formulate the joint quantization and power allocation problem based on the optimum scalar quantization (optimum in LMMSE sense) and subsequently we outline a solution to the joint quantization and power allocation problem and show its effectiveness with some numerical examples.

III. Joint Quantization and Power Allocation

The FC employs the LMMSE estimation rule to form the estimate $\hat{s}_{ouq}$ of the source s based on the received messages $\{m_i'\}_{i=1}^N$ from the sensors, that is

$$\hat{s}_{ouq} = \sum_{i=1}^N v_i m_i' \tag{4}$$

where $\{v_i\}_{i=1}^N$ are the LMMSE weighting coefficients. The corresponding estimation distortion measured as mean-squared error between the estimate and the source can be written as

$$\begin{aligned} D_{ouq} &= \mathbb{E}_{\{s, s_i, n_i, w_i | h_i, \forall i\}}\left[(s - \hat{s}_{ouq})^2\right], \\ &= \sigma_s^2 - 2\sum_{i=1}^N v_i a_i + \sum_{i=1}^N v_i^2 b_i + \sum_{i=1}^N \sum_{j \neq i}^N v_i' v_j' c_{ij}, \end{aligned} \tag{5}$$

with a_i, b_i and c_{ij} defined as follows:

$$\begin{aligned} a_i &= \sum_{l_i=1}^{M_i} \sum_{k_i=1}^{M_i} m_{l_i}' p\left(m_{l_i}' | m_{k_i}\right) \int_s \int_{s_i} s f_{s,s_i}(s, s_i) \\ &\qquad p\left(m_{k_i} | s_i\right) ds ds_i, \\ b_i &= \sum_{l_i=1}^{M_i} \sum_{k_i=1}^{M_i} (m_{l_i}')^2 p\left(m_{l_i}' | m_{k_i}\right) \int_{s_i} f_{s_i}(s_i) p\left(m_{k_i} | s_i\right) ds_i, \\ c_{ij} &= \sum_{l_i=1}^{M_i} \sum_{k_i=1}^{M_i} \sum_{\nu_j=1}^{M_j} \sum_{\kappa_j=1}^{M_j} m_{l_i}' m_{\nu_j}' p\left(m_{l_i}' | m_{k_i}\right) p\left(m_{\nu_j}' | m_{\kappa_j}\right) \\ &\qquad \int_{s_i} \int_{s_j} f_{s_i, s_j}(s_i, s_j) p\left(m_{k_i} | s_i\right) p\left(m_{\kappa_j} | s_j\right) ds_i ds_j, \end{aligned} \tag{6}$$

where $f_{s_i}(s_i)$, $f_{s,s_i}(s, s_i)$ and $f_{s_i,s_j}(s_i, s_j)$ are probability density functions, $p\left(m_{k_i} | s_i\right)$ denotes the probability of quantizing to m_{k_i} for given s_i and $p\left(m_{l_i}' | m_{k_i}\right)$ is channel transition probability - the probability of receiving m_{l_i}' when m_{k_i} is transmitted. The quantization probability $p\left(m_{k_i} | s_i\right)$ is given by

$$p\left(m_{k_i} | s_i\right) = \frac{1}{\sqrt{2\pi\sigma_{n_i}^2}} \int_{u_{k_i}}^{u_{k_i+1}} e^{\frac{-(x_i - s_i)^2}{2\sigma_{n_i}^2}} dx_i, \tag{7}$$

for $i = 1, \ldots, N$ and $k_i = 1, \ldots, M_i$. Moreover, we can show that

$$\begin{aligned} &\int_s \int_{s_i} s f_{s,s_i}(s, s_i) p(m_{k_i} | s_i) ds ds_i = \phi_i \left(e^{-\frac{u_{k_i}^2}{2\sigma_i^2}} - e^{-\frac{u_{k_i+1}^2}{2\sigma_i^2}} \right), \\ &\int_{s_i} f_{s_i}(s_i) p(m_{k_i} | s_i) ds_i = \frac{1}{2}\left[\mathrm{erf}\left(\frac{u_{k_i+1}}{\sqrt{2}\sigma_i}\right) - \mathrm{erf}\left(\frac{u_{k_i}}{\sqrt{2}\sigma_i}\right)\right], \\ &\int_{s_i} \int_{s_j} f_{s_i, s_i}(s_i, s_j) p(m_{k_i} | s_i) p(m_{\kappa_j} | s_j) ds_i ds_j = \frac{0.5}{\sqrt{2\pi}} \int_{\frac{u_{\kappa_j}}{\sigma_j}}^{\frac{u_{\kappa_j+1}}{\sigma_j}} \\ &\left[\mathrm{erf}\left(\delta_{ij}\bar{x}_j - \eta_{ij} u_{k_i}\right) - \mathrm{erf}\left(\delta_{ij}\bar{x}_j - \eta_{ij} u_{k_i+1}\right)\right] e^{\frac{-\bar{x}_j^2}{2}} d\bar{x}_j, \end{aligned} \tag{8}$$

where $\sigma_i^2 = \sigma_{s_i}^2 + \sigma_{n_i}^2$, $\phi_i = \frac{\sigma_s \sigma_{s_i} \rho_{si}}{\sqrt{2\pi}\sigma_i}$, $\bar{x}_j = \frac{x_j}{\sigma_j}$, $\delta_{ij} = \frac{\sigma_{s_i}\sigma_{s_j}\rho_{ij}}{\psi_{ij}}$, $\eta_{ij} = \frac{\sigma_j}{\psi_{ij}}$ and $\psi_{ij} = \sqrt{2\left(\sigma_{s_i}^2\sigma_{n_j}^2 + \sigma_{n_i}^2\sigma_j^2 + \sigma_{s_i}^2\sigma_{s_j}^2\left(1 - \rho_{ij}^2\right)\right)}$.

The channel transition probabilities $p\left(m_{l_i}' | m_{k_i}\right)$ depend on the binary coding scheme used to encode the indices corresponding to the quantization values m_{k_i}, the modulation type used to transmitted the encoded bits, channel gain g_i and the statistics of the receiver noise w_i. For ideal communication channels from the sensors to the FC, the channel transition probabilities become $p\left(m_{l_i}' | m_{k_i}\right) = \delta_{l_i k_i}$ for $l_i, k_i = 1, \ldots, M_i$ and $i = 1, \ldots, N$, where $\delta_{l_i k_i}$ is equal to one for $l_i = k_i$ and zero otherwise. Therefore, for ideal channels, the coefficients a_i, b_i and c_{ij}, defined in

(6), reduce to

$$a_i = \sum_{k_i=1}^{M_i} m_{k_i} \int_s \int_{s_i} s f_{s,s_i}(s, s_i) p\left(m_{k_i}|s_i\right) ds ds_i,$$

$$b_i = \sum_{k_i=1}^{M_i} (m_{k_i})^2 \int_{s_i} f_{s_i}(s_i) p\left(m_{k_i}|s_i\right) ds_i,$$

$$c_{ij} = \sum_{k_i=1}^{M_i} \sum_{\kappa_j=1}^{M_j} m_{k_i} m_{\kappa_j} \int_{s_i} \int_{s_j} f_{s_i,s_j}(s_i, s_j) p\left(m_{k_i}|s_i\right) p\left(m_{\kappa_j}|s_j\right) ds_i ds_j. \quad (9)$$

In the ensuing development, we focus on the case of non-ideal channels. Nevertheless, the formulations also apply to the case of ideal-channels. To determine v_i, take derivative of D_{ouq} with respect to v_i and set it equal to zero, that is $\frac{\partial D_{ouq}}{\partial v_i} = 0$, which gives

$$v_i = \frac{a_i - \sum_{i \neq j}^{N} v_j c_{ij}}{b_i}, \; i = 1, \ldots, N. \quad (10)$$

Substituting (10) in (5) we can write

$$D_{ouq} = \sigma_s^2 - \sum_{i=1}^{N} v_i a_i. \quad (11)$$

Using matrix-vector notation, (10) and (11) can be written in a more compact form as follows:

$$\mathbf{v} = \mathbf{U}^{-1}\mathbf{a}, \quad (12)$$

$$D_{ouq} = \sigma_s^2 - \mathbf{a}^T \left[\mathbf{U}^{-1}\right]^T \mathbf{a} = \sigma_s^2 - \mathbf{a}^T \mathbf{U}^{-1}\mathbf{a}, \quad (13)$$

where $\mathbf{v} = [v_1, \ldots, v_N]^T$, $\mathbf{a} = [a_1, \ldots, a_N]^T$, $\mathbf{U} = \mathbf{C} \circ \mathbf{B}$ with $\mathbf{C} \circ \mathbf{B}$ denoting the Hadamard or Schur product of $\mathbf{C}$ and $\mathbf{B}$, $[\mathbf{B}]_{ij} = 1$ for $i \neq j$, $[\mathbf{B}]_{ij} = b_i$ for $i = j$, $[\mathbf{C}]_{ij} = c_{ij}$ for $i \neq j$ and $[\mathbf{C}]_{ij} = 1$ for $i = j$.

The formulation in (4)-(13) is entirely general for the scalar quantization scheme which covers both non-uniform and uniform scalar quantization processes. For given M_i the non-uniform quantization function $\mathcal{Q}_i$ is fully specified by the quantization boundaries $\left\{u_{k_i}\right\}_{k_i=1}^{M_i+1}$ and the quantization values $\left\{m_{k_i}\right\}_{k_i=1}^{M_i}$. However, the uniform quantization function $\mathcal{Q}_i$ is completely specified by the quantization interval Δ_i. In this case, the quantization boundaries can be written in terms of Δ_i as follows:

$$u_{k_i} = (2k_i - 2 - M_i)\frac{\Delta_i}{2}, \quad (14)$$

for $k_i = 2, \ldots, M_i$ with u_{1_i} and u_{M_i+1} respectively denoting the greatest lower-bound and the lowest upper-bound on x_i. The corresponding quantization values are given by

$$m_{k_i} = (2k_i - 1 - M_i)\frac{\Delta_i}{2}, \quad (15)$$

for $k_i = 1, \ldots, M_i$. Inspired by the simplicity of the uniform quantization, henceforth we consider this scheme. Nevertheless, even for this supposedly simple yet fundamental quantization scheme we will see that the problem of joint quantization and power allocation is still quite challenging to solve.

Since we do not assume any entropy coding, therefore, to encode M_i indices corresponding to M_i quantization levels, we need $L_i = \log_2 M_i$ code bits. For joint quantization and power allocation, we consider the following optimization problem:

$$\begin{aligned} \min_{L_i, \Delta_i, P_i, \forall i} \quad & D_{ouq} \\ \textit{s.t.} \quad & \sum_{i=1}^{N} P_i \leq P_{tot}, \\ & L_i \in \mathbb{Z}_+, \; \Delta_i, P_i \in \mathbb{R}_+, \; \forall i. \end{aligned} \quad (16)$$

The optimization problem is a nonlinear nonconvex mixed integer programming problem which is hard to solve. However, for given $\{L_i\}_{i=1}^N$ we can derive an iterative procedure based on the Lagrangian multipliers method to solve the problem for $\{\Delta_i\}_{i=1}^N$ and $\{P_i\}_{i=1}^N$ [8]. To this purpose, the associated Lagrangian function can be written as follows:

$$f(\Delta_i, P_i, \lambda, \xi_i, \eta_i) = D_{ouq} + \lambda\left(\sum_{i=1}^{N} P_i - P_{tot}\right) - \sum_{i=1}^{N} \left(\xi_i \Delta_i + \eta_i P_i\right), \quad (17)$$

where λ, $\{\xi_i\}_{i=1}^N$ and $\{\eta_i\}_{i=1}^N$ are Lagrangian multipliers. The corresponding Karush-Kuhn-Tucker (KKT) optimality conditions can be written as

$$\frac{\partial f}{\partial \Delta_i} = -\mathbf{a}^T \mathbf{U}^{-1} \mathring{\mathbf{a}}_i + \mathbf{a}^T \mathbf{U}^{-1} \mathring{\mathbf{U}}_i \mathbf{U}^{-1}\mathbf{a} - \mathring{\mathbf{a}}_i^T \mathbf{U}^{-1}\mathbf{a} - \xi_i = 0, \quad (18)$$

$$\frac{\partial f}{\partial P_i} = -\mathbf{a}^T \mathbf{U}^{-1} \breve{\mathbf{a}}_i + \mathbf{a}^T \mathbf{U}^{-1} \breve{\mathbf{U}}_i \mathbf{U}^{-1}\mathbf{a} - \breve{\mathbf{a}}_i^T \mathbf{U}^{-1}\mathbf{a} + \lambda - \eta_i = 0, \quad (19)$$

$$\xi_i \Delta_i = 0, \; \xi_i \geq 0, \; \Delta_i \geq 0, \quad (20)$$

$$\lambda\left(\sum_{i=1}^{N} P_i - P_{tot}\right) = 0, \; \lambda \geq 0, \; \sum_{i=1}^{N} P_i \leq P_{tot}, \quad (21)$$

$$\eta_i P_i = 0, \; \eta_i \geq 0, \; P_i \geq 0. \quad (22)$$

In (18), $\mathring{\mathbf{a}}_i$ is a column vector with i^{th} element as $\frac{\partial a_i}{\partial \Delta_i}$ and all other elements equal to zero, and $\mathring{\mathbf{U}}_i$ is a matrix with the following properties:

$$\left[\mathring{\mathbf{U}}_i\right]_{jk} = \begin{cases} 0 & \text{for } j \neq i \text{ and } k \neq i, \\ \frac{\partial [\mathbf{U}]_{jk}}{\partial \Delta_i} = \frac{\partial [\mathbf{C}]_{jk}}{\partial \Delta_i} & \text{for } j \text{ or } k = i \text{ and } j \neq k, \\ \frac{\partial [\mathbf{U}]_{jk}}{\partial \Delta_i} = \frac{\partial [\mathbf{B}]_{jk}}{\partial \Delta_i} & \text{for } j = k = i. \end{cases} \quad (23)$$

Similarly in (19), $\breve{\mathbf{a}}_i$ is a column vector with i^{th} element as $\frac{\partial a_i}{\partial P_i}$ and all other elements equal to zero, and $\breve{\mathbf{U}}_i$ is a matrix with the following properties:

$$\left[\breve{\mathbf{U}}_i\right]_{jk} = \begin{cases} 0 & \text{for } j \neq i \text{ and } k \neq i, \\ \frac{\partial [\mathbf{U}]_{jk}}{\partial P_i} = \frac{\partial [\mathbf{C}]_{jk}}{\partial P_i} & \text{for } j \text{ or } k = i \text{ and } j \neq k, \\ \frac{\partial [\mathbf{U}]_{jk}}{\partial P_i} = \frac{\partial [\mathbf{B}]_{jk}}{\partial P_i} & \text{for } j = k = i. \end{cases} \quad (24)$$

Eq. (18) through (24) show that a closed form analytical solution for $\{\Delta_i\}_{i=1}^N$ and $\{P_i\}_{i=1}^N$ is not possible. Therefore, we may resort to numerical methods to solve this system of equations iteratively. To this purpose, an iterative procedure can be used wherein we solve for $\{\Delta_i\}_{i=1}^N$ for given $\{P_i\}_{i=1}^N$ and vice versa until there is no appreciable change in the distortion.

IV. Numerical Examples and Discussion

The foregoing optimization problem is applicable to any kind of binary coding scheme used to encode the quantization indices and any digital modulation scheme used to transmit the quantization bits. However, for the sake of illustration and simplicity, we focus on the natural binary code and the BPSK modulation scheme. Moreover, we assume that the power P_i of sensor i is equally divided among its quantization bits L_i. Consequently the bit-error probability is given by $\epsilon_i = \frac{1}{2}\left(1 - \text{erf}\left(\sqrt{g_i^2 P_i / 2L_i\sigma_{w_i}^2}\right)\right)$. In this particular case, we can easily compute the channel transition probabilities $p\left(m'_{il}|m_{ik}\right)$ from ϵ_i.

For $N = 3$ we assume $\sigma_s^2 = \sigma_{s_i}^2 = \sigma_{w_i}^2 = g_i = 1$, $\sigma_{n_i}^2 = 0.01\ \forall i$, $(\rho_{s1}, \rho_{s2}, \rho_{s3}) = (0.9048, 0.0067, 0.2231)$ and $(\rho_{12}, \rho_{13}, \rho_{23}) = (0.0067, 0.2019, 0.0054)$. Fig. 2 shows the achieved MSE distortion versus P_{tot} for $(L_1, L_2, L_3) = 1$, $(L_1, L_2, L_3) = (2, 1, 1)$, $(L_1, L_2, L_3) = (3, 1, 2)$, $(L_1, L_2, L_3) = (4, 2, 4)$ and $(L_1, L_2, L_3) = 6$. Moreover, Fig. 3 and Fig. 4 respectively plots the power allocation among the sensors $\{P_i\}_{i=1}^N$ and the variations of the quantization step-sizes $\{\Delta_i\}_{i=1}^N$ with P_{tot} for $(L_1, L_2, L_3) = 1$ and $(L_1, L_2, L_3) = (3, 1, 2)$. Note that in the figures $\log(.) = \log_{10}(.)$. From Fig. 2 we can see that, to minimize the distortion, it is better to quantize with less number of bits at low power and vice-versa. The figure also shows that at high power, with increasing the quantization bits, the achieved distortion approaches the lower bound distortion D_0. For given quantization bits, Fig. 3 and Fig. 4 show that, compared to other sensors, the sensor with better correlation properties quantizes with small quantization step-size and transmits with more power. Moreover, at sufficiently large power P_{tot}, the step-size of each sensor becomes constant and the power is equally divided among the sensors. The figures also show that for each sensor the quantization step-size decreases with increasing the number of quantization-bits.

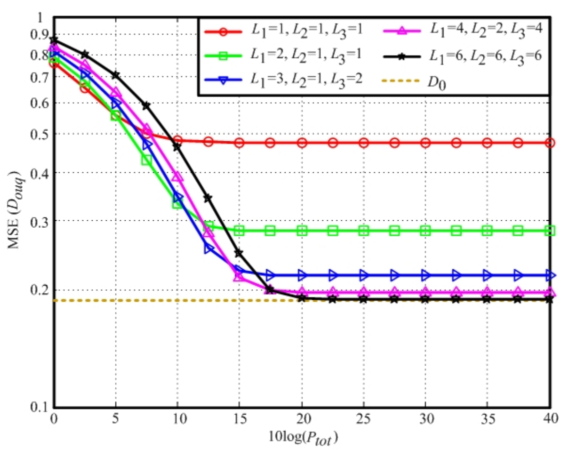

Fig. 2: Achieved reconstruction distortion.

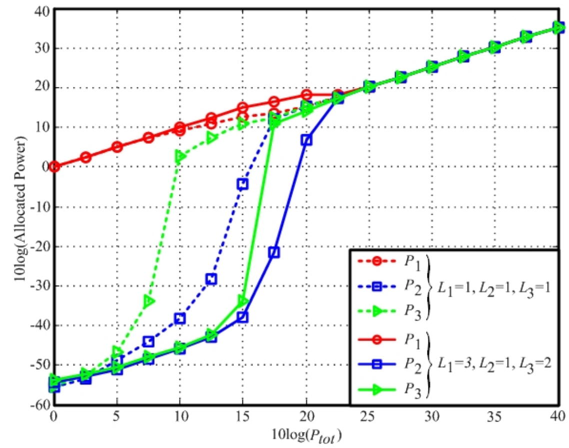

Fig. 3: Allotted power.

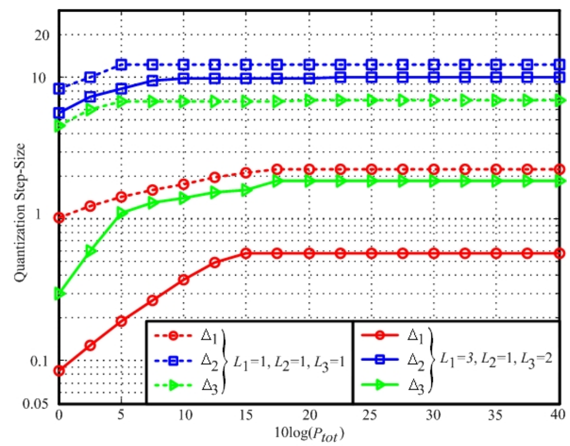

Fig. 4: Quantization step-size.

V. Conclusions

In this work, we have proposed a design to jointly quantize the sensor observations and allocate power to transmit the observations to the FC with the goal to reconstruct the source with minimum distortion. The design incorporates the spatial correlation, the observation noises and the channels quality. The design does not impose any statistical structure on the quantization noises. Moreover, it does not make any simplifying assumption about the contribution of the channel bit-errors to the distortion. In the numerical examples we have seen that to minimize the distortion sensors having better correlation properties compared to other sensors quantize their observation with finer resolution and transmit at higher power. Moreover, at sufficiently high power all sensors transmit with equal power and for given quantization bits, the quantization step-size becomes invariant with the power.

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